



UP - TGT

प्रशिक्षित स्नातक शिक्षक

उत्तर प्रदेश माध्यमिक शिक्षा सेवा चयन बोर्ड

गणित

भाग - 2



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बीजगणितीय तादात्म्या (ALGEBRAIC IDENTITIES)

□ महत्वपूर्ण नियम →

$$\textcircled{1} \quad (a+b)^2 = a^2 + b^2 + 2ab$$

$$\textcircled{2} \quad (a-b)^2 = a^2 + b^2 - 2ab$$

$$\textcircled{3} \quad (a+b)^2 = (a-b)^2 + 4ab$$

$$\textcircled{4} \quad (a-b)^2 = (a+b)^2 - 4ab$$

$$\textcircled{5} \quad a^2 - b^2 = (a+b)(a-b)$$

$$\textcircled{6} \quad (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$\textcircled{7} \quad (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$\textcircled{8} \quad (a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$

$$\textcircled{9} \quad (a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

$$\textcircled{10} \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$\textcircled{11} \quad a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= \frac{1}{2} (a+b+c) [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

जोि त $a^3 + b^3 + c^3 - 3abc = 0$

If $\left\{ \begin{array}{l} \text{(i)} \quad a+b+c = 0 \quad (a \neq b \neq c) \\ \text{(ii)} \quad a^2 + b^2 + c^2 - ab - bc - ca = 0 \quad \text{i.e. } a = b = c \end{array} \right\}$

$$\textcircled{12} \quad a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\textcircled{13} \quad \text{If } ax^2 + bx + c = 0$$

then, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

मुद्द महत्वपूर्ण नियम \rightarrow

$$\textcircled{14} \quad \text{यदि } x + \frac{1}{x} = a \quad \text{तो}$$

$$(i) \quad x^2 + \frac{1}{x^2} = a^2 - 2$$

$$(ii) \quad x^4 + \frac{1}{x^4} = b^2 - 2 \quad \text{तो } a^2 - 2$$

$$\textcircled{15} \quad \text{यदि } x - \frac{1}{x} = a \quad \text{तो}$$

$$(i) \quad x^2 + \frac{1}{x^2} = a^2 + 2$$

$$(ii) \quad x^4 + \frac{1}{x^4} = b^2 - 2, \text{ जहाँ } b = a^2 + 2$$

$$\textcircled{16} \quad x^4 - \frac{1}{x^4} = a, \quad \text{तब}$$

$$(i) \quad x^2 + \frac{1}{x^2} = \sqrt{a+2} = b \quad (ii) \quad x + \frac{1}{x} = \sqrt{b+2} \quad (iii) \quad x - \frac{1}{x} = \sqrt{b-2}$$

$$\textcircled{17} \text{ यदि } x + \frac{1}{x} = 2 \text{ तो } x = 1$$

$$\textcircled{18} \text{ यदि } x + \frac{1}{x} = -2 \text{ तो } x = -1$$

$$\textcircled{19} \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$\textcircled{20} \quad x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

$$\textcircled{21} \text{ यदि } x + \frac{1}{x} = -1 \text{ तो } x^3 = 1$$

$$\textcircled{22} \text{ यदि } x + \frac{1}{x} = +1 \text{ तो } x^3 = 1$$

$$\textcircled{23} \text{ यदि } x + \frac{1}{x} = 13 \text{ तो } x^3 + \frac{1}{x^3} = 0, x^6 = -1 \quad \boxed{\text{पर } (x^6 + 1) = 0}$$

$$\textcircled{24} \text{ यदि } \begin{aligned} ax + by &= m \\ bx - ay &= n \end{aligned} \quad \text{तो}$$

$$(a^2 + b^2)(x^2 + y^2) = m^2 + n^2$$

Note - यदि वस्तविक संख्याओं के वर्गों का योग 0 है तो वे सभी संख्याएँ 0 की होंगी। i.e.

$$\text{यदि } (x-a)^2 + (y-b)^2 + (z-c)^2 = 0, \text{ तो}$$

$$x-a = 0 \Rightarrow x = a$$

$$y-b = 0 \Rightarrow y = b$$

$$z-c = 0 \Rightarrow z = c$$

$$\text{यदि } x^2 + y^2 + z^2 = 0, \text{ तो } x=0, y=0 \text{ और } z=0$$

Exercise

① यदि $a = -5, b = 6, c = 10$, तो -

$$\frac{a^3 + b^3 + c^3 - 3abc}{(ab + bc + ca - a^2 - b^2 - c^2)}$$

का मान क्योगा?

$$(ab + bc + ca - a^2 - b^2 - c^2)$$

[ans = 1]

② यदि $x - \frac{1}{x} = 6$ तो $x^2 + \frac{1}{x^2}$ का मान क्योगा? [ans = 38]

③ यदि $x^2 + \frac{1}{x^2} = 51$ तो $x - \frac{1}{x}$ का मान क्योगा? [ans = ±7]

④ $x - \frac{3}{x}$ का मान ज्ञात कीजिये यदि $x + \frac{3}{x} = 4$ [ans = ±2]

⑤ $x - \frac{1}{x} = 6$, तो $x^3 - \frac{1}{x^3}$ का मान क्योगा? [ans = 234]

⑥. $x + \frac{1}{x} = 2$, तो $x^3 + \frac{1}{x^3}$ का मान क्योगा? [ans = 2]

⑦ $x + \frac{1}{x} = 2$ तो $x^{17} + \frac{1}{x^{17}}$ का मान क्योगा? [ans = 2]

⑧ यदि $x+y+z=15, xy+yz+zx=85$, तो $x^2+y^2+z^2$ का मान क्योगा? [ans = 55]

⑨ $x^2+y^2+z^2=576$ और $xy+yz+zx=50$, तो $x+y+z$ का मान ज्ञात करें? [ans = ±26]

⑩ यदि $5x - \frac{5}{x} = 10$ तो $x^2 + \frac{1}{x^2}$ का मान ज्ञात करें?

[ans = 6]

11) यदि $x = \frac{a^2 - 25}{a^2 - 36}$ और $y = \frac{a+5}{a+6}$, तो $\frac{x}{y}$ का मान ज्ञात करें ?

$$\boxed{\text{ans} = \frac{a-5}{a-6}}$$

12) यदि $x = a^{2/3} - a^{-2/3}$ तो $x^3 + 3x$ का मान ज्ञात करें ?

$$\boxed{\text{ans} = a^2 - \frac{1}{a^2}}$$

13) $\frac{x^2 - y^2 - 2yz - z^2}{x^2 + y^2 + 2xy - z^2} = ?$

$$\boxed{\text{ans} = \frac{x-y-z}{x+y-z}}$$

14) यदि $x = 27$ और $\sqrt[3]{x} + \sqrt[3]{y} = \sqrt[3]{729}$, तो y का मान ज्ञात करें ?

$$\boxed{\text{ans} = 216}$$

15) यदि $(a-b) = 3$, $(b-c) = 5$ और $(c-a) = 1$

तो $\frac{a^3 + b^3 + c^3 - 3abc}{a+b+c}$ का मान जीत करें ?

$$\boxed{\text{ans} = 17.5}$$

16) यदि $a+b = 5$, $a^2 + b^2 = 13$ तो $a-b$ का मान क्योंगा

(जबकि $a > b$)

$$\boxed{\text{ans} = 1}$$

17) यदि $a + \frac{1}{a+2} = 0$, तो $(a+2)^3 + \frac{1}{(a+2)^3}$ का मान क्योंगा ?

$$\boxed{\text{ans} = 2}$$

18) यदि $a = 25$, $b = 15$, $c = -10$ हैं

$\frac{a^3 + b^3 + c^3 - 3abc}{(a-b)^2 + (b-c)^2 + (c-a)^2}$ का मान क्योंगा ?

$$\boxed{\text{ans} = 15}$$

19) यदि $x + \frac{1}{16x} = 1$, तो $64x^3 + \frac{1}{64x^3}$ का मान ज्ञात कीजिये ?

Ans = 52

20) यदि $x + \frac{1}{x} = 6$, तो $\frac{3x}{2x^2 + 2 - 5x}$ का मान ज्ञात कीजिये ?

Ans = 3

21) यदि $\frac{4p}{p^2 - 4p + 4} = \frac{1}{8}$, $p \neq 0$, तो $p + \frac{4}{p}$ का मान क्या होगा ?

Ans = 36

22) यदि $x^2 - 3x + 6 = 0$, तो $x^2 + \frac{56}{x^2}$ का मान बताइये ?

Ans = -3

23) यदि $4x^2 + y^2 + 4z^2 + 3 = 4z + 2y - 4x$ तो $x + y + z$ का मान ज्ञात कीजिये ?

Ans = 1

24) $\frac{3-5x}{x} + \frac{3-5y}{y} + \frac{3-5z}{z} = 0$, तो $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ का मान क्या होगा ?

Ans = 5

25) यदि $x^2 - 3x + 1 = 0$ और $x > 1$, तो $(x - \frac{1}{x})$ का मान क्या होगा ?

Ans = $\pm\sqrt{5}$

26) यदि $x = 2 + \sqrt{3}$, तो $\frac{x^6 + x^4 + x^2 + 1}{x^3}$ का मान ज्ञात कीजिये ?

Ans = 56

27) यदि $x^4 + \frac{1}{x^4} = 23$, तो $(x - \frac{1}{x})^2$ का मान ज्ञात कीजिये ?

Ans = 3

⑧ यदि x वास्तविक संख्या है और $x^4 + \frac{1}{x^4} = 119$ हो तो

$\left(x + \frac{1}{x}\right)$ का मान ज्ञात कीजिये?

Ans = ± 3

⑨ यदि $a = 3 + 2\sqrt{2}$, तो $\frac{a^6 + a^4 + a^2 + 1}{a^3}$ का मान ज्ञात कीजिये?

Ans = 204

⑩ यदि $n^2 + 3n + 1 = 0$, तो $n^3 + \frac{1}{n^3}$ का मान ज्ञात कीजिये?

Ans = -10

Solution

$$\textcircled{1} \quad \frac{a^3 + b^3 + c^3 - 3abc}{-(a^2 + b^2 + c^2 - ab - bc - ca)} = \frac{(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)}{-(a^2 + b^2 + c^2 - ab - bc - ca)}$$

$$= - (a+b+c)$$

$$= - (-5 - 6 + 10)$$

$$= - (-1) = \boxed{1}$$

$$\textcircled{2} \quad x - \frac{1}{x} = 6$$

$$\therefore x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 = 6^2 + 2 = 36 + 2 = \boxed{38}$$

$$\textcircled{3} \quad x^2 + \frac{1}{x^2} = 51$$

$$\therefore x - \frac{1}{x} = \sqrt{\left(x^2 + \frac{1}{x^2}\right) - 2} = \sqrt{51 - 2} = \sqrt{49} = \boxed{\pm 7}$$

$$\textcircled{4} \quad \left(x - \frac{3}{n}\right)^2 = \left(x + \frac{3}{n}\right)^2 - 4 \times n \times \frac{3}{n}$$

$$\left(x - \frac{3}{n}\right)^2 = 4^2 - 12 = 16 - 12 = \boxed{4}$$

$$\left(\frac{x-3}{n}\right)^2 = 4 \Rightarrow \boxed{\left(\frac{x-3}{n}\right) = \pm 2}$$

$$\textcircled{5} \quad x^3 - \frac{1}{n^3} = \left(x - \frac{1}{n}\right)^3 + 3\left(x - \frac{1}{n}\right)$$

$$= 6^3 + 3 \cdot 6$$

$$= 216 + 18$$

$$= \boxed{234}$$

⑥ जैसा कि हम जानते हैं, यदि $x + \frac{1}{x} = 2 \Rightarrow x = 1$ जो कि इसे सिर्फ़ दरता है।

$$\therefore n^3 + \frac{1}{n^3} = 1+1 = \boxed{2}$$

Alternatively -

$$\begin{aligned} \therefore n^3 + \frac{1}{n^3} &= \left(n + \frac{1}{n}\right)^3 - 3\left(n + \frac{1}{n}\right) \\ &= 2^3 - 3 \times 2 \\ &\Rightarrow 8 - 6 = \boxed{2} \end{aligned}$$

⑦ $x + \frac{1}{x} = 2 \Rightarrow x = 1$ (इसे सिर्फ़ दरता है)

$$\therefore x^17 + \frac{1}{x^{17}} = 1+1 = \boxed{2}$$

⑧ $x+y+z = 15$

$$\Rightarrow (x+y+z)^2 = (15)^2$$

$$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 225$$

$$\Rightarrow x^2 + y^2 + z^2 = 225 - 2 \times 85 = 225 - 170 = \boxed{55}$$

⑨ $(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$$= 576 + 2(50) = 676$$

$$\therefore x+y+z = \pm \sqrt{676} = \boxed{\pm 26}$$

⑩ $5x - \frac{5}{x} = 10, x - \frac{1}{x} = 2$

$$(x - \frac{1}{x})^2 = 22 \Rightarrow x^2 + \frac{1}{x^2} - 2 = 4, \boxed{x^2 + \frac{1}{x^2} = 6}$$

$$⑪ \frac{y}{x} = \frac{\left(\frac{a^2 - 25}{a^2 - 36}\right)}{\left(\frac{a+5}{a+6}\right)} = \frac{\frac{(a+5)(a-5)}{(a+6)(a-6)}}{\frac{(a+5)}{(a+6)}} = \boxed{\frac{a-5}{a-6}}$$

$$⑫ x = a^{2/3} - a^{-2/3}$$

दोषी पक्षी का घन करने पर -

$$x^3 = (a^{2/3} - a^{-2/3})^2$$

$$n^3 = (a^{2/3})^3 - (a^{-2/3})^3 - 3 \cdot a^{2/3} \cdot a^{-2/3} (a^{2/3} - a^{-2/3})$$

$$\Rightarrow n^3 = a^{-2} - b^2 - 3 \times 1 (x)$$

$$\Rightarrow n^3 + 3x = a^2 - a^{-2} = a^2 - \frac{1}{a^2}$$

$$\boxed{Ans = a^2 - \frac{1}{a^2}}$$

$$⑬ \text{ दिया गया व्यंजक} - \frac{x^2 - (y^2 + 2yz + z^2)}{(y^2 + yz + 2xy) - z^2} =$$

$$= \frac{n^2 - (y+z)^2}{(n+y)^2 - z^2} = \frac{[x + (y+z)] [x - (y+z)]}{(n+y+z) (n+y-z)}$$

$$= \boxed{\frac{x-y-z}{n+y-z}}$$

$$⑯ \quad x = 27$$

$$\sqrt[3]{x} + \sqrt[3]{y} = \sqrt[3]{729} \Rightarrow 3 + \sqrt[3]{y} = 9$$

$$\Rightarrow \sqrt[3]{y} = 9 - 3 = 6 \quad \therefore y = (6)^3 = \boxed{216}$$

$$⑯ \quad \frac{a^3 + b^3 + c^3 - 3abc}{a+b+c}$$

$$= \frac{1}{2} (a+b+c) \left[(a-b)^2 + (b-c)^2 + (c-a)^2 \right]$$

$$\therefore = \frac{1}{2} \left[(a-b)^2 + (b-c)^2 + (c-a)^2 \right]$$

$$= \frac{1}{2} [9 + 25 + 1] = \frac{35}{2} = \boxed{17.5}$$

$$⑯ \quad (a+b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow 25 = 13 + 2ab$$

$$\Rightarrow 2ab = 25 - 13 = 12$$

$$\Rightarrow \therefore (a-b)^2 = a^2 + b^2 - 2ab = 13 - 12 = 1$$

$$\Rightarrow \boxed{ab = 1}$$

$$⑯ \quad a + \frac{1}{a+2} = 0 \Rightarrow a+2 + \frac{1}{a+2} - 2 = 0$$

विनाशकी पर

$$\Rightarrow \left(a+2 + \frac{1}{a+2} \right)^3 = (2)^3$$

$$\Rightarrow (a+2)^3 + \frac{1}{(a+2)^3} + 3(a+2) \times \frac{1}{(a+2)} \cdot \left(a+2 + \frac{1}{a+2} \right) = 8$$

$$\Rightarrow (a+2)^3 + \frac{1}{(a+2)^3} + 3 \times 2 = 8$$

$$\Rightarrow \cancel{(a+2)^3} + \frac{1}{(a+2)^3} = 8 - 6 = 2 \quad \boxed{\text{Ans=2}}$$

$$\begin{aligned}
 18) \quad & a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \\
 & = \frac{1}{2} (a+b+c) [(a-b)^2 + (b-c)^2 + (c-a)^2] \\
 \text{लेखक} = & \frac{\frac{1}{2} (a+b+c) [(a-b)^2 + (b-c)^2 + (c-a)^2]}{(ab)^2 + (b-c)^2 + (c-a)^2} \\
 & = \frac{1}{2} (a+b+c) = \frac{1}{2} [25+15-10] = \boxed{15}
 \end{aligned}$$

$$19) \quad x + \frac{1}{16x} = 1$$

दोनों तरफ 4 से गुणा करने पर-

$$4x + \frac{1}{4x} = 4$$

$$\begin{aligned}
 \text{अब } & 64n^3 + \frac{1}{64x^3} = (4n)^3 + \frac{1}{(4x)^3} \\
 & = (4x + \frac{1}{4x})^3 - 3(4x + \frac{1}{4x}) \\
 & = 4^3 - 3 \times 4 \\
 & = 64 - 12 = \boxed{52}
 \end{aligned}$$

$$20) \quad \text{दिया शुभा, } x + \frac{1}{x} = 6,$$

$$\begin{aligned}
 \Rightarrow \frac{3x}{2x^2 + 2 - 5x} &= \frac{3x}{x \left[\left(2x + \frac{2}{x} \right) - 5 \right]} \\
 &= \frac{3}{2 \left[x + \frac{1}{x} \right] - 5} = \frac{3}{2 \times 6 - 5} = \boxed{\frac{3}{7}}
 \end{aligned}$$

21) दिया गया व्यंजक - $\frac{p^2 - 4p + 4}{4p} = 0$

$$\Rightarrow \frac{p^2 - 4p + 4}{p} = 32$$

$$\Rightarrow \frac{p^2}{p} - \frac{4p}{p} + \frac{4}{p} = 32$$

$$\Rightarrow p - 4 + \frac{4}{p} = 32$$

$$\Rightarrow p + \frac{4}{p} = \boxed{36}$$

22) $x^2 - 3x + 6 = 0 \Rightarrow x^2 + 6 = 3x$
दोनों पक्षों की x की आगे करने पर

$$x + \frac{6}{x} = 3$$

दोनों पक्षों का वर्ग करने पर-

$$\Rightarrow \frac{x^2 + 36}{x^2} = 9 + 2x \cdot \frac{6}{x} = 9.$$

$$\Rightarrow x^2 + \frac{36}{x^2} = 9 - 12$$

$$\Rightarrow x^2 + \frac{36}{x^2} = -3$$

$$\boxed{\text{ans} = -3}$$

23) दिया गया व्यंजक - $4x^2 + y^2 + 4z^2 + 3 - 4z + 2y + 4x = 0$

$$\Rightarrow (2x+1)^2 + (y-1)^2 + (2z-1)^2 = 0$$

$$\Rightarrow 2x+1 = 0 \Rightarrow x = -\frac{1}{2}$$

$$y-1=0 \Rightarrow y=1$$

$$x+y+z = -\frac{1}{2} + 1 + \frac{1}{2} = \boxed{1}$$

$$2z-1=0 \Rightarrow \boxed{z=\frac{1}{2}}$$

$$\begin{aligned}
 & \textcircled{24} \cdot \frac{3-5x}{5} + \frac{3-5y}{y} + \frac{3-5z}{z} = 0 \\
 \Rightarrow & \frac{3}{x} - \frac{5x}{x} + \frac{3}{y} - \frac{5y}{y} + \frac{3}{z} - \frac{5z}{z} = 0 \\
 \Rightarrow & \frac{3}{x} + \frac{3}{y} + \frac{3}{z} - 15 = 0 \\
 \Rightarrow & 3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 15 \\
 \Rightarrow & \boxed{\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 5}
 \end{aligned}$$

$$\begin{aligned}
 & \textcircled{25} \quad x^2 - 3x + 1 = 0 \\
 \Rightarrow & x(x-3+\frac{1}{x}) = 0 \quad \Rightarrow \quad x + \frac{1}{x} = 3 \\
 \therefore & \left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4 \\
 & = 9 - 4 = 5 \\
 \therefore & \boxed{x - \frac{1}{x} = \pm \sqrt{5}}
 \end{aligned}$$

$$\begin{aligned}
 & \textcircled{26} \quad \because x = 2 + \sqrt{3} \\
 \therefore & \frac{1}{x} = \frac{1}{2 + \sqrt{3}} = \frac{2 - \sqrt{3}}{(2 + \sqrt{3})(2 - \sqrt{3})} = \frac{2 - \sqrt{3}}{4 - 3} \\
 & = 2 - \sqrt{3} \quad \therefore x + \frac{1}{x} = 4 \\
 \text{लेख} & = \frac{x^6 + x^4 + x^2 + 1}{x^3} \\
 & = x^3 + x + \frac{1}{x} + \frac{1}{x^3} = \left(x^3 + \frac{1}{x^3}\right) + \left(x + \frac{1}{x}\right) \\
 & = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) + \left(x + \frac{1}{x}\right) = 4^3 - 3 \times 4 + 4 = 64 - 12 + 4 = \boxed{56}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{27} \quad x^4 + \frac{1}{x^4} = 23 &\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 23 \\
 &\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 23 + 2 = 25 \\
 \therefore x^2 + \frac{1}{x^2} &= 5 \quad \therefore \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \\
 &= 5 - 2 = \boxed{3}
 \end{aligned}$$

$$\textcircled{28} \quad x^4 + \frac{1}{x^4} = 119 = a \quad \text{माना}$$

$$\begin{aligned}
 \therefore x^2 + \frac{1}{x^2} &= \sqrt{a+2} = \sqrt{119+2} = \sqrt{121} \\
 &= \pm 11 = b \quad \text{माना} \\
 \therefore x - \frac{1}{x} &= \pm \sqrt{b-2} \\
 &= \pm \sqrt{11-2} = \pm \sqrt{9} \\
 &= \boxed{\pm 3}
 \end{aligned}$$

$$\textcircled{29} \quad a = 3 + \sqrt{2}, \quad \frac{1}{a} = 3 - 2\sqrt{2}$$

$$[\because (3)^2 - (2\sqrt{2})^2 = 1]$$

$$\begin{aligned}
 \therefore a + \frac{1}{a} &= 6 \\
 \text{अब, } \frac{a^6 + a^4 + a^2 + 1}{a^3} &= \frac{a^6}{a^3} + \frac{a^4}{a^3} + \frac{a^2}{a^3} + \frac{1}{a^3} \\
 &= \left(a^3 + \frac{1}{a^3}\right) \left(a + \frac{1}{a}\right) \\
 &= \left(a + \frac{1}{a}\right)^3 - 3ax\frac{1}{a} \left(a + \frac{1}{a}\right) + \left(a + \frac{1}{a}\right) \\
 &= (6)^3 - 3 \times 6 + 6 \\
 &= 216 - 18 + 6 \\
 &= \boxed{204}
 \end{aligned}$$