



# UGC-NET

## ECONOMICS

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# UGC NET - ECONOMICS

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## Unit - 1

### Micro Economics

#### Theory of Consumer Behaviour : Basic Themes

##### Introduction

It is generally observed that market aggregate demand curve for a commodity is downward sloping, given other things. Our problem is to investigate economic rationality behind this for a commodity of all individual consumers. The market demand basically depends on the characteristics of demand for a commodity by individual consumers, and the demand for a commodity of an individual consumer depends upon the behavior of the consumer. Clearly, to investigate economic rationality behind the law of demand, we shall start with the analysis of consumer behavior.

##### The Basic Themes

There are different approaches to analyse the consumer behaviour. But in all approaches, it is assumed that the consumer is rational. This means that the consumer's objective is to maximise her utility by choosing one commodity bundle from among all the commodity bundles (money income and the prices of the commodities are given to the consumer).

##### Consumer Choice Concerning Utility

Consumers can't maximise her utility unless she can measure it. Hence, utility must be a measurable concept. The measurement is undertaken differently in different approaches. In traditional frame, we have two types of measurement of utility,

1. Cardinal analysis
2. Ordinal analysis

##### Cardinal Theory: An Introduction

In cardinal approach, utility is measured cardinally or numerically in terms of money. The consumer not only knows which one is preferred but also by what amount. The assumptions of this approach is given below:

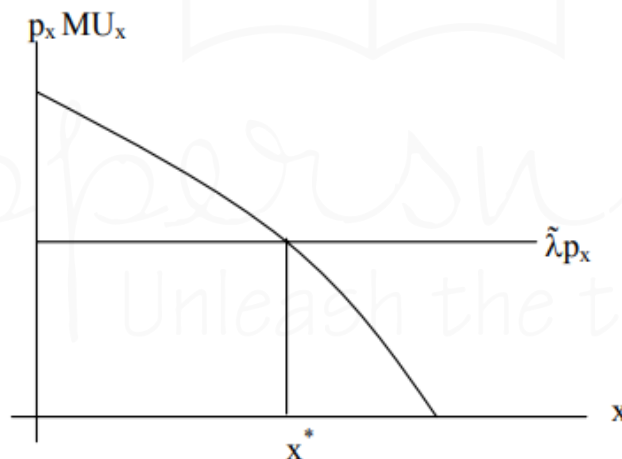
1. **Consumer is rational. Implication:** The consumer's objective is to maximise her utility by choosing one of the commodity bundle from all other available commodity bundles at given prices of commodities and money income.
2. If the taste and preferences are given, the total utility of the consumer depends on the quantity of consumption.
3. **Goods are good. Implication:** Let 'U' denote utility level of the consumer and let 'x' be the consumption bundle. As 'x' increases (decreases), 'U' increases (decreases). Therefore, marginal utility is positive.
4. **Marginal utility of 'x' is diminishing.** Implication: As 'x' increases (decreases),  $MU_x$  decreases (increases). Therefore,  $MU_x$  curve is downward sloping
5. **Utility is measured cardinally or numerically in terms of money.** Implication: Since it is measured numerically consumer not only knows which commodity bundle is preferred but also by how much amount.

**6. Marginal utility of money is constant.** Implication:  $MU_m = \lambda$  where  $\lambda$  is positive and constant. That means as money income increases (decreases) by one unit, utility increases (decreases) by  $\lambda$  unit.

### Consumer Equilibrium

According to our assumption for 'x' units consumption of the commodity, gross utility obtained by the consumer is  $U(x)$ . But for this, the consumer must spend  $p_x \cdot x$  units of money income if  $p_x$  be the price of the commodity 'x', which is given to the consumer. Since from assumption 6,  $\lambda$  represents fall in utility due to one unit fall in money income, the net utility of the consumer is given by  $N(x) = U(x) - \lambda p_x \cdot x$ , where  $\lambda$  and  $p_x$  are given to the consumer. So consumer's objective is to maximise  $N(x)$  by choosing 'x'. For that we take the first derivative of  $N(x)$  and set that equal to zero,  $\frac{dN(x)}{dx} = 0$ . Or, we get  $\frac{dU(x)}{dx} - \lambda p_x = 0$ . From this first order condition, we can derive the optimum value of 'x' which is (say)  $x^* = x^*(p_x, \lambda)$ . The second order condition for utility Maximization requires

$$\frac{\partial^2 N(x)}{\partial x^2} = \frac{\partial^2 U(x)}{\partial x^2} < 0, \text{ Which is ensured by the assumption of falling } MU_x.$$



**Fig. : Consumer Equilibrium in Cardinal Theory**

### Codinal Theory: A Short Note

In ordinal approach, utility is measured ordinally i.e., qualitatively (not numerically or quantitatively). Alternatively, consumer can rank her preferences according to the order she wants to compare but not in terms of the different amount. It is a qualitative measure and therefore more realistic measurement of utility or satisfaction.

There are two different approaches of ordinal theory, viz.,

1. Indifference curve approach
2. Revealed preference approach

### Indifference Curve Approach

Indifference curve is constructed by taking utility level constant, so different indifference curves imply different level of utility for same consumer. The equilibrium is achieved when indifference curve become tangent to the budget line.

## Revealed Preference Approach

In revealed preference approach, consumer equilibrium can be found by ranking different bundle of goods in the commodity space. Given the budget constraint, consumer chooses the best bundle for which her utility will maximise. This theory was originally constructed by the famous economist Paul. A. Samuelson.

## Introduction to Demand Analysis

It is generally seen that market demand curve is downward sloping. Market demand curve (or sometimes called Aggregate demand curve) is nothing but the aggregation of individual demand curves. Individual demand curve can be constructed by joining different consumer equilibrium for different prices (remember that consumer can't alter the market prices, it is given to the consumer). In neo-classical consumer theory, price is exogenous variable, so demand curve can be obtain only if we change the price exogenously and join all the equilibrium points. From next on our objective is to find out the consumer demand curve, for which we will adopt ordinal theory and in that, we will take indifference curve approach.

## Ordinal Theory: Indifference Curve Approach

In indifference curve approach consumer is assumed to be rational, so that consumer's objective is to maximise her utility by choosing a commodity bundle among all other available commodity bundles (under budget constraint) where total utility ('U') depends on quantity consumption given Her taste and preferences. Therefore, in a two-commodity world (say  $x_1$  and  $x_2$ ) utility function is given by  $U = U(x_1, x_2)$  and it depends on taste and preferences of the consumer, which is specified by axioms given below:

### 1. Axiom of Reflexiveness: Consumer's choice is reflexive.

Implication: 'R' denotes weak preference relation. Suppose there are two goods  $x_1$  and  $x_2$  and suppose  $x_1$  is weakly preferred to  $x_2$  i.e.,  $x_1 R x_2$  which implies that either  $x_1$  is strictly preferred over  $x_2$  (it is denoted by  $x_1 P x_2$ ) or  $x_1$  is indifference to  $x_2$  (it is denoted by  $x_1 I x_2$ ), where 'P' and 'I' implies strict preference relation and indifference respectively.

The set constituted by all commodity bundles or vector is known as commodity set (X). Any one commodity bundle is denoted by 'x' is weakly preferred (i.e., either strictly preferred or indifferent) over any other commodity bundle (i.e., in respect to 'x'). Therefore, we have  $x R x$ .

Clearly, any one commodity bundle may be indifferent to another commodity bundle i.e., there is a possibility of indifference or same level of utility between the commodity bundles.

None of the commodity bundles are not preferred i.e., consumer can choose any commodity bundle. So choice set of this consumer is specified by the commodity set 'X'.

### 2. Axiom of completeness: Consumer's choice is complete.

Implication: Since consumer is rational, she must have a unique preference relation. That means the consumer choice is either  $x_1 R x_2$  or  $x_2 R x_1$ . Alternatively, consumer's choice is consistent or comparable. For unique preference relation, consumer choice must be transitive, where transitivity implies that if  $x_1 R x_2$  and  $x_2 R x_3$  then  $x_1 R x_3$ , where  $x_3$  is another commodity.

### 3. Axiom of continuity: Consumer's preference relation (R) is continuous.

i. **Axiom of non-satiation:** Consumer's choice is non-satiated in all goods.

Implication: Non-satiation means larger the consumption of a good leads to larger satisfaction or utility or lower the consumption lower is the satisfaction or utility. Non-satiation of all goods (which means "goods are good" or "more is better") means any commodity bundle 'A' is preferred over another commodity bundle 'B' only if bundle 'A' consists larger quantity of at least one good and no less quantity of any other goods. Notational if  $A \succ B$ , then A is preferred over B or  $A \succ B$  where B is any other commodity bundle.

ii. **Axiom of convexity:** Consumer choice is such that indifference curve is strictly convex to the origin (i.e., utility function is quasi-concave).

iii. **Axiom of selfishness:** Consumer choice is selfish.

Implication: Consumer's choice is self-guided. Any other consumer does not influence it.

### Concept of Preference, Utility Function and Indifference Curve

Consumer preference ('R') specified by the above axioms can be represented by a function where total utility ('U') depends on quantity consumption ( $x_1, x_2$ ), which satisfied all other axioms. The function  $U = U(x_1, x_2)$  is known as Consumer Behaviour utility function. Since consumer is rational, her objective is to maximise the utility specified by the utility function  $U = U(x_1, x_2)$  subject to her budget constraint. To solve the consumer utility Maximization problem, we use a graphical tool, which is known as Indifference curve.

Meaning and definition of indifference curve: Different combination of goods  $x_1$  and  $x_2$  along which consumer is indifferent (or consumer has same level of utility) give a curve in commodity-commodity plane known as indifference curve. Therefore, along the indifference curve utility or satisfaction remains unchanged.

Existence of indifference curve: Because of axiom of Reflexiveness consumer can choose a commodity bundle over another commodity bundle i.e., consumer may be indifferent between any commodity bundle and such a choice might be continuous. So, indifference curve may exist anywhere in the commodity space.

## Derivation of Indifference Curve

### Graphical Presentation

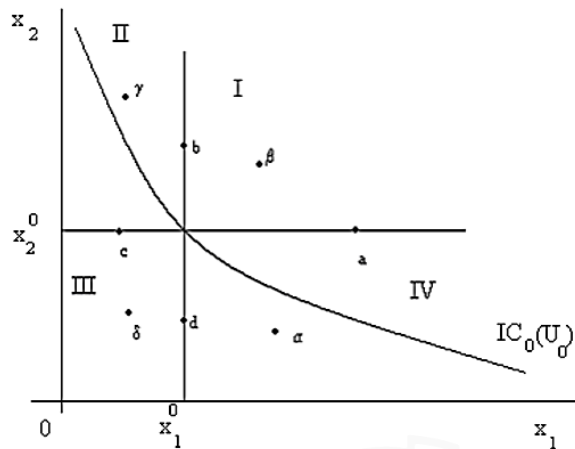


Fig.: A Typical Indifference Curve

Consider any commodity bundle denoted by point A in the above figure which consist  $x_1^0$  and  $x_2^0$  amount of good I and good II respectively and from which consumer obtains particular level of utility, say  $U_0$ . We compare the commodity bundle 'A' with other commodity bundle in the commodity space. For that we divide the entire commodity plane into four phases from 'A'.

Consider any point in phase I say  $\beta$ , where we have large quantity of both  $x_1$  and  $x_2$  compared to point 'A'. Again, if we consider any point say 'a' in horizontal line in phase I, we have larger quantity of  $x_1$  with same quantity of  $x_2$  compared to point 'A'. Similarly, for any point 'b' in vertical axis, we have larger  $x_2$  with same  $x_1$ . That means in phase I including the borderlines, we have larger quantity of at least one commodity and no less quantity of any other commodity compared to 'A'. Thus, we have larger utility in phase I including the borderlines compared to 'A'.

By similar logic, we have lower consumption of at least one good and no larger consumption of any other good in phase III including the borderlines compared to point 'A'. Hence, we have lower level of utility in phase III including the borderlines compared to 'A' by the axiom of non-satiation for all goods.

Clearly, in phases III, including borderlines, and I utility is not constant between the commodity bundles compared to point 'A'. So, indifference curve (along which utility is constant) can't pass through phases I and III including their borderlines.

Consider any point in phase IV excluding borderlines, say  $\alpha$ . We have larger  $x_1$  (for which utility is larger) and lower  $x_2$  (for which utility is lower) compared to 'A'. Since both goods are non-satiated, utility of point  $\alpha$  may be larger, lower or equal compared to point 'A'. Similarly, for any point in phase II excluding the borderlines, say  $\delta$ , we have larger consumption of  $x_2$  but lower of  $x_1$  compared to point 'A'. Therefore, by axiom of non-satiation in all goods, utility at point  $\delta$  may be larger, lower or equal compared to 'A'.

Clearly, only in phases II and IV excluding the borderlines, there is a possibility of the same level of utility between the bundles compared to point 'A'. So, indifference curve, along which utility remains unchanged, must pass through the phase II and phase IV, excluding their lines. Thus,



indifference curve is necessarily downward sloping where all goods are non-satiated given that a consumer choice is continuous, reflexive and complete.

### Mathematical Presentation

Consider the utility function  $U = U(x_1, x_2)$ . Differentiating totally, we get the following:

$dU = U_1dx_1 + U_2dx_2 = 0$  (as along the indifference curve utility is constant,  $dU = 0$ ). Therefore,

$$\frac{dx_2}{dx_1} = -\frac{U_1(x_1, x_2)}{U_2(x_1, x_2)}$$

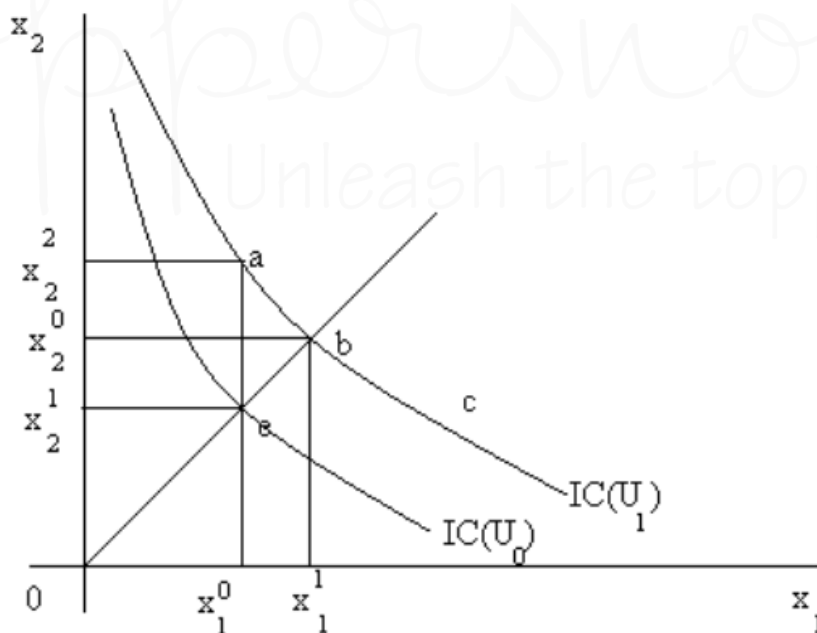
Which is the slope of the indifference curve. It is negative since  $U_1(x_1, x_2) > 0$  and  $U_2(x_1, x_2) > 0$  by assumption of non-satiation of all goods. Thus, indifference curve is downward sloping because all goods are non-satiated; choice is continuous, reflexive and complete.

### Economic meaning

All goods are non-satiated i.e., larger (lower) consumption leads to larger (lower) utility. Hence, for given  $x_2$ , as  $x_1$  increases, utility increases. Thus, to maintain same level, utility must be reduced, which is possible by reducing  $x_2$ . Hence, as  $x_1$  increases,  $x_2$  must decrease in order to maintain same level of utility. That is why indifference curve is downward sloping.

### Properties of indifference curve

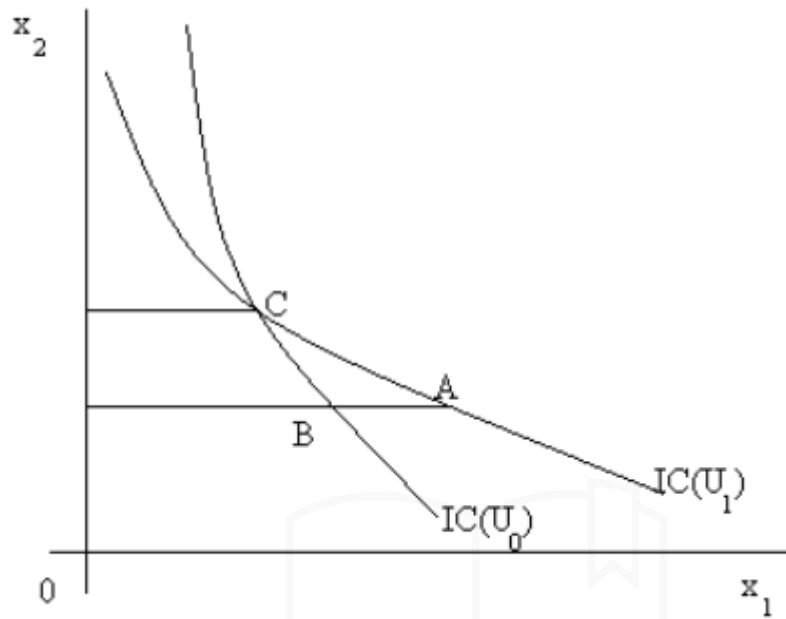
**Property I:** Higher indifference curve gives higher utility.



**Fig. : Higher Indifference Curve gives Higher Level of Utility**

**Explanation:** Since all goods are non-satiated, larger consumption of any good leads to larger utility. Thus, a commodity bundle, which consists of larger quantity of at least one good and no less consumption of any other goods, gives larger utility compared to any other commodity bundle. Consequently, higher indifference curve represents higher consumption of at least one commodity and no less consumption of any other commodity.

**Property II: Indifference curves cannot intersect with each other.**



**Fig. : Indifference Curves Can't Intersect Each Other**

**Explanation:** Suppose two indifference curves intersect each other. By definition, along the indifference curve, utility is constant. So, consumer is indifferent between points 'A' and 'C' that lie on the same indifference curve. Similarly, consumer is indifferent between points 'B' and 'C', as they also lie on the same indifference curve. So, AIC and BIC, where 'I' denotes indifference. Now, from transitivity we have AIB i.e., point 'A' and point 'B' give the same utility to the consumer. But for given  $x_2$ ,  $x_1$  is larger in point 'A' compared to point 'B'. So, by the assumption of non-satiation, we have point 'A' that gives larger utility to consumer as compared to point 'B'. This contradicts the fact that point 'A' and 'B' gives the same level of utility to the consumer (as we have proved above). Therefore, when all goods are non-satiated and transitivity holds, indifference curves can't intersect.

**Utility Maximisation**

**Graphical Presentation**

Let consider a two-commodity world,  $x_1$  and  $x_2$  representing good I and good II respectively.  $p_1$  and  $p_2$  are the prices of good I and good II respectively, where the prices are given to the consumer, i.e., prices are exogenously given and consumer can't change them. Money income of the consumer is  $M$ , which is also exogenously given to the consumer. Note that  $p_1x_1 + p_2x_2$  is the total expenditure of the consumer when she consumes  $x_1$  units of good I and  $x_2$  units of good two. The total expenditure of the consumer can't exceed her money income, therefore  $p_1x_1 + p_2x_2 \leq M$  (a)

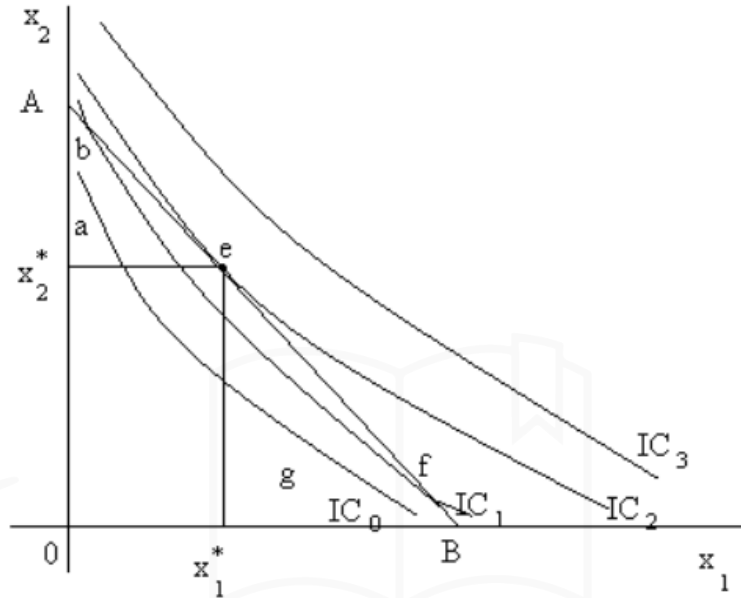
Equation (a) is known as consumer budget constraint. Let  $U = U(x_1, x_2)$  is the utility function of the consumer. Therefore, consumer must solve the following Maximisation problem (UMP):

Problem UMP:  $\text{Max } U(x_1, x_2)$

subject to  $x_1 > 0$

$x_2 > 0$

and  $p_1x_1 + p_2x_2 \leq M$



**Fig. : Derivation of Consumer Equilibrium**

As consumer objective is to maximise her utility and as larger consumption leads to larger utility, she always wants to consume more of any goods. But she also has to spend some amount of her income to consume larger amount of goods. So ultimately in equilibrium she will spend all her income and  $M = p_1x_1 + p_2x_2$ .

**Consumer Behaviour** Now suppose that the line segment AB represents the budget line. Along AB  $p_1x_1 + p_2x_2 = M$  holds. Let initial indifference curve of the consumer is  $IC_0$ . In  $IC_0$ , there are many points along that indifference curve such that  $p_1x_1 + p_2x_2 \leq M$  holds. Therefore, utility maximising consumer will spend more as she moves to higher indifference curve (say  $IC_1$ ). In  $IC_1$  there are still such points along the indifference curve such that  $p_1x_1 + p_2x_2 \leq M$  holds, so again consumer spends more. This process will continue as long as consumer reaches an indifference curve where for no point along the indifference curve  $p_1x_1 + p_2x_2 \leq M$  holds and at least one point of the indifference curve is on the budget line. At that point, we have consumer equilibrium,  $C(x_1, x_2) = (x_1^*(M, p_1, p_2), x_2^*(M, p_1, p_2))$  (in Figure 1.5.3 point 'e' is the equilibrium point). Note that at equilibrium, slope of the indifference curve is equal to the slope of the budget line. Therefore, at equilibrium we have

1. Budget constraint holds with equality sign.
2. Slope of the indifference curve is equal to the slope of the budget line.

## Mathematical Presentation

Consumer's objective is to maximise her utility by solving UMP. To solve UMP, we set the Lagrange function of the corresponding problem, which is,

$$L(x_1, x_2) = U(x_1, x_2) + \lambda (M - p_1x_1 - p_2x_2)$$

Our objective is to maximise this Lagrange function by choosing  $x_1$ ,  $x_2$  and  $\lambda$ . For that we differentiate the Lagrange function by  $x_1$ ,  $x_2$  and  $\lambda$ , and set all equal to zero.

$$\frac{dL(x_1, x_2)}{dx_1} = \frac{dU(x_1, x_2)}{dx_1} - \lambda p_1 = 0 \quad \text{----- (f}_1\text{)}$$

$$\frac{dL(x_1, x_2)}{dx_2} = \frac{dU(x_1, x_2)}{dx_2} - \lambda p_2 = 0 \quad \text{----- (f}_2\text{)}$$

$$\frac{dL(x_1, x_2)}{d\lambda} = M - p_1x_1 - p_2x_2 = 0 \quad \text{----- (f}_3\text{)}$$

From equation (f1) and (f2), we get,

$$\frac{dU(x_1, x_2)}{dx_1} / \frac{dU(x_1, x_2)}{dx_2} = p_1 / p_2. \text{ Note } \frac{dU(x_1, x_2)}{dx_1} / \frac{dU(x_1, x_2)}{dx_2} \text{ is the slope of the indifference}$$

curve and  $p_1 / p_2$  is the slope of the budget line. So, at equilibrium we have a slope of the indifference curve that is equal to the slope of the budget line. Again, from equation (f3) we get  $M = p_1x_1 + p_2x_2$ , so budget equation holds with equality sign.

## Concepts of Income and Substitution Effects

Change in demand for a good due to one unit change in price of that good for given prices and money income is known as own price effect for that good. Thus, own price effect  $= \frac{dx_1}{dp_1}$  and it consists of own substitution effect and income effect for a price change.

Own Substitution Effect: Change in demand quantity for a good (say  $x_1$ ) due to change in its own price under constant real income (in terms of utility) is called substitution effect for that good

and can be written as  $(\frac{dx_i}{dp_i})_{\bar{U}, p_j}$ .

Income Effect: Income effect for a good (say  $x_1$ ) represents change in demand quantity for that good for a change in real income. So income effect  $(\frac{dx_i}{dM})_{\bar{p}}$ , which is positive for a normal good, negative for inferior good and zero for neutral good.

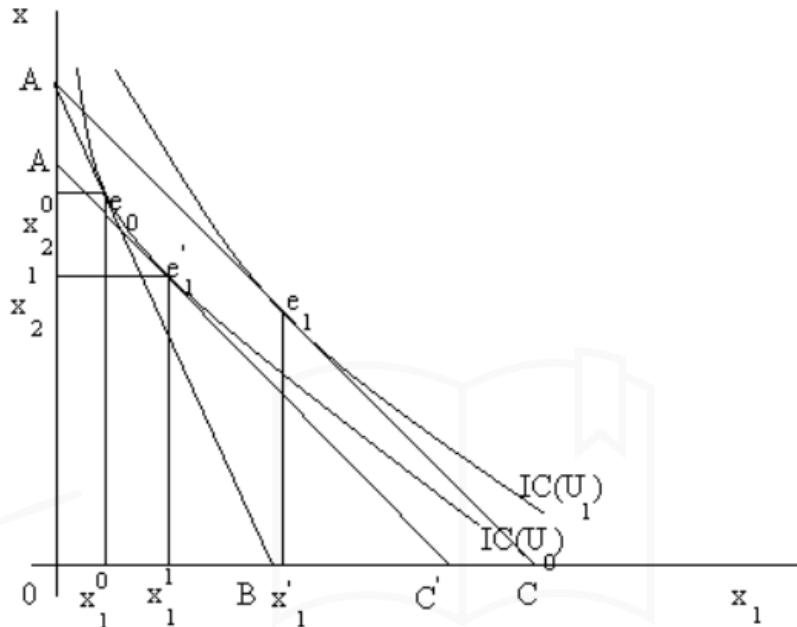
Income Effect For A Price Change: For given money income, as price of any one good change one unit then real income ( $M/p_i$ ) changes for which demand for the good changes by income effect. It is known as income effect for a price change. Thus, income effect for a price change

$= -x_i (\frac{dx_i}{dM})$ . Note that income effect and income effect for a price change have opposite sign and different magnitude.

## Slutsky's Theorem

### Graphical Presentation

We prove here that own price effect is the sum of own substitution effect and income effect for a price change, which is known as Slutsky's theorem. This is shown in the figure given below:



**Fig. : Slutsky's Theorem**

At initial prices and money income, budget line is AB and according to the condition of the equilibrium,  $e_0$  is the initial equilibrium point. The consumer gets  $U_0$  level of utility. Suppose at constant income and  $p_2$ ,  $p_1$  decreases (say by one unit). Consequently, the intercept of the budget line ( $M/p_2$ ) remains unchanged but absolute slope of the budget line ( $p_1/p_2$ ) decreases. The new budget line becomes flatter with the same intercept. It is denoted by AC line. New equilibrium can be achieved at any point on the new budget line AC (and therefore own price effect can take any algebraic sign). Suppose the equilibrium takes place at point  $e_1$ . Hence, as  $p_1$  decreases, for given  $p_2$  and  $M$ , demand for good 1 increases from  $x_{10}$  to  $x_{11}$ . This is the own price effect for  $x_1$  and here it is negative. A part of this change is due to change in real income (since for given  $p_2$  and  $M$  as  $p_1$  decreases, real income increases) and another part is originated at constant real income. To decompose these effects, we reduce money income ( $M$ ) of the consumer in such a way that real income in terms of utility remains unchanged. After such reduction of  $M$ , intercept of the new budget line AC, i.e., ( $M/p_2$ ) decreases with the same slope ( $p_1/p_2$ ) for given  $p_1$  and  $p_2$ . Hence the new budget line shifts parallelly downwards subject to the fact that after the shift, it is tangent to the previous indifference curve. The consumer can attain the same level of utility and the real income remains constant in terms of utility after adjusting money income and utility is also maximised. After adjustment of money income, budget line is  $A'C'$  along which real income in terms of utility remains constant after change in  $p_1$  for given  $p_2$ . This budget line is known as compensated budget line. Under such budget line equilibrium will necessarily take place at point  $e_1'$ . Hence under constant real income in terms of utility, as  $p_1$  decreases for given  $p_2$ ,  $x_1$  increases (from  $x_{10}$  to  $x_{11}'$ ) by substituting  $x_2$  (from  $x_{20}$  to  $x_{21}$ ). This is known as own price substitution effect for  $x_1$  which is negative and indifference curve is

downward sloping strictly convex to the origin. But as  $x_1$  increases from  $x_{10}$  to  $x_{11}$  and real income also increases, the demand for good 1 increases from  $x_{10}$  to  $x_{11}$  through a rise in real income. This would indicate that by income effect for a price change,  $x_1$  is a normal good. Clearly, we have own price effect consists of own substitution effect and income effect for a price change, where own substitution effect is negative but income effect for a price change can take any algebraical sign depending on the good is normal, superior or inferior.

### Mathematical Presentation

We already know from the first order conditions of utility Maximisation that,

$$\frac{dL(x_1, x_2)}{dx_1} = \frac{dU(x_1, x_2)}{dx_1} - \lambda p_1 = 0 \quad \text{----- (a)}$$

$$\frac{dL(x_1, x_2)}{dx_2} = \frac{dU(x_1, x_2)}{dx_2} - \lambda p_2 = 0 \quad \text{----- (b)}$$

$$\frac{dL(x_1, x_2)}{d\lambda} = M - p_1 x_1 - p_2 x_2 = 0 \quad \text{----- (c)}$$

We then totally differentiate these equations and get:

$$U_{11} dx_1 + U_{12} dx_2 - p_1 d\lambda = \lambda dp_1 \quad \text{----- (e)}$$

$$U_{21} dx_1 + U_{22} dx_2 - p_2 d\lambda = \lambda dp_2 \quad \text{----- (f)}$$

$$-p_1 dx_1 - p_2 dx_2 + 0 \cdot d\lambda = -dM + x_1 dp_1 + x_2 dp_2 \quad \text{----- (g)}$$

By using Cramer's rule we have,

$$dx_1 = \begin{pmatrix} \lambda dp_1 & U_{12} & -p_1 \\ \lambda dp_2 & U_{22} & -p_2 \\ -dM + dp_1 + dp_2 & -p_2 & 0 \end{pmatrix} / |D|$$

$$\text{where, } |D| = \begin{pmatrix} U_{11} & U_{12} & -p_1 \\ U_{21} & U_{22} & -p_2 \\ -p_1 & -p_2 & 0 \end{pmatrix} \text{ and } U_{ij} = \frac{\partial^2 U(x_i, x_j)}{\partial x_i \partial x_j}.$$

Or, we can write,

$$dx_1 = \frac{\lambda dp_1 D_{11} + \lambda dp_2 D_{21} + (-dM + x_1 dp_1 + x_2 dp_2) D_{31}}{|D|} \quad \text{----- (h)}$$

where  $D_{ij}$  is the co-factor of the  $i$ th row and  $j$ th column of the determinant  $|D|$ . For income effect we know  $dp_1 = dp_2 = 0$ , therefore we have from equation (h),

$$\frac{\partial x_1}{\partial M} = \frac{-D_{31}}{|D|}, \text{ or } \left( \frac{\partial x_1}{\partial M} \right)_P = \frac{p_2 U_{12} - p_1 U_{22}}{|D|} \quad \text{----- (i)}$$

Now for own price effect we have  $dM=dp_2=0$ . So from equation (h) we get,

$$dx_1 = \frac{\lambda D_{11} \partial p_1 + x_1 D_{31} \partial p_1}{|D|} \text{ or, } \left(\frac{dx_1}{\partial p_1}\right)_{\bar{M}, \bar{p}_2} = \frac{\lambda D_{11}}{|D|} + x_1 \frac{\lambda D_{31}}{|D|} \text{----- (j)}$$

Lastly, to find out own substitution effect we consider utility is constant in terms of income so,  $-dM+x_1 dp_1+x_2 dp_2=0$  and  $dp_2=0$ . We have from equation

$$(h), \left(\frac{dx_1}{\partial p_1}\right)_{\bar{U}, \bar{p}_2} = \frac{\lambda D_{11}}{|D|} \text{----- (k)}$$

Therefore, from equation (i), (j) and (k), we get,

$$\left(\frac{\partial x_1}{\partial p_1}\right)_{\bar{M}, \bar{p}_2} = \left(\frac{\partial x_1}{\partial p_1}\right)_{\bar{U}, \bar{p}_2} - x_1 \left(\frac{\partial x_1}{\partial M}\right)_{\bar{p}},$$

which is the Slutsky's equation.

### Compensated Demand Curve

Compensated demand function for a commodity (say  $x_1$ ) of an individual consumer represents demand quantity for that good (which is purchased by the consumer) as a function of price of that good and prices of other goods under constant real income and constant other things.

Notational, it is given by  $x_1=x_1(p_1, p_2, y)$ , where  $y$  is the real income.

Demand curve for a good showing the relationship between demand quantity for that good and its own price given other things and given real income is known as compensated demand curve along which real income is constant (real income is defined by the ratio between money income and price level). Along the demand curve price of that, good changes, so money income should be proportionately adjusted or compensated such that real income is constant. That is why the corresponding demand function and demand curve is known as compensated demand function and compensated demand curve.

There are two different approaches to the measurement of real income, viz.,

- **Hicksian Approach:** In Hicksian approach, real income is measured in forms of utility. A constant real income means a constant utility. Thus, demand quantity for a good purchased by a consumer as a function of prices of all goods under constant utility and constant other things is known as compensated Hicksian demand function.

Demand curve for a commodity showing the relationship between quantity demand for that commodity and it's own price under constant other things and constant real income in terms of utility is known as compensated Hicksian demand curve.

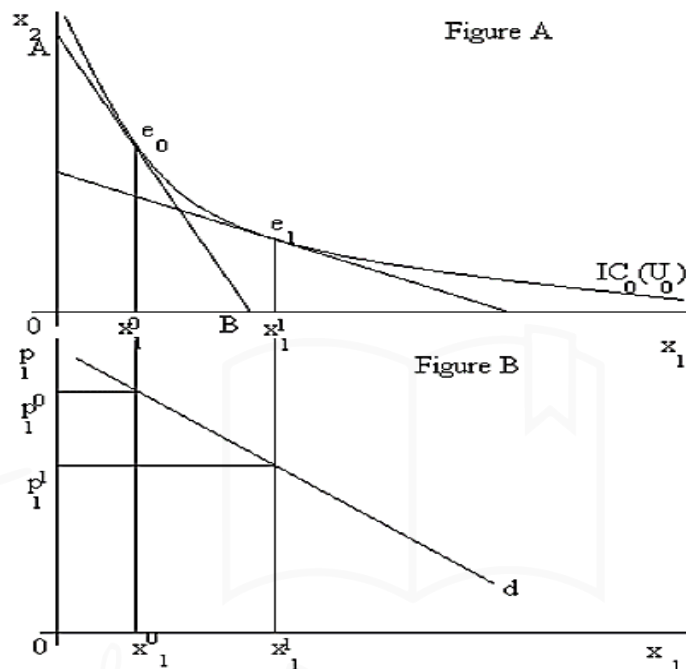
- **Slutsky's Approach:** In this approach, real income is measured in terms of purchasing power. A constant real income means a constant purchasing power (it is denoted by  $yp$ ). Demand quantity for a good purchased by a consumer as a function of prices of all goods under constant other things and constant purchasing power is known as compensated Slutsky's demand function and corresponding demand curve is known as compensated Slutsky's demand curve.



Below we discuss the Hicksian approach graphically.

### Derivation of compensated demand curve:

Hicksian compensated demand function for  $x_1$  is given by  $x_1 = x_1(p_1, p_2, U)$ , where Hicksian compensated demand curve for a good represent the relationship between price of that good with its own demand quantity for given prices of other goods and real income in terms of utility.



**Fig. : Derivation of Compensated Demand Curve**

We now derive this graphically. Suppose, initial equilibrium is attained at  $e_0$  in Figure A where price of good one is  $p_{10}$  and price of good two is  $p_{20}$  respectively and utility is fixed at  $U_0$ . Corresponding indifference curve is  $IC_0(U_0)$ . Compensated Hicksian demand for  $x_1$  is at  $x_{10}$ . Expenditure line is  $AB$  at initial equilibrium with absolute slope  $p_{10}/p_{20}$ .

Plot this  $x_{10}$  and  $p_{10}$  in Figure B. Suppose, for given utility and  $p_2$ ,  $p_1$  decreases to  $p_{11}$ . Therefore, absolute slope of the budget line decreases, i.e., expenditure line become flatter. Since utility is constant, the indifference curve remains the same as before. Therefore, expenditure is minimised for given utility at point  $e_1$  in Figure A, as indifference curve is downward sloping strictly convex to the origin. So compensated Hicksian demand for good 1 increases to  $x_{11}$  plot  $p_{11}$  and  $x_{11}$  in Figure B. By joining all such pair of  $p_1$  and  $x_1$  in Figure B, we have a downward sloping curve in  $p_1$ - $x_1$  plane, for given  $p_2$  and utility. This downward sloping demand curve is the Hicksian compensated demand curve. This is shown in the above Figure B.

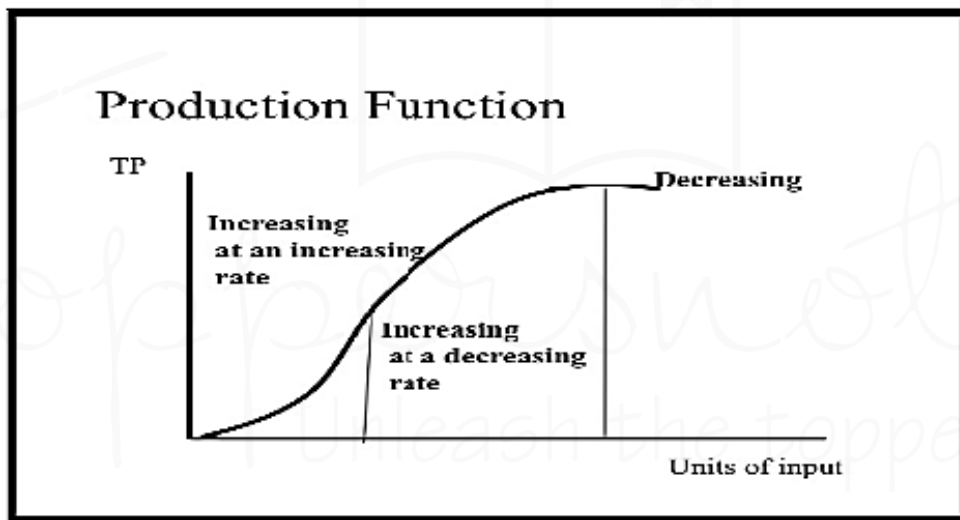


## Theory of Cost and Production Analysis

### Production Function:

In economics, a production function relates physical output of a production process to physical inputs or factors of production. It is a mathematical function that relates the maximum amount of output that can be obtained from a given number of inputs - generally capital and labor. The production function, therefore, describes a boundary or frontier representing the limit of output obtainable from each feasible combination of inputs.

Firms use the production function to determine how much output they should produce given the price of a good, and what combination of inputs they should use to produce given the price of capital and labor. When firms are deciding how much to produce they typically find that at high levels of production, their marginal costs begin increasing. This is also known as diminishing returns to scale - increasing the quantity of inputs creates a less-than-proportional increase in the quantity of output. If it weren't for diminishing returns to scale, supply could expand without limits without increasing the price of a good.



**Figure: Production function**

### Iso quant's:

An iso quant (equal quantity) is a curve that shows the combinations of certain inputs such as Labor (L) and Capital (K) that will produce a certain output Q. Mathematically, the data that an iso quant projects is expressed by the equation

$$f(K,L) = Q$$

This equation basically says that the output that this firm produces is a function of Labor and Capital, where each iso quant represents a fixed output produced with different combinations of inputs. A new iso quant emerges for every level of output.

The Marginal Rate of Technical Substitution (MRTS) equals the absolute value of the slope. The MRTS tells us how much of one input a firm can sacrifice while still maintaining a certain output level. The MRTS is also equal to the ratio of Marginal Productivity of Labor (MPL): Marginal

Productivity of Capital (MPK). The mathematical form of how Labor (L) can be substituted for Capital (K) in production is given by:

$$\text{MRTS (L for K)} = -dK/dL = MP_L/MP_K$$

### **Iso costs:**

An iso cost line (equal-cost line) is a Total Cost of production line that recognizes all combinations of two resources that a firm can use, given the Total Cost (TC). Moving up or down the line shows the rate at which one input could be substituted for another in the input market. For the case of Labor and Capital, the total cost of production would take on the form:

$$TC = (WL) + (RK)$$

TC= Total Cost, W=Wage, L= Labor, R= Cost of Capital, K= Capital

Example:

A company producing widgets encounters the following costs- cost of capital is \$25000, labor cost is \$15000, and the total cost the firm is willing to pay is \$150,000. Show the iso cost line graphically.

The equation represented by the data is:  $150,000 = (15000)L + (25000)K$

Setting  $L=0$ , we find the y-intercept to be  $K=6$ . Setting  $K=0$ , we find the x-intercept to be 10

### **MRTS OR MRS: Marginal Rate of Technical Substitution:**

The principle of marginal rate of technical substitution (MRTS or MRS) is based on the production function where two factors can be substituted in variable proportions in such a way as to produce a constant level of output.

#### **Prof. Salvatore defines MRTS thus:**

“The marginal rate of technical substitution is the amount of an input that a firm can give up by increasing the amount of the other input by one unit and still remain on the same iso quant.”

The marginal rate of technical substitution between two factors K (capital) and L (labour),  $MRTS_{LK}$  is the rate at which L can be substituted for K in the production of good X without changing the quantity of output. As we move along an iso quant downward to the right, each point on it represents the substitution of labour for capital.

MRTS is the loss of certain units of capital which will just be compensated for by additional units of labour at that point. In other words, the marginal rate of technical substitution of labour for capital is the slope or gradient of the iso quant at a point. Accordingly, the slope of  $MRTS_{LK} = -\Delta K/\Delta L$ . This can be understood with the aid of the iso quant schedule.