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Physics

Volume - 3

SIERRA INNOVATIONS

Index

SIMPLE HARMONIC MOTION

Oscillations:

Periodic Motion:

* That motion which repeats itself after a regular time interval.
* That regular time interval is called time period of the motion.

Example: Rotation of planet around the sun, rotation of earth, uniform circular motion.

* If a particle repeats its motion after every fixed interval of time, which is called periodic time the motion is said to be periodic.

  Its angular frequency is given as $\omega = \frac{2\pi}{T}$

Ques.: Find ω for second hand in a watch $\omega_S = \frac{2\pi}{60} = \frac{\pi}{30} \frac{rad}{30 sec}$

Find ω for minute hand in a watch $\omega_m = \frac{2\pi}{60 \times 60} = \frac{2\pi}{3600} = \frac{\pi}{1800}$

Find ω for revolution of earth $\omega_E = \frac{2\pi}{365 \times 86400}$

Oscillatory:

* That periodic motion which is about a fixed point, like to-and-fro, back-and-forth, up-and-down.

Example:

* The needle of a sewing machine.
* The motion of a ball in bowl.

Note: All oscillatory motion are periodic but all periodic motion are not oscillatory.

Harmonic Functions:

* Those mathematical trigonometric functions which are periodic and continuous.

$\boxed{y = \sin kx}$ $\boxed{y = \cos kx}$

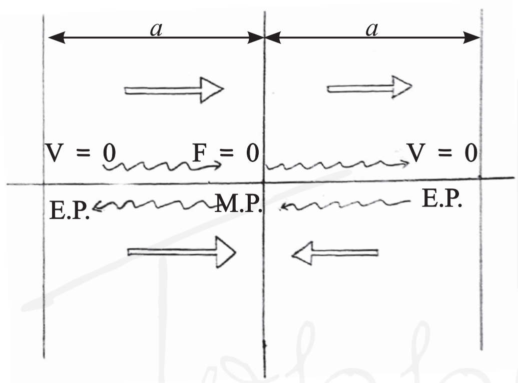
Simple Harmonic Motion:

* If a particle moves up and down (back and forth) about a mean position (also called equilibrium position) in such a way that a restoring force/torque acts on a particle which is proportional to displacement, then motion is called simple Harmonic Motion (SHM).

Exmaple: Motion of body syspended by a spring.

* For SHM

Equation of SHM: Every SHM can be best represented as a projection of a particle in circular motion on its diameter.



$$x = A\sin\theta = A\sin\omega t \qquad ...(1)$$

$$v = \frac{dx}{dt} = A\omega \cdot \cos\omega t \text{ (velocity)}$$

$$a = \frac{dv}{dt} = -\omega^2 \cdot A \cdot \sin\omega t \text{ (acceleration)}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = -\omega^2 x \text{ (from eqn. (1))}$$

$$\Rightarrow \quad \boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

* Called "Basic Differential Equation" of motion of a particle in SHM.

Simple Harmonic Motion (SHM)

Linear SHM

1. On a straight line

$$\boxed{x = A\sin(\omega t + \phi)}$$

2. Condition of SHM:

$$\boxed{a = -\omega^2 x}$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

Angular SHM

1. On a circular arc

$$\boxed{\theta = \theta_0 \sin(\omega t + \phi)}$$

2. Condition of SHM:

$$\boxed{\alpha = -\omega^2\theta}$$

$$\boxed{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0}$$

3. Restoring Force:

$$F = -m\omega^2 x$$

$$k = m\omega^2$$

4. Time Period:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

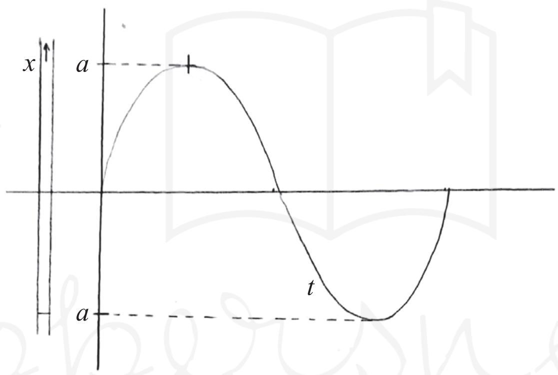
3. Restoring Torque:

$$\tau = -I\omega^2\theta$$

4. Time Period:

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

Ques.: Find the amplitude at $t = \frac{3T}{4}$.



at $$t = \frac{3T}{4}$$

$$x = a \sin \frac{2\pi}{T} \times \frac{3T}{4}$$

$$x = a \sin \frac{3\pi}{2}$$

Ques.: A particle starts performing SHM and starts from mean position towards the extreme. Find time taken by particle from 0 to $a/2$

Solns.:

$$x = \frac{a}{2}$$

$$= \frac{a}{2} = a \sin(\omega t)$$

$$= \sin\frac{\pi}{6} = \sin\frac{2\pi t}{T}$$

$$= \frac{\pi}{6} = \frac{2\pi t}{T}$$

$\Rightarrow$ $$t = \frac{T}{12} \text{ sec}$$

SHM:-

(1) Displacement: The distance of oscillating particle from its mean position at any time.

(2) Time period: The constant interval of time after which the motion is repeated.

(3) Amplitude: Max. value of displacement on either side of mean position (A) or (a) is called amplitude.

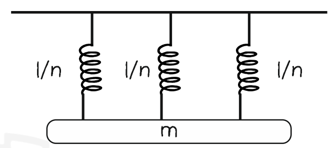
$$T = \frac{2\pi}{\omega} \qquad n = \frac{1}{T} = \frac{\omega}{2\pi}$$

* Parallel spring

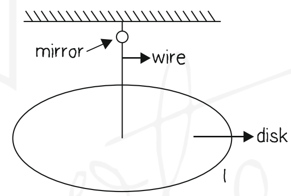
$$K_{eq} = nk + nk + \dots$$

$$= nk\,[1 + 1 + 1 + \dots]$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2p\sqrt{\frac{m}{n^2K}} = \frac{T}{n}$$



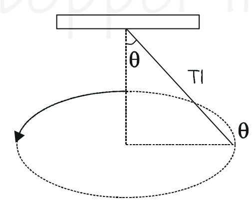
Torsional orcillator



$$T = 2\pi\sqrt{\frac{\ell}{C}}$$

$$C = \frac{\eta\pi r^4}{2\ell}$$

Conical Pendulum.



$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

Floating block

$$T = 2\pi\sqrt{\frac{m}{A\rho g}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{Ld}{\rho g}}$$

or $T = 2\pi\sqrt{\frac{h}{g}}$

Phase:

* The 'ωt' as a whole or '$\omega t + \theta$' as a whole gives the phase of the particle.
* Phase at any time gives the location & direction of velocity, it also tells the total distance covered by the particle.

  For eg: If a particle has a phase $\frac{\pi}{3}$ & $\frac{2\pi}{3}$ at

  Two different time then one particle is moving away from the mean position while the other is moving towards mean position.

$$= \frac{\pi}{3}, \frac{2\pi}{3}$$

$$= x = a \sin \omega t$$

$$v = a\ \omega \cos \omega t$$

$$= x^2 = a \sin\left(\frac{\pi}{3}\right), x_2 = a \sin\left(\frac{2\pi}{\sqrt{3}}\right)$$

$$= x_1 = \frac{\sqrt{3}a}{2}$$

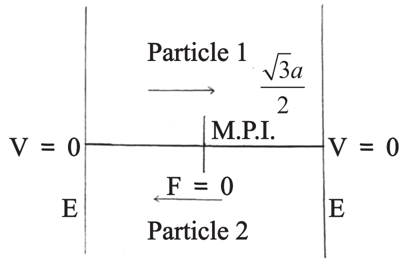
$$x_2 = \frac{\sqrt{3}a}{2}$$

$$V_1 = a\omega \cos\left(\frac{\pi}{3}\right)$$

$$V_2 = a\omega \cos\left(\frac{2\pi}{3}\right)$$

$$V_1 = \frac{aw}{2}$$

$$V_2 = \frac{-aw}{2}$$



- Initial phase = choice of $t = 0$
- Even if particle has some initial phase, its mean position does not change.

Initial Phase:

The phase at time $t = 0$ is known as the initial phase.

Let the particle's equation is

$x^1 = a \sin(\omega t + \theta)$ then on putting

$t = 0$ let the particle is at x^1,

then the initial phase will be

$$= x^1 = a \sin(\omega \times 0 + \theta)$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{x^1}{a}\right) = \theta$$

Initial phase does not represent the mean position. Mean position is that where force = 0,

acc^n = 0, $x = 0$

For eg:

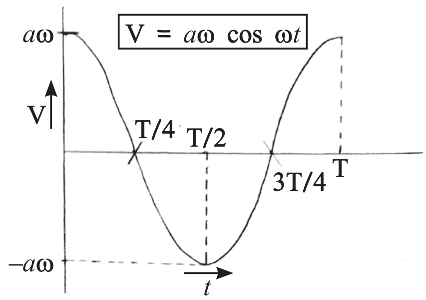
Let the particle equation is $x + 1 = 4 \sin \omega t$

In that case $x = -1$ will be the mean position and particle's amplitude co-ordinate are

$x = 5$ and $x = 3$

Velocity in SHM:

* Maximum velocities are $\pm a\omega$
* Maximum velocity occurs at mean position and at extreme position, it is zero.



We know, $x = a \cdot \sin \omega t$

$$\Rightarrow \quad \sin \omega t = \frac{x}{a}$$

$$v = a\omega \cos \omega t$$

$$= a\omega\sqrt{1-\sin^2\omega t}$$

$$= a\omega\sqrt{1-\frac{x^2}{a^2}}$$

$$= \omega\sqrt{a^2-x^2}$$

$$v = w\sqrt{a^2-x^2}$$

at $x = 0,\ v = \pm\ a\omega$

at $x = \pm\ a,\ v = 0$

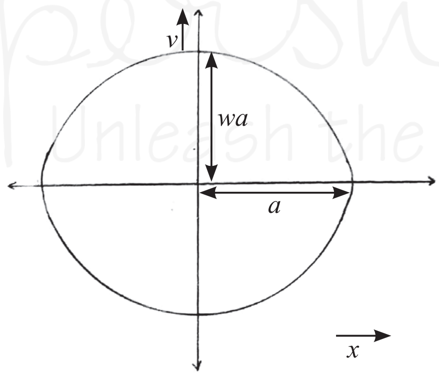
$\Rightarrow$ $V^2 = \omega^2(a^2 - x^2)$

$\Rightarrow$ $V^2 = \omega^2 a^2 - \omega^2 x^2$

$\Rightarrow$ $V^2 + \omega^2 x^2 = \omega^2 a^2$

$\Rightarrow$ $\dfrac{V^2}{(wa)^2} + \dfrac{x^2}{a^2} = 1$

Graph b/w v & x will be an ellipse.



Ques.: The maximum speed of a particle performing SHM is 10 m/s, find speed when the particle is at distance half of the amplitude.

Solns.: $10\ \text{m/s} = a\omega$

$$V = W\sqrt{a^2-\frac{a^2}{4}}$$

$$= W\sqrt{\frac{3a^2}{4}}$$

$$= \frac{\sqrt{3}}{2}\ a\omega$$

$$= \frac{\sqrt{3}}{2} \times 10$$
$$= 5\sqrt{3} \text{ m/s}$$

Ques.: The time period of a particle performing SHM is 'T' find the time, taken by the particle to complete $\frac{3^{th}}{8}$ oscillation.

Solns.: In complete oscillation, a particle travels a distace $4\,A$, hence $\frac{3}{8}$th oscillation would mean a distance of $\frac{3A}{2}$.

We can devide the distance in two parts.

Part 1: A distance A is travelled from mean position to extreme position. During this time taken = $\frac{T}{4}$.

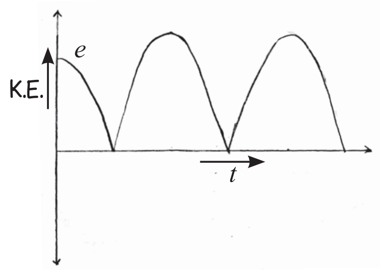
Part 2: $\frac{A}{2}$ is travelled from extreme position towards mean position.

$$y = A \sin \omega t$$
$$\frac{A}{2} = A \sin \omega t$$
$$\Rightarrow \quad \frac{1}{2} = \sin\left(\frac{2\pi}{T}t\right)$$
$$\Rightarrow \quad \frac{5\pi}{6} = \frac{2\pi}{T}t$$

(Looking for solution b/w $t = \frac{T}{4}$ & $t = \frac{T}{2}$)

$$\Rightarrow \quad \boxed{t = \frac{5T}{12}}$$

1. Kinetic Energy:



$$V = a\omega \cos \omega t$$

$$= KE = \frac{1}{2} ma^2\omega^2 \cos^2 \omega t$$

$$\text{KE at time } t = E \cos^2 \omega t$$

Time period for variation of KE = (T/2)

Time period for variation of

$$x \rightarrow T$$

$$\text{Energy} \rightarrow T/2$$

$$V \rightarrow T$$

* Avg. value of KE with time

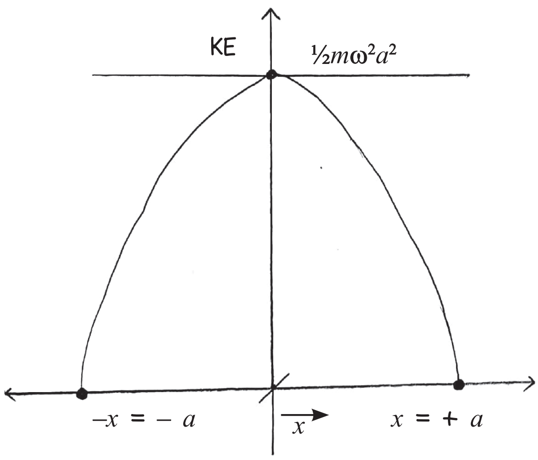
$$= \frac{1}{T}\int_0^T E \cdot \cos^2 \omega t \cdot dt$$

$$= \frac{E}{2T}\int_0^T \{1 + \cos 2\omega t\} \cdot dt$$

$$= \frac{E}{2T} \times T$$

$$\boxed{KE_{avg} = \frac{E}{2} = \frac{1}{4} m\omega^2 A^2}$$

* KE with distance:



$$KE = \frac{1}{2} mw^2(a^2 - x^2)$$

$$KE = \frac{1}{2} mw^2 a^2 - \frac{1}{2} mw^2x^2$$

Avg. value of KE with $x = \dfrac{<KE>= \int_0^a \frac{1}{2}mw^2a^2dx - \int_0^a \frac{1}{2}mw^2x^2dx}{\int_0^a dx}$

$$= \frac{1}{a}\left[\frac{1}{2}mw^2a^2(a) - \frac{1}{2}mw^2\frac{a^3}{3}\right]$$

$$= \frac{3}{6} mw^2 a^2 - \frac{1mw^2}{6} a^2$$

$$= \frac{1}{3} mw^2 a^2$$

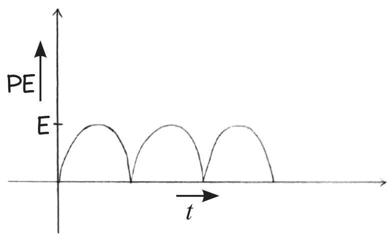
2. Potential Energy:

P.E. with time:

$$U = \frac{1}{2} mw^2a^2 \sin^2 wt$$

$$= E \sin^2 wt$$

Avg value of PE with time



$$= \int_0^T \frac{(E\sin^2 wt)dt}{\int_0^T dt}$$

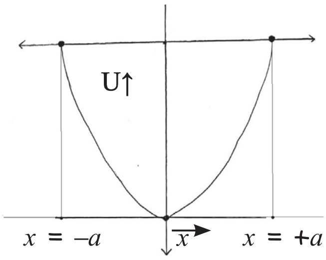
$$= (E/2)$$

E/2

PE with distance

$$U = \frac{1}{2}mw^2x^2$$

$\langle U \rangle$

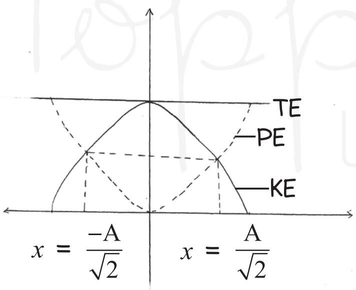


Avg PE with distance

$$\langle U \rangle = \frac{\int_0^a U dX}{\int_0^a dx} = \frac{1}{8} m w^2 \; a^2$$

Time period for variation of PE: T/2

3. Total Mechanical Energy:



$$E = KE + PE$$

$$= \frac{1}{2} m\omega^2 (A^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$

$$E = \frac{1}{2} m\omega^2 A^2$$

The curves representing KE, PE and total energy

Ques.: A body of mass 1 kg is executing simple harmonic motion which is given by $x = 6 \cos\left(100t + \frac{\pi}{4}\right)$cm. What is the

i. Amplitude of displacement
ii. Angular frequency
iii. Initial Phase
iv. Velocity
v. Acceleration
vi. Maximum kinetic energy

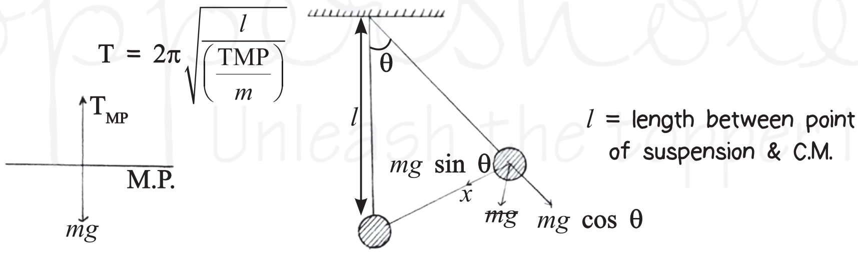
Solns.: The given equation is $x = 6\cos(100t + \frac{\pi}{4})$
comparing it with $x = A\sin(\omega t + \phi)$

i. Amplitude = 6.0 cm
ii. Angular frequency $\omega = 100\ s^{-1}$
iii. Initial phase $= \frac{\pi}{4}$
iv. Velocity $V = \omega\sqrt{A^2 - x^2} = 100\sqrt{36 - x^2}$ cm/s
v. Acceleration $= -\omega^2 x = -(100)^2 x = -10^4\ x$
vi. $KE_{max} = \frac{1}{2}mA^2\omega^2 = \frac{1}{2} \times 1 \times (0.06)^2 \times (100)^2 = 18\ J$

Angular SHM:

- In this type of SHMm particle oscillates in a circular arc.
- It can be a point mass or a rigid body.
- In case of rigid body, oscillation of centre of mass should be considered.

1. Simple Point Mass (Simple Pendulum):



(Restoring force) R.F. $= mg\sin\theta = mg(\theta) = mg\left(\frac{x}{l}\right)$

$$acc^n = \frac{gx}{l} = w^2 x$$

$$w = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Concept of Effective Length:

$$T = 2\pi\sqrt{\frac{Leff.}{g}}$$

Example:

2. Compound Pendulum (Rigid Body):

$$I = I_{cm} + ml^2 \quad \Rightarrow \quad I = mK^2 + ml^2$$

$$T = 2\pi\sqrt{\frac{M(K^2 + l^2)}{mgl}} = 2\pi\sqrt{\frac{\left(\frac{K^2}{l} + l\right)}{g}}$$

$$\Rightarrow \qquad L_{eff.} = l^2 + \frac{K^2}{l}$$

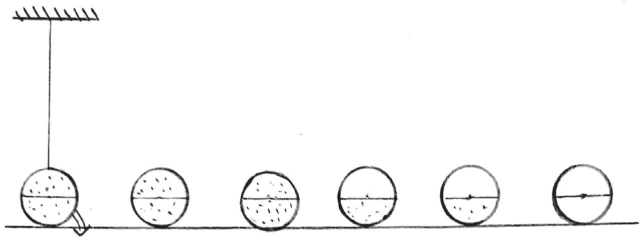
(2) $T = 2\pi\sqrt{\frac{L_{eff.}}{g}}$ (3) T ↑ ses gradually then ↓ ses suddenly

* A child on a swing stands up suddenly then T ↓ ses.

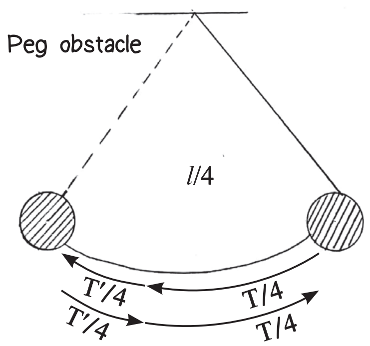
Working Rule for SHM:

1. Find MP (Where $F = 0$)
2. Displace the body from M.P.
3. Find new forces at new position
4. Find the force or its component towards MP, we call it restoring force.

$$\frac{F}{m} = \text{Acc}^n = w^2 x$$



T increases and then decreases and again reaches to same initial value.



Ques.: If the period of oscillations of a simple pendulum is 4 sec, find its length. If the velocity of the bob in the mean position is 40 cm/s, find its amplitude, $g = 9.8\ \text{m/s}^2$.

Solns.: Period $T = 4$ sec

Velocity at mean position $V_{max} = 40$ cm/s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T^2 = 4\pi^2\frac{l}{g}$$

$$\Rightarrow \quad l = \frac{T^2 g}{4\pi^2} = \frac{(4)^2(9.8)}{4(\pi)^2}$$

$$\Rightarrow \quad \boxed{l = 3.97\ \text{m}}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}\text{rad/s}$$

$$V_{max} = \omega A$$

$$\Rightarrow \quad A = \frac{V_{max}}{\omega} = \frac{40}{\pi} \times 2$$

$$\Rightarrow \quad \boxed{A = 25.5\ \text{cm}}$$

Ques.: Time period of simple pendulum on earth surface Is T, now the penduleum is taken upto a height = H = R/2 where R is the radius of earth.

Solns.:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

g^1 at R/2

$$= g^1 = \frac{g}{\left(1+\frac{R}{2R}\right)^3} = \frac{4g}{9}$$

$$T^1 = 2\pi\sqrt{\frac{9l}{4g}}$$

$$T^1 = \frac{3T}{2}$$

Concept of g_{eff}:

* It is based on the aceleration of frame of reference in which the simple pendulum is suspended.

* To calculate geff, we need to calculate tension at the equilibrium position divide by the mass.

$$g_{eef} = \frac{\text{Tension}}{\text{Mass}}$$

Lift Cases:

$$\text{T-}mg = ma$$

$$\frac{\text{Tension}}{\text{Mass}} = g + a$$

$$g_{eff} = g + a$$

$$\text{T} = 2\pi\sqrt{\frac{l}{g+a}}$$

Moving upwards with acc.:

$$\text{T} = 2\pi\sqrt{\frac{l}{g+a}}$$

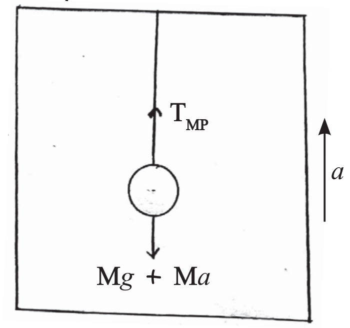
Moving with constant velocity:

$$\text{T} = 2\pi\sqrt{\frac{l}{g}}$$

Free fall:

$$g_{eff} \to 0$$
$$\text{T} \to \infty$$

* Time peiod of pendulum is independent from mass.



$$= \left[\frac{\text{T}_{MP}}{m} = g\right]$$

$$= \text{T} = 2\pi\sqrt{\frac{l}{(\text{T}_{MP}/m)}}$$