Toppersnotes

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NEET - UG

NATIONAL TESTING AGENCY

Physics

Volume - 2

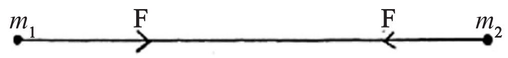
SIERRA INNOVATIONS

Contents

GRAVITATION

Newton's Law of Gravitation:

* It states that every particle in the universe attracts all other particles with a force which is proportional to the product of their masses and inversely proportional to the square of the distance between them.



$$\left.\begin{array}{l} F \propto m_1 m_2 \\ F \propto \dfrac{1}{r^2} \end{array}\right] \quad F \propto \frac{m_1 m_2}{r^2}$$

$$F = \frac{Gm_1m_2}{r^2}$$

G = Gravitational constant

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

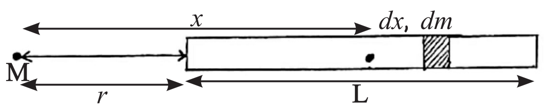
$\vec{r}$ = Position vector of 'm_2' w.r.t 'm_1'.

$$\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = F = \frac{Gm_1m_2}{r^2}$$

$$\vec{F}_{21} = \left(\frac{Gm_1m_2}{r^2}\right)\left(-\hat{r}\right)$$

$$\vec{F}_{21} = \frac{-Gm_1m_2}{r^3}\vec{r}$$

$$\vec{F}_{12} = \frac{Gm_1m_2}{r^3}\vec{r}$$



$$F \neq \frac{GMm}{\left(r + \frac{L}{2}\right)^2} \qquad \because \qquad L \to m$$

$$dF = \frac{GM \cdot dm}{x^2} \qquad L \to \frac{M}{L}dx = dm$$

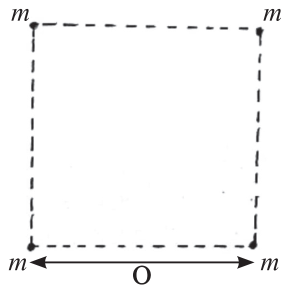
$$F = \int_r^{(r+L)} \frac{Gm}{x^2}\frac{M}{L}dx = \frac{GMm}{L}\int_r^{(r+L)}\frac{dx}{x^2}$$

$$F = \frac{GMm}{L}\left[\frac{-1}{x}\right]_r^{r+L} = \frac{GMm}{L}\left[\frac{1}{r} - \frac{1}{(r-L)}\right]$$

$$F = \frac{GMm}{L}\left[\frac{r+L-r}{r(r+L)}\right] = \frac{GMm}{r(r+L)}$$

Ques.: Three masses, each equal to m are placed at three corners of a square of side a. Calculate the force of attraction on the mass placed at fourth corner.

Solns.:

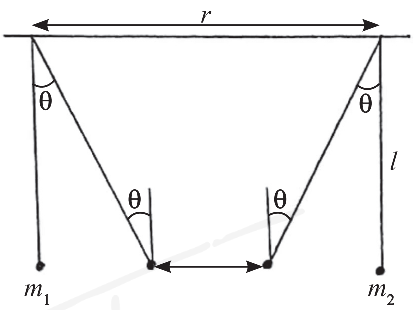


$$F = \frac{Gmm}{a^2}$$

$$F' = \frac{Gmm}{(\sqrt{2a})^2}$$

$$\boxed{F = 2F + F'}$$

Ques.:



$$F = \frac{Gm^2}{(r')^2}$$

$$F = T\sin\theta \qquad \ldots(1)$$

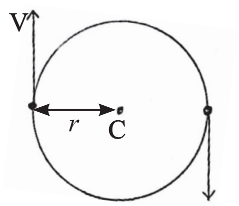
$$mg = T\cos\theta \qquad \ldots(2)$$

$$T^2 = F^2 + (mg)^2 \Rightarrow T = \sqrt{F^2 + (mg)^2}$$

$$\tan\theta = \frac{F}{mg} = \frac{G\cancel{m_1}m_2}{(r')^2 \times \cancel{m_1} \times g} = \frac{Gm_2}{(r')^2 \times g}$$

Ques.: 2 particles, each of mass m goes around in a circular motion their mutual gravitational attraction. Find the value of speed with which particle is doing circular motion.

Solns.:



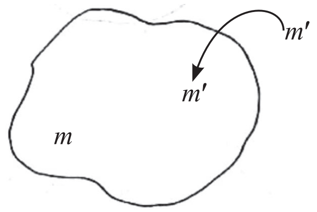
$$F = \frac{Gm^2}{r^2}$$

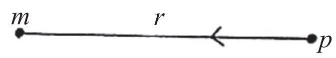
$$\frac{\cancel{M} \times v^2}{r} = \frac{Gm^{\cancel{2}}}{(2r)^2}$$

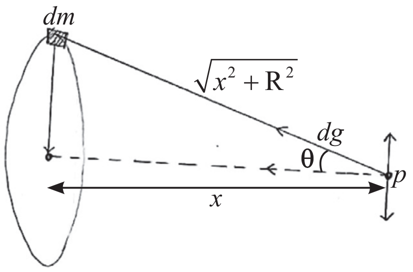
$$v^2 = \frac{Gm}{4r} \Rightarrow \boxed{\frac{1}{2}\sqrt{\frac{Gm}{r}} = v}$$

Gravitational Field:

* It is region in surrounding of mass where if any other mass comes, it experiences gravitational force.







$$g = \frac{Gm}{r^2}$$

$$dg = \frac{Gdm}{(x^2+R^2)}$$

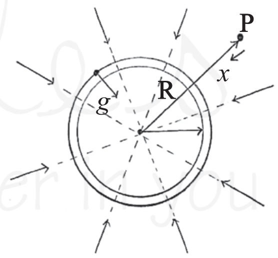
$$g_p = \int dg \cos\theta$$

$$g_p = \int \frac{Gdm}{R^2+x^2}\cdot\frac{x}{(x^2+R^2)^{1/2}} = \frac{Gx}{(R^2+x^2)^{3/2}}(m)$$

$$\boxed{g_p = \frac{Gmx}{(R^2+x^2)^{3/2}}}$$

Sphere:

Case-1: Hollow Sphere:

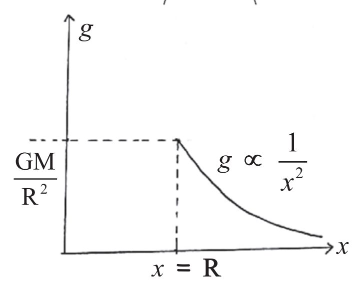


1. $x > R$ (outside)

$$g_p = \frac{Gm}{x^2}$$

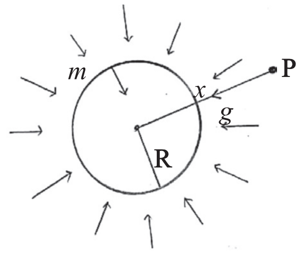
2. $x = R$ (surface)

$$g_s = \frac{Gm}{R^2}$$



3. $x < R$ (inside)

$$g_{in} = 0$$



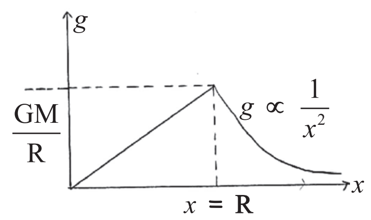
Case-2: Solid Sphere:

1. $x > R$

$$g_p = \frac{Gm}{x^2}$$

2. $x = R$

$$g_s = \frac{Gm}{R^2}$$

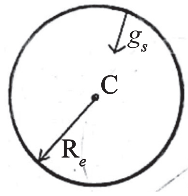


3. $$x < R$$

$$g_{in} = \frac{Gmx}{R^3} \Rightarrow gm \propto x$$

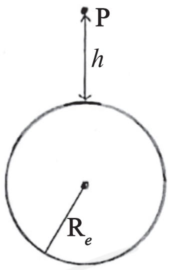
Earth:

I. $$g_s = \frac{GMe}{R_e^{\,2}}$$



g_s (according to gravity) = $9.8\ \text{m/s}^2$

II.



$$g_p = \frac{GM_e}{(R_e + h)^2}$$

$$g_p = \frac{GM_e}{R_e^{\,2}\left(1 + \frac{h}{R_e}\right)^2}$$

$$\Rightarrow \quad g_p = \frac{g_s}{\left(1 + \frac{h}{R_e}\right)^2}$$

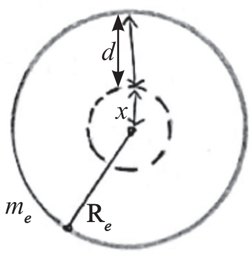
$(1 + x)^n = 1 + nx$ if $h \lll R_e$

$$g_p = g_s\left(1 + \frac{h}{R_e}\right)^{-2}$$

$$\because \quad \frac{h}{R_e} \lll 1$$

$$\Rightarrow \quad \boxed{g_p = g_s\left(1 - \frac{2h}{R}\right)}$$

III. $$g_{in} = \frac{G \cdot M_e \cdot x}{R_e^{\,3}}$$



$$\boxed{x = R_e - d}$$

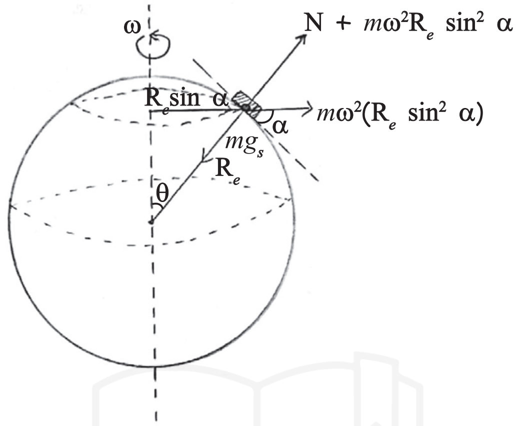
$$g_{in} = \frac{GM_e(R_e - d)}{R_e^{\,2} \cdot (R_e)} = g_s\left(1 - \frac{d}{R_e}\right)$$

$$\boxed{g_{in} = g_s\left(1 - \frac{d}{R_e}\right)}$$

Effect of Rotation of Earth on the Value of *g*:

$$N + m\omega^2 R_e \sin^2 \alpha = Mg_s$$

$$mg_{eff} = N = mg_s - m\omega^2 R_e \sin^2 \alpha$$



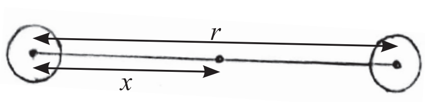
$$g_{eff} = g_s - \omega^2 R_e \sin^2 \alpha$$

At equator, $\alpha = 90°$

$\therefore$ $g_{eff} = g_s - \omega^2 \cdot R_e \quad (9.6 \text{ m/s}^2)$

At poles $\alpha = 0°$

$\therefore$ $g_{eff} = g_s \quad (9.8 \text{ m/s}^2)$



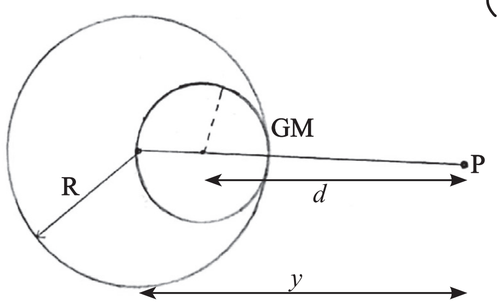
$$g_e = g_m$$

$$\frac{GM_e}{x^2} = \frac{GMm}{(r-x)^2}$$

$$x = \ldots\ldots\ldots$$

$$(1 + x)^n = 1 + nx$$

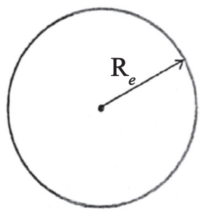
$$x <<< 1$$



$$g_{big} - g_{small}$$

$$= \frac{GM}{y^2} - \frac{GM}{8d^2}$$

$$g_s = \frac{GM_e}{R_e^{\ 2}}$$



If radius of earth shrinks by 1% mass remains same.

$$g' = \frac{GM_e}{(0.99R_e)^2} = \frac{GM_e}{R_e^2(0.99^2)}$$

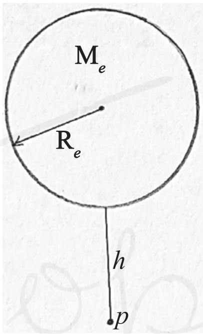
$$g' = \frac{g_s}{(1-0.01)^2} = g_s(1-0.01)^{-2}$$

$$g' = g_s(1 + 0.02) = g_s(1.02)$$

$$\boxed{g' = g_s\left(\frac{102}{100}\right)}$$

Ques.: At what height above the earth's surface value of gravitationla force will be half of its value at the surface of the earth?

Solns.:



$$g_s = \frac{GM_e}{R_e^2}$$

$$\frac{g_s}{2} = g_p = \frac{g_s}{\left(1+\frac{h}{R_e}\right)^2}$$

$$\Rightarrow \quad \sqrt{2} = 1 + \frac{h}{R_e}$$

$$\Rightarrow \quad \boxed{h = (\sqrt{2} - 1)R_e}$$

Ques.: With what angular velocity earth will rotate so that app. value of g at its equator becomes 0?

Solns.:

$g_{\text{eff}} = g_s$ (at surface)

$g_{\text{eff}} = g_s - \omega^2 R_e \sin^2 \alpha$

1 ($\because$ at equator $\alpha = 90°$)

$$g_s - \omega^2 R_e = 0$$

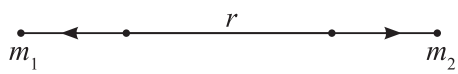
$$\Rightarrow \quad \omega = \sqrt{\frac{g}{R_e}}$$

Gravitational Potential Energy or Gravitational Interaction Energy:

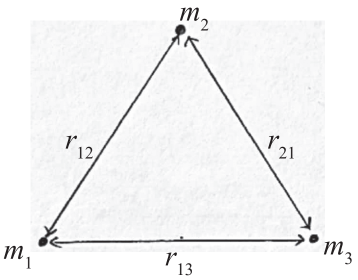
* Defined as the amount of work required to bring the particle from infinity to desired.

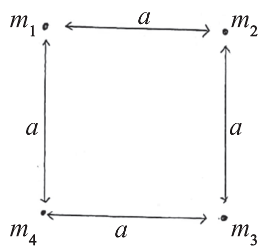
Case-I:

$$U = \frac{-Gm_1m_2}{r}$$



Cose-II:





Multiple particle system:

$$U = \frac{-Gm_1m_2}{r_{12}} \frac{-Gm_2m_3}{r_{23}} \frac{-Gm_3m_1}{r_{13}}$$

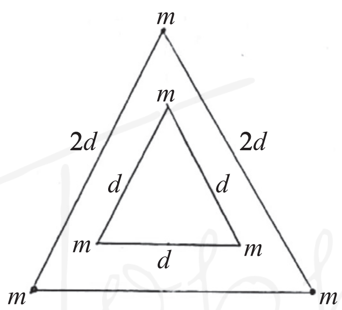
$$U = \frac{-Gm_1m_4}{a} \frac{-Gm_1m_2}{\sqrt{2}a} \frac{-Gm_2m_3}{\sqrt{2}a}$$

$$\frac{-Gm_4m_3}{a} \frac{-Gm_1m_3}{\sqrt{2}a} \frac{-Gm_2m_4}{\sqrt{2}a}$$

Ques.: Find W done in ↑sing the sides of triangle form d to $2d$?

Solns.:

$\Rightarrow$



$$W = U_f - U_i$$

$$U_i = \frac{-Gm^2}{d} - \frac{Gm^2}{d} - \frac{Gm^2}{d} = \frac{-3Gm^2}{d}$$

$$U_f = \frac{-3Gm^2}{2d}$$

$$\text{Work done} = \frac{-3Gm^2}{2d} + \frac{3Gm^2}{d} = \frac{-3Gm^2 + 6Gm^2}{2d}$$

$$\text{Work done} = \frac{3Gm^2}{2d}$$

Ques.: Find the velocity of m_1 & m_2 when the separation between them becomes d.

m_1 $\cdot$ (rest) $\qquad v_1 \longrightarrow \infty \longleftarrow v_2$ $\qquad m_2$ $\cdot$ (released from rest.)

$$m_1 \longleftarrow d \longrightarrow m_2$$

Solns.:

$$U_f = \frac{-Gm_1m_2}{d},\ U_i = 0,\ K_i = 0$$

$$E_i = 0$$

$$\boxed{E_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} = 0} \qquad \ldots(1)$$

$\because \quad E_i = E_f$

$\Rightarrow \quad \because \quad$ Ext. force is absent, $P_i = P_f$

$$0 + 0 = m_1v_1 - m_2v_2$$

$$m_1v_1 = m_2 \cdot v_2$$

$$\Rightarrow \qquad v_1 = \frac{m_1 m_2}{m_1}$$

$$\frac{1}{2} m_1 \left(\frac{m_2^{\cancel{2}} v_2^2}{m_1^2} \right) + \frac{1}{2} \cancel{m}_2 v_2^2 - \frac{G m_1 \cancel{m}_2}{d} = 0$$

$$\frac{1}{2} \frac{m_2 v_2^2}{m_1} + \frac{1}{2} v_2^2 = \frac{G m_1}{d}$$

$$\Rightarrow \qquad v_2^2 = \frac{2 G m_1}{d \left(\frac{m_2 + m_1}{m_1} \right)}$$

$$v_2 = \sqrt{\frac{2 G m_1^2}{d(m_2 + m_1)}} = m_1 \sqrt{\frac{2G}{d(m_2 + m_1)}}$$

$$v_1 = m_2 \sqrt{\frac{2G}{d(m_2 + m_1)}}$$

$$0 = \frac{1}{2} m_1 v_1^2 - \frac{G m_1 m_2}{d}$$

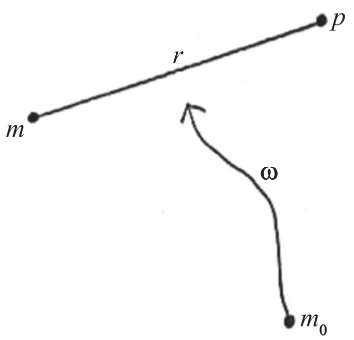
$$\Rightarrow \qquad \frac{1}{2} v_1^2 = \frac{G m_2}{d}$$

$$\Rightarrow \qquad \boxed{v_1 = \sqrt{\frac{2 G m_2}{d}}}$$

Gravitational Potential:

* W done in bringing a unit mass from ∞ to the given location.

Interactional Energy of Unit Mass:



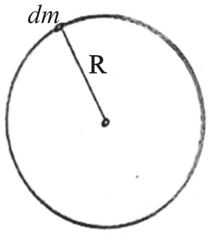
$$m_0 \rightarrow U$$

$$1 \rightarrow \frac{U}{m_0} = V_p$$

$$V_p = \frac{G m m_0}{r m_0} = \frac{-G m}{r}$$

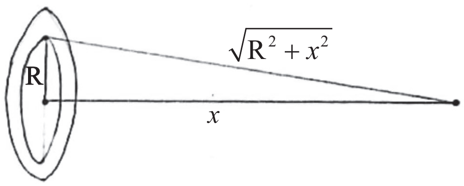
Ring:

$\Rightarrow$



$$dV = \frac{-G \cdot dm}{R}$$

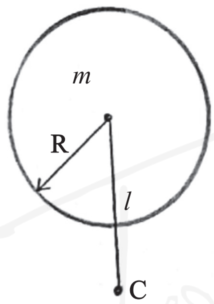
$$V = \int dV = \frac{-Gm}{R}$$



$$dV = \frac{-Gdm}{\sqrt{R^2 + x^2}}$$

$$\int dV = V = \frac{-Gm}{\sqrt{R^2 + x^2}}$$

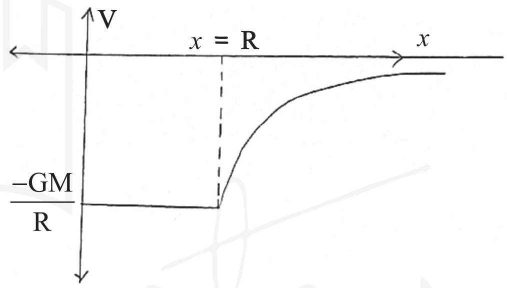
Hollow Sphere:



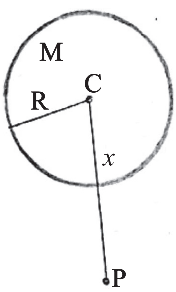
(i) $x > R, \; V_p = \dfrac{-Gm}{x}$

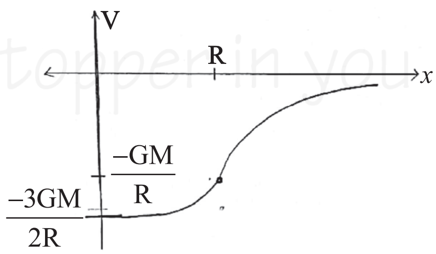
(ii) $x = R, \; V_p = \dfrac{-Gm}{R}$

(iii) $x = R, \; Vp = \dfrac{-Gm}{R}$



Solid Sphere:





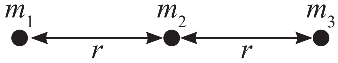
(i) $x > R, \; V_p = \dfrac{-Gm}{x}$

(ii) $x = R, \; V_p = \dfrac{-Gm}{R} = V_s$

(iii) $x < R, \; V_p = \dfrac{-Gm(3R^2 - x^2)}{2R^3}$

(iv) $x = 0, \; V_p = \dfrac{-3}{2}\dfrac{Gm}{R} = \dfrac{-3}{2} V_s$

Golden Key Point:



$\Rightarrow$

$$V = \frac{U}{m}$$

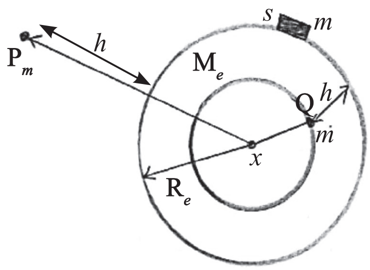
$$U = mV$$

$$U_p = m_2 V_p$$

$$U_p = m_2 \left[\frac{-Gm_1}{r} \frac{-Gm_3}{r} \right]$$

Potential Energy of a Body in Earth's Gravitational Field:

$\Rightarrow$



$$V = \frac{U}{m}$$

$$U = mV$$

$$U_s = mV_s = m\left(\frac{-GM}{R_e}\right) = \frac{-GMm}{R_e}$$

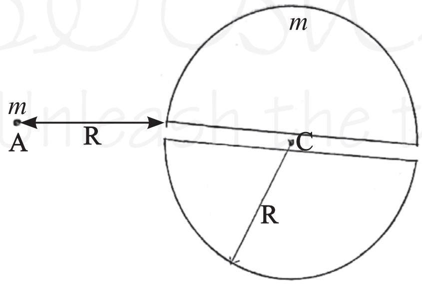
$$U_p = mV_p$$

$\Rightarrow$ $$U_p = m\left[\frac{-GM}{Re + h}\right]$$

$$U_Q = U_{in} = m\left[\frac{-GM}{2R^3}(3R^2 - x^2)\right]$$

$$\boxed{x = R_e - h}$$

Find the velocity of particle when it just crosses the centre.



$\Rightarrow$ $$mV_A = mV_C + \frac{1}{2}mV^2$$

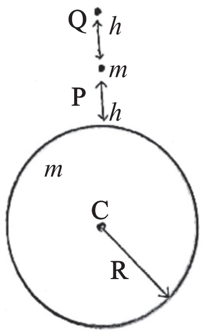
$$\frac{-Gm}{\cancel{2}R} = \frac{-3Gm}{\cancel{2}R} + \frac{V^2}{\cancel{2}}$$

$$V^2 = \frac{-Gm}{R} + \frac{3Gm}{R} = \frac{2Gm}{R}$$

$\Rightarrow$ $$\boxed{V = \sqrt{\frac{2Gm}{R}}}$$

Ques.: Find out the work done in shifting from P to Q?

Solns.:



$$\Rightarrow \quad U_f = \frac{-GMm}{R+2h}, \; U_i = \frac{-GMm}{R+h}$$

$$\Rightarrow \quad W = U_f - U_i$$

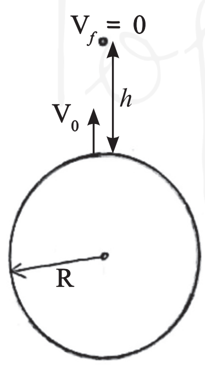
$$\Rightarrow \quad W = \frac{-GMm}{R+2h} + \frac{GMm}{R+h}$$

$$W = GMm\left(\frac{-\cancel{R} - h + \cancel{R} + 2h}{(R+2h)(R+h)}\right)$$

$$= \frac{GMmh}{(R^2 + 3Rh + 2h^2)}$$

Ques.: If a particle is projected from the surface of earth with velocity V_0, find the maximum height to which the particle will rise?

Solns.:



$$\Rightarrow \quad \frac{1}{2}m\, V_0^2 \; \frac{-GM\cancel{m}}{R} = \frac{-GM\cancel{m}}{R+h}$$

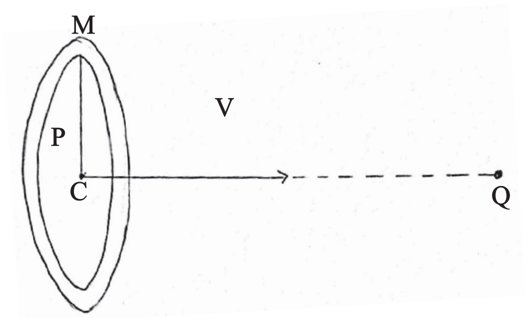
$$\frac{1}{2}V_0^2 = GM\left(\frac{-\cancel{R} + \cancel{R} + h}{R^2 + Rh}\right) = \frac{GMh}{(R^2 + Rh)}$$

$$\Rightarrow \quad \boxed{V_0 = \sqrt{\frac{2GMh}{R^2 + Rh}}}$$

Ques.: From the centre of ring, a point mass is projected such that it will escape to infinity. Find out that velocity?

Solns.:

$$\Rightarrow \quad \varepsilon_1 = K + P$$



$$= \frac{1}{2}mV^2 + m(V_i) = 0$$

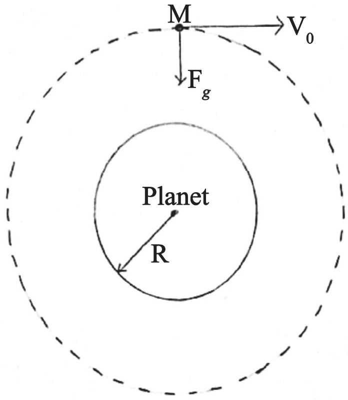
$$= \frac{1}{2}mV^2 + m\left(\frac{-Gm}{R}\right) = 0$$

$$V = \sqrt{\frac{Gm}{R}}$$

Satellite:

$r \longrightarrow$ orbital radius

$V \longrightarrow$ orbital speed



$$F_g = \frac{mV_0^2}{r}$$

$$\Rightarrow \quad \frac{GMm}{r^2} = \frac{-mV_0^2}{r}$$

$$V_0 = \sqrt{\frac{GM}{r}}$$

Note: It is independent of the mass of the satellite.

Orbital Period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{V} = \frac{2\pi \cdot r^{3/2}}{\sqrt{GM}}$$

$$\Rightarrow \quad \boxed{T \propto r^{3/2}} \quad \text{(Kepler laws)}$$

Energy of Satellite:

KE of satellite;

$$K = \frac{1}{2} mV_0^2 = \frac{1}{2}\frac{mGM}{r}$$

$$K = \frac{GMm}{2r}$$

$$\text{PE of sat.} = P = \frac{-GMm}{r}$$

$$\text{Total energy} = E_T = \frac{-GMm}{2r}$$

$$\boxed{|E_T| = |KE| = \frac{1}{2`}|PE|}$$

Ques.: A satellite is revolving around earth in orbital radius in $4R_e$. Find its orbital speed & time period?

Solns.: ⇒

$$V_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gr^2}{4Re}}$$

$$= \sqrt{\frac{gR_e \times R_e}{4R_e}} = \sqrt{\frac{gR_e}{4}}$$

$$V_0 = \frac{1}{2}\sqrt{GR_e}$$

Time period: $$\frac{2\pi(4\times R_e)^{3/2}}{\sqrt{gR_e{}^2}} = \frac{2\pi(4R_e)^{3/2}}{\sqrt{gR_e{}^2}}$$

$$= 16\pi\sqrt{\frac{R_e}{g}}$$

Ques.: Two satellites A & B of same mass are orbiting around earth at altitudes R & 3R. Calculate the ratio of KE of A & B?

Solns.: ⇒

$$KE_A = \frac{GMm}{R+Re},$$

$$KE_B = \frac{GMm}{3R+Re}$$

$$\frac{KE_A}{KE_B} = \frac{3R+Re}{R+Re} = \frac{4R}{2R} = \frac{2}{1}$$

⇒ $2 : 1$

Ques.: A satellite of mass 2×10^3 kg is to be shifted from an orbit of radius 2Re to 3Re. Find out the minimum energy required to shift?

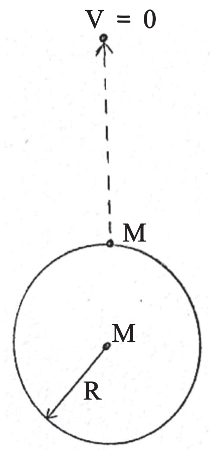
Solns.: ⇒

$$E = \frac{-GMm}{2\times 2R_e} + \frac{GMm}{6R_e}$$

$$= +\ GMm\left[\frac{-3+2}{12R_e}\right]$$

$$\boxed{E = \frac{-GMm}{12R_e}}$$

Bounded Motion and Escape Velocity:



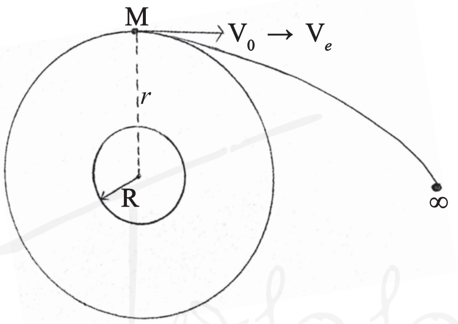
* Total energy of mass 'm' at the surface = K + P

$$E_T = \frac{1}{2}mV^2 - \frac{GMm}{R} < 0$$

If somehow we implant KE such that its total energy = 0

$$\frac{1}{2}mV_e^2 \; \frac{-GMm}{R} = 0$$

$$\Rightarrow \qquad V_{escape} = \sqrt{\frac{2GM}{R}}$$



Planet = Earth

$$V_{escape} = \sqrt{\frac{2GM}{R_e}} = \sqrt{2g_s \cdot R_e} = 11.2 \text{ Km/sec.}$$

$$V_0 = -\sqrt{\frac{GM}{r}}$$

$$TE = \frac{-GMm}{2r}$$

If some how speed of satellite is increased from $V_0 \rightarrow V_{escape}$

$$\frac{1}{2}mV_e^2 - \frac{GMm}{r} = 0$$

$$V_{es} = \sqrt{\frac{2GM}{r}}$$

$$V_{es} = \sqrt{2V_0}$$

Increase in speed of satellite = ΔV

$$= \sqrt{2}V_0 - V_0 = V_0(\sqrt{2} - 1)$$

$$= V_0 \times 0.414$$

$$\boxed{\Delta V = 41.4\,\%\,V_0}$$

* If orbital velocity of a satellite is increased by 41.4%, then it escapes from the gravitational field of earth.

Ques.: An artificial satellite is moving around earth in a circular orbit such that its speed is equal to half of the escape velocity from the earth. Find the height of the satellite.

Solns.: $\Rightarrow$

$$V_0 = \frac{1}{2}V_{escape}$$

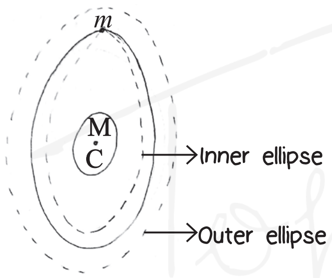
$$\sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{\frac{GM}{R}}$$

$$\frac{\cancel{GM}}{R+h} = \frac{1}{\cancel{4}_2}\left(\frac{\cancel{2}\,\cancel{GM}}{R}\right)$$

$$2R = R + h$$

$$\boxed{R = h}$$

Motion of Satellite in an Elliptical Path:



If $V_0 < V_s < V_{esc}$

$$\frac{mV_0^2}{r} > F_g$$

If $V_s < V_0$

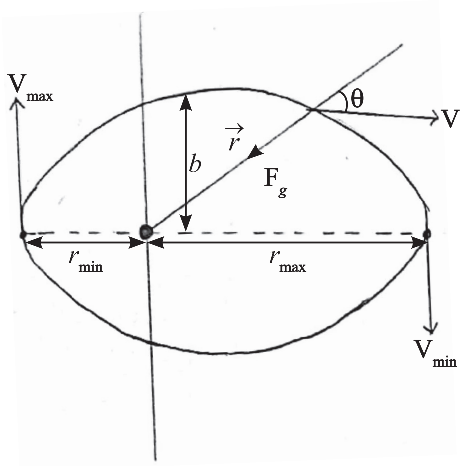
$$\boxed{F_g > \frac{mV_0^2}{r}}$$

Angular Momentum in Case of Satellite Motion:

The only force acting is gravitational force.

$$\vec{\tau}\,F_g = 0$$

$$\Rightarrow \quad L_i = L_f$$



$$L = m(\vec{r} \times \vec{v}) = mvr\sin\theta$$

$$L = mvr\sin\theta = mV_{max}\,r_{min}$$

$$= mV_{min}\,r_{max} = \text{constant}$$

$$G = \frac{1}{2}mV^2 - \frac{GMm}{r} = \frac{1}{2}mV_{min}^2 - \frac{GMm}{r_{max}}$$

$$= \frac{1}{2}mV_{max}^2\ \frac{-GMm}{r_{min}}$$