



UPSC – CSAT

Civil Services Exam

Union Public Service Commission

Part - 2

Numerical Ability



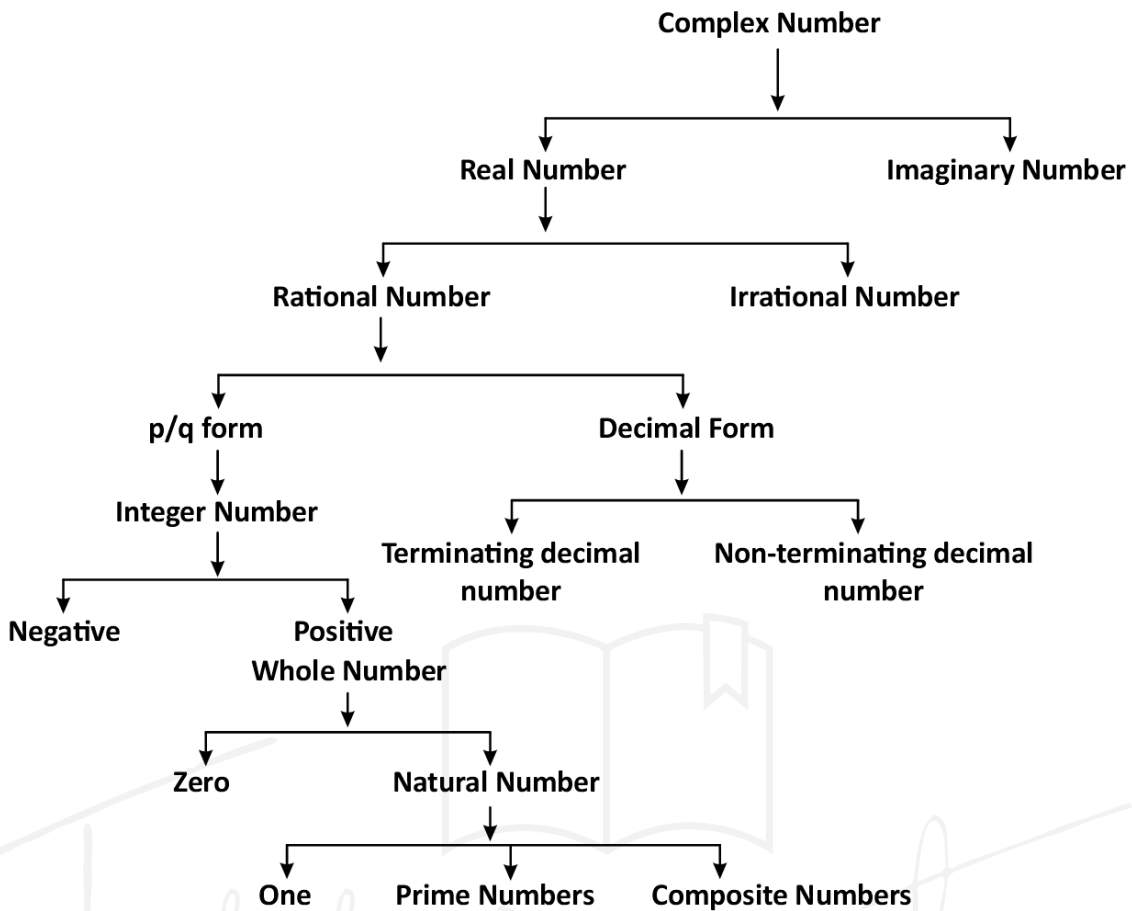
UPSC – CSAT (ENGLISH)

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Number System



Complex Number (Z)

Z = Real numbers + Imaginary numbers

$$Z = a + ib$$

Where, a = Real numbers.

b = Imaginary numbers.

Real Numbers

Rational and irrational numbers together are called real numbers. These can be represented on the number line.

Imaginary Numbers

Numbers that can not be represented on the number line.

Integer Numbers

A set of numbers which includes whole numbers as well as negative numbers, is called integer numbers, it is denoted by I.

$$I = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

Natural Numbers

The numbers which are used to count things are called natural numbers.

$$N = \{1, 2, 3, 4, 5, \dots\}$$

Whole Numbers

When 0 is also included in the family of natural numbers, then they are called whole numbers.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

The product of four consecutive natural numbers is always exactly divisible by 24.

Even Numbers

Numbers which are completely divisible by 2 are called even numbers.

$$n^{\text{th}} \text{ term} = 2n$$

$$\text{Sum of first } n \text{ even natural numbers} = n(n+1)$$

$$\text{Sum of square of first } n \text{ even natural numbers} = \frac{2n(n+1)(2n+1)}{3}$$

$$\left\{ n = \frac{\text{Last term}}{2} \right\}$$

Odd Numbers

The numbers which are not divisible by 2 are odd numbers.

$$\text{Sum of first } n \text{ odd numbers} = n^2$$

$$\left\{ n = \frac{\text{Last term} + 1}{2} \right\}$$

Natural Numbers

$$\text{Sum of first } n \text{ natural numbers} = \frac{n(n+1)}{2}$$

$$\text{Sum of square of first } n \text{ natural numbers} = \frac{n(n+1)(2n+1)}{6}$$

$$\text{Sum of cube of first } n \text{ natural numbers} = \left[\frac{n(n+1)}{2} \right]^2$$

The difference of the squares of two consecutive natural numbers is equal to their sum.

Example - $11^2 = 121$

$$12^2 = 144$$

$$11 + 12 \rightarrow 23 \quad \text{Difference } 144 - 121 = 23$$

Prime Numbers – Which have only two forms - $1 \times$ numbers

E.g. - $\{2, 3, 5, 7, 11, 13, 17, 19, \dots\}$

Where, 1 isn't a Prime Number.

- The digit 2 is only even prime number.
- 3, 5, 7 is the only pair of consecutive odd prime numbers.
- Total prime numbers between 1 to 25 = 9
- Total prime numbers between 25 to 50 = 6
- There are total of 15 prime numbers between 1-50.
- There are total of 10 prime numbers between 51 – 100.
So there are total 25 prime numbers from 1-100.
- Total prime numbers from 1 to 200 = 46
- Total prime numbers from 1 to 300 = 62
- Total prime numbers from 1 to 400 = 78
- Total prime numbers from 1 to 500 = 95

Co-prime Numbers

Numbers whose HCF is only 1.

E.g. - (4,9), (15, 22), (39, 40)

HCF = 1

Perfect Number

A number whose sum of its factors is equal to that number (except the number itself in the factors)

E.g. - 6 → 1, 2, 3 → Here 1 + 2 + 3 → 6

28 → 1, 2, 4, 7, 14 → 1 + 2 + 4 + 7 + 14 → 28

Rational Numbers

Numbers that can be written in the form of P/Q, but where Q must not be zero and P and Q must be integers.

E.g. - $\frac{2}{3}$, $\frac{4}{5}$, $\frac{10}{-11}$, $\frac{7}{8}$

Irrational Numbers

These cannot be displayed in P/Q form.

E.g. - $\sqrt{2}$, $\sqrt{3}$, $\sqrt{11}$, $\sqrt{19}$, $\sqrt{26}$...

Perfect square numbers

↓

Unit Digit which can be of square

0

1

4

5 or 25

6

9

Which can't be square

2 —

3 —

7 —

8 —

- The last two digits of the square of any number will be the same as the last two digits of the square of numbers 1-24.

Note: Therefore, everyone must remember the squares of 1-25.

Convert to Binary and Decimal –

1. Convert Decimal Number to Binary Number

To find the binary number equivalent to a decimal number, we continuously divide the given decimal number by 2 until we get 1 as the final quotient.

E.g.

2	89	$2 \times 44 = 88 ; 89 - 88 = 1$
2	44	$2 \times 22 = 44 ; 44 - 44 = 0$
2	22	$2 \times 11 = 22 ; 22 - 22 = 0$
2	11	$2 \times 5 = 10 ; 11 - 10 = 1$
2	5	$2 \times 2 = 4 ; 5 - 4 = 1$
2	2	$2 \times 1 = 2 ; 2 - 2 = 0$
	1	Final quotient

Hence, binary number equivalent to 89 = $(1011001)_2$

2. Convert Binary to Decimal Number

In binary system the value of 1 when it moves one place to its left every time it doubles itself and wherever 0 comes its value is 0.

E.g.

1	0	1	1	0	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Now

$$\begin{aligned}
 (1011001)_2 &= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
 &= 64 + 0 + 16 + 8 + 8 + 0 + 1 \quad \{2^0 = 1\} \\
 &= 89
 \end{aligned}$$

Finding the Number of Divisors or Number of Factors

First we will do the prime factorization of the number and write it as Power and multiply by adding

One to each power, then the number of divisors will be obtained.

E.g. By how many total numbers can 2280 be completely divided?

Sol. $2280 = 2^3 \times 3^1 \times 5^1 \times 19^1$

$$\begin{aligned}
 \text{Number of divisors} &= (3 + 1) (1 + 1) (1 + 1) (1 + 1) \\
 &= 4 \times 2 \times 2 \times 2 = 32
 \end{aligned}$$

Find the unit's digit

1. When the number is in the form of power –

When the unit digit of Base is 0, 1, 5 or 6, the unit digit of the result remains the same for any natural power. When the unit digit of base is 2, 3, 4, 7, 8, or 9, divide the power by 4 and put the same power on the unit digit of the base as the remainder. When the power is rounded off to 4, then the 4th power will be placed on the unit digit of the base.

2. In the form of simplification –

Write the unit digit of each number and simplify it according to the symbol, the result that will come will be its unit digit answer.

Divide by Power of Numbers (Finding the Divisor)

1. If $a^n + b^n$ is given –

If n is odd, then $(a+b)$ will be its divisor.

2. If $a^n - b^n$ is given –

Divisor (when n is odd) $\rightarrow (a-b)$

Divisor (when n is even) $\rightarrow (a - b)$ or $(a + b)$ or both.

1. If $a^n \div (a - 1)$ then the remainder always be 1.
2. $a^n \div (a + 1)$
 - {
 - If n is an even then the remainder always be 1.
 - If n is an odd then the remainder always be a .
3. If $(a^n + a) \div (a - 1)$ then the remainder always be 2 .
4. $(a^n + a) \div (a + 1)$
 - {
 - If n is an even then the remainder always be zero (0).
 - If n is an odd then the remainder always be $(a - 1)$

Terminating Decimal

Those numbers which end after a few digits after the decimal like - 0.25, 0.15, 0.375 can be written in a fraction number.

Non-Terminating Decimal

Those numbers which continue after the decimal and can be of two types.

0.3333, 0.7777, 0.183183183.....

Repeating	Numbers that never end after the decimal, but repeat, till infinity. It can be written in fractions.
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Non Repeating Decimal	Numbers that never end after the decimal point, but they do not repeat their numbers.
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Recurring Decimal Fraction

That decimal fraction is the repetition of one or more digits after the decimal point, then one or more digits are repeated after the dot.

Eg. $\frac{1}{3} = 0.333\dots$, $\frac{22}{7} = 3.14285714\dots$. To represent such fractions, a line is drawn over the repeating digit.

$$0.\overline{3524} = \frac{3524 - 35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$$

$$\frac{22}{7} = 3.14285714\dots = 3.\overline{142857}$$

It is called bar.

- Convert pure recurring decimal fraction to simple fraction as follows –

$$0.\overline{P} = \frac{P}{9}$$

$$0.\overline{pq} = \frac{pq}{99}$$

$$0.\overline{pqr} = \frac{pqr}{999}$$

- Convert a mixed recurring decimal fraction to an ordinary fraction as follows –

$$0.p\overline{q} = \frac{pq - p}{90}$$

$$0.pq\overline{r} = \frac{pqr - pq}{900}$$

$$0.\overline{pqr} = \frac{pqr - pq}{990}$$

$$0.pqrs\overline{r} = \frac{pqrs - pq}{9900}$$

Example - (i) $0.\overline{39} = \frac{39}{99} = \frac{13}{33}$

(ii) $0.6\overline{25} = \frac{625 - 6}{990} = \frac{619}{990}$

(iii) $0.3\overline{524} = \frac{3524 - 35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$

Symbol of the Roman Method

1	→	I
2	→	II
3	→	III
4	→	IV
5	→	V
6	→	VI
7	→	VII
8	→	VIII
9	→	IX
10	→	X

20	→	XX
30	→	XXX
40	→	XL
50	→	L
100	→	C
500	→	D
1000	→	M

Rule of Divisibility

Rule of 2	The last digit is an even number or zero (0) as - 236, 150, 1000004
Rule of 3	If the sum of the digits of a number is divisible by 3, then the whole number will be divisible by 3. E.g. 729, 12342, 5631
Rule of 4	Last two digits are zero or divisible by 4. E.g. 1024, 58764, 567800
Rule of 5	The last digit is zero or 5. E.g. 3125, 625, 1250
Rule of 6	If a number is divisible by both 2 and 3 then it is also divisible by 6. E.g. 3060, 42462, 10242
Rule of 7	After multiplying the last digit of a number by 2 and subtracting it from the remaining number, if the number is a multiple of 0 or 7 or if any digit is repeated in a multiple of 6, then the number will be divisible by 7. E.g. 222222, 444444444444, 7854
Rule of 8	If the last three digits of a number are divisible by 8 or the last three digits are '000' (zero). E.g. 9872, 347000
Rule of 9	If the sum of the digits of a number is divisible by 9, then the whole number will be divisible by 9.
Rule of 10	The last digit should be zero (0).
Rule of 11	If the difference between the sum of digits at odd places and sum of digits at even places is zero (0) or 11 or a multiple of 11. E.g. 1331, 5643, 8172659
Rule of 12	Composite form of divisible by 3 and 4.
Rule of 13	Repeating the digit 6 times, or multiplying the last digit by 4 and adding it to the remaining number, if the number is divisible by 13, then the whole number will be divisible by 13. E.g. 222222, 17784

Examples

Q.1 If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of that number, then what will be $\frac{5}{3}$ of that number?

- (a) 12 (b) 18 (c) 15 (d) 20

Sol. (d)

Let the number = x

According to the question,

$$\Rightarrow \frac{9x - 2x}{12} = 7$$

$$\Rightarrow 7x = 7 \times 12$$

$$\Rightarrow x = 12$$

Hence, $\frac{5}{3}$ part of the number

$$= \frac{x - 5}{3} \Rightarrow \frac{12 \times 5}{3} = 20$$

Q.2 If the sum of two numbers is a and their product is a then their reciprocals will be –

- (a) $\frac{1}{a} + \frac{1}{b}$ (b) $\frac{b}{a}$ (c) $\frac{a}{b}$ (d) $\frac{a}{ab}$

Sol. (c)

Let the two numbers be P and Q respectively.

$$P + Q = a$$

$$PQ = b$$

$$\frac{1}{P} + \frac{1}{Q} \Rightarrow \frac{Q + P}{PQ} = \frac{a}{b}$$

Q.3 The sum of two numbers is 75 and their difference is 25, then what will be the product of those two numbers?

- (a) 1350 (b) 1250 (c) 1000 (d) 125

Sol. (b)

Let the greater number is x and smaller number is y.

$$\therefore x + y = 75 \quad \dots(i)$$

$$\text{and, } x - y = 25 \quad \dots(ii)$$

$$2x = 100 \text{ (By adding the equation i and ii)}$$

$$x = 50$$

Putting the value of x in eqn. (i),

$$50 + y = 75$$

$$y = 75 - 50 = 25$$

Hence, the product of both the numbers = $xy = 50 \times 25 = 1250$

Q.4 Divide 150 into two parts such that the sum of their reciprocal is $\frac{3}{112}$. Calculate both parts.

- (a) 50, 90 (b) 70, 80 (c) 60, 90 (d) 50, 100

Sol. (b)

Let the first part is x then its second part be $(150 - x)$.

According to the question,

$$\Rightarrow \frac{1}{x} + \frac{1}{(150-x)} = \frac{3}{112}$$

$$\Rightarrow \frac{150-x+x}{x(150-x)} = \frac{3}{112}$$

$$\Rightarrow 3x(150-x) = 150 \times 112$$

$$\Rightarrow 150x - x^2 = \frac{150 \times 112}{3}$$

$$\Rightarrow x^2 - 150x + 5600 = 0$$

$$\Rightarrow x^2 - 70x - 80x + 5600 = 0$$

$$\Rightarrow x(x-70) - 80(x-70) = 0$$

$$\Rightarrow (x-80)(x-70) = 0$$

$$\therefore x = 80 \text{ or } 70$$

If the first part = 80 then the second part = $150 - 80 \Rightarrow 70$

If the first part = 70 then the second part = $150 - 70 \Rightarrow 80$

Q.5 If the sum of any three consecutive odd natural numbers is 147, then the middle number will be –

- (a) 47 (b) 48 (c) 49 (d) 51

Sol. (c)

x = Suppose an odd number.

According to the question,

$$(x) + (x + 2) + (x + 4) = 147$$

$$3x + 6 = 147$$

$$x = \frac{141}{3} = 47$$

Hence, the middle number = $(x + 2) = 47 + 2 = 49$

Q.6 If the product of first three and last three of 4 consecutive prime numbers is 385 and 1001, then find the greatest prime number.

Sol. Let a, b, c & d are four prime numbers.

$$abc = 385 \quad (i)$$

$$bcd = 1001 \quad (ii)$$

$$\frac{abc}{bcd} = \frac{385}{1001} = \frac{5}{13}$$

Greatest prime number = 13

Trick:

Sum of first n odd numbers = n^2

$$1 + 3 + 5 + \dots + 99 = ?$$

$$? = \left(\frac{99+1}{2} \right)^2 = 2500 \text{ Ans.}$$

Q.7 What will be the sum of the even numbers between 50 and 100?

Sol. $52 + 54 + 56 + \dots + 98$

$$= (2 + 4 + 6 + \dots + 98) - (2 + 4 + 6 + \dots + 50)$$

$$n = \frac{98}{2} = 49, \quad n = \frac{50}{2} = 25$$

$$= 49 \times 50 = 2450, \quad 25 \times 50 = 1250$$

$$\therefore ? = 2450 - 1250 = 1200 \text{ Ans.}$$

Q.8 What will be the sum of odd numbers between 50 and 100?

Sol. $51 + 53 + \dots + 99$

$$= (1 + 3 + 5 + \dots + 99) - (1 + 3 + 5 + \dots + 49)$$

$$= \frac{99+1}{2} = \frac{100}{2} = 50, \quad \frac{49+1}{2} = \frac{50}{2} = 25$$

$$\therefore ? = (50)^2 - (25)^2$$

$$= 2500 - 625 = 1875 \text{ Ans.}$$

Q.9 In a division method, the divisor is 12 times the quotient and 5 times the remainder. Accordingly, if the remainder is 36, then what will be the dividend?

(a) 2706

(b) 2796

(c) 2736

(d) 2826

Sol. (c)

Remainder = 36

$$\therefore \text{Divisor} = 5 \times 36 = 180$$

$$\therefore \text{Quotient} = \frac{180}{12} = 15$$

$$\therefore \text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$= 180 \times 15 + 36$$

$$= 2700 + 36$$

$$= 2736$$

Q.10 What is the unit digits of $(3694)^{1739} \times (615)^{317} \times (841)^{491}$

- (a) 0 (b) 2 (c) 3 (d) 5

Sol. Unit digit in $(3694)^{1739} = 4$; Unit digit in 4 = Unit digits in $\left\{ (4^2)^{896} \times 4 \right\}$

= Unit digit in $(6 \times 4) = 4$

Unit digit in $(615)^{317} =$ Unit digit in $(5)^{317} = 5$

Unit digit in $(841)^{491} =$ Unit digit in $(1)^{491} = 1$

$5 \times 4 \times 1 = 20$, Unit digit = 0

Q.11 What will be written in the form of $\frac{p}{q}$ of 18.484848....?

- (a) $\frac{462}{25}$ (b) $\frac{610}{33}$ (c) $\frac{200}{11}$ (d) $\frac{609}{33}$

Sol. Let $x = 18.484848.....$ then,

$100x = 1848.484848.....$

On subtracting, $99x = 1830 \Rightarrow x = \frac{1830}{99} = \frac{610}{33}$

Hence, the required form as $\frac{p}{q}$ of 18.484848..... = $\frac{610}{33}$

Q.12 Put $\frac{\overline{0.936} - \overline{0.568}}{\overline{0.45} + \overline{2.67}}$ in the form of rational number.

Sol. $\overline{0.936} = \frac{936}{999}$, $\overline{0.568} = \frac{568}{999}$

$\therefore (\overline{0.936} - \overline{0.568}) = \left(\frac{936}{999} - \frac{568}{999} \right) = \frac{(936 - 568)}{999} = \frac{368}{999}$

$\overline{0.45} = \frac{45}{99}$, $\overline{2.67} = 2 + \overline{0.67} = 2 + \frac{67}{99} = \frac{198 + 67}{99} = \frac{265}{99}$

$\therefore (\overline{0.45} + \overline{2.67}) = \left(\frac{45}{99} + \frac{265}{99} \right) = \frac{(45 + 265)}{99} = \frac{310}{99}$

Given expression = $\left(\frac{\overset{184}{\cancel{368}}}{\underset{111}{\cancel{999}}} \times \frac{\overset{11}{\cancel{99}}}{\underset{155}{\cancel{310}}} \right) = \frac{2024}{17205}$

Q.13 What will be the common factor of $\left\{ (127)^{127} + (97)^{127} \right\}$ and $\left\{ (127)^{97} + (97)^{97} \right\}$?

- (a) 127 (b) 97 (c) 30 (d) 224

Sol. $(x + y)$ is one of the factor of $(x^m + y^m)$ If m is an odd.

\therefore The factor of $\left\{ (127)^{127} + (97)^{127} \right\} = (127 + 97) = 224$

Similarly, the factor of $\left\{ (127)^{97} + (97)^{97} \right\} = (127 + 97) = 224$

Hence, the common factor of both is 224.