Toppersnotes
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UGC-NET

Paper - 2

NATIONAL TESTING AGENCY (NTA)

ELECTRONIC SCIENCE

Paper 2 – Volume 3

SIERRA INNOVATIONS

Index

Unit – 3 (2)

Unit – 3 (2)

Chapter-01
Signal defination & Classifications

Signal → A signal is a f^n which contains some information.

System → A sys. is interconnection of devices (or) componants which converts signal from one form to another form.

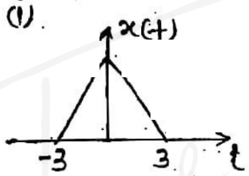
Classification of signals →

1) Even & odd signals →

i) **Even** → This are symmetrical (or) mirror image about y-axis.

i.e. → $\boxed{x(t) = x(-t)}$ → time reversal

Eg:- (i)



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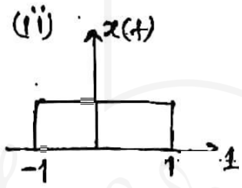

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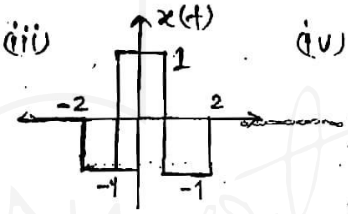


(iv) $x(t) = \cos\omega_0 t$ (even)

$t = -t$

$x(-t) = \cos\omega_0(-t)$

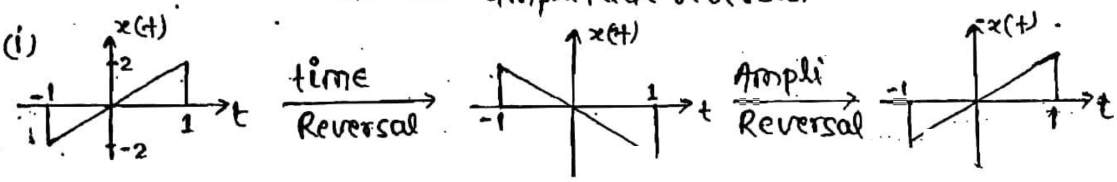
$= \cos\omega_0 t$

$x(-t) = x(t)$

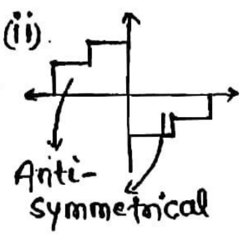
* **Odd** → This are antisymmetrical about y-axis.

i.e. $\boxed{\begin{array}{c} x(-t) = -x(t) \\ \text{(or)} \\ x(t) = -x(-t) \end{array}}$ — time reversal; amplitude reversal

Eg:- (i)



(ii)



(iii) $x(t) = \sin\omega_0 t$ → odd signal.

$(t = -t)$

$x(-t) = \sin\omega_0(-t)$

$x(-t) = -\sin\omega_0 t$

$\boxed{x(-t) = -x(t)}$

* The avg. value of an odd signal is 0; but converse of this statement is not true.

Important points →

Important points →

(1.) Even × Even = Even ; $t^2 \times t^4 = t^6$

(2.) Even × odd = odd ; $t^2 \times t^3 = t^5$

(3.) Odd × odd = Even ; $t^3 \times t^5 = t^8$

(4.) Even ± Even = Even

$x(t) = t^2 + \cos t$

$x(-t) = t^2 + \cos t = x(t)$

(5.) Odd + Odd = Odd

$x(t) = \sin t + t^3$

$x(-t) = -\sin t - t^3$

$\boxed{x(-t) = -x(t)}$

(6.) Even + odd = Neither even nor odd.

$x(t) = t^2 + \sin t$

$x(-t) = t^2 - \sin t$

$\boxed{x(-t) \neq x(t)}$

* Any signal can be deuided into 2 part in which one part will be even & the other part will be odd.

i.e. $\boxed{x(t) = x_E(t) + x_O(t)}$

where;

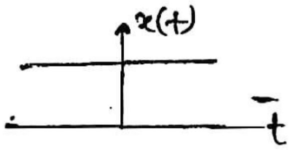
$x_E(t)$ = even part of $x(t)$ = $\dfrac{x(t)+x(-t)}{2}$

$x_O(t)$ = Odd part of $x(t)$ = $\dfrac{x(t)-x(-t)}{2}$

Eg. → $x(t) = 2$ = dc signal

$\downarrow t = -t$

$x(-t) = 2 = x(t)$ [Even signal]



dc signal is a Even signal.

(2.) $f(k) = \sin(k^2)$

$\downarrow k = -k$

$f(-k) = \sin(k^2) = f(k)$ [Even signal]

(3.) $f(r) = \sin \pi/2$

$= 1$

$f(r) = f(-r)$ [Even signal]

(4.) Find $x_E(t)$ & $x_O(t)$ of the signal.

$$x(t) = 3 - \frac{t^2}{\sin t} + \frac{\cos t}{t} - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t$$

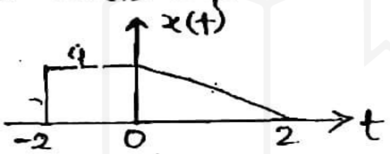
$$E - \frac{E}{O} + \frac{E}{O} - \frac{E}{E} + O \times O$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

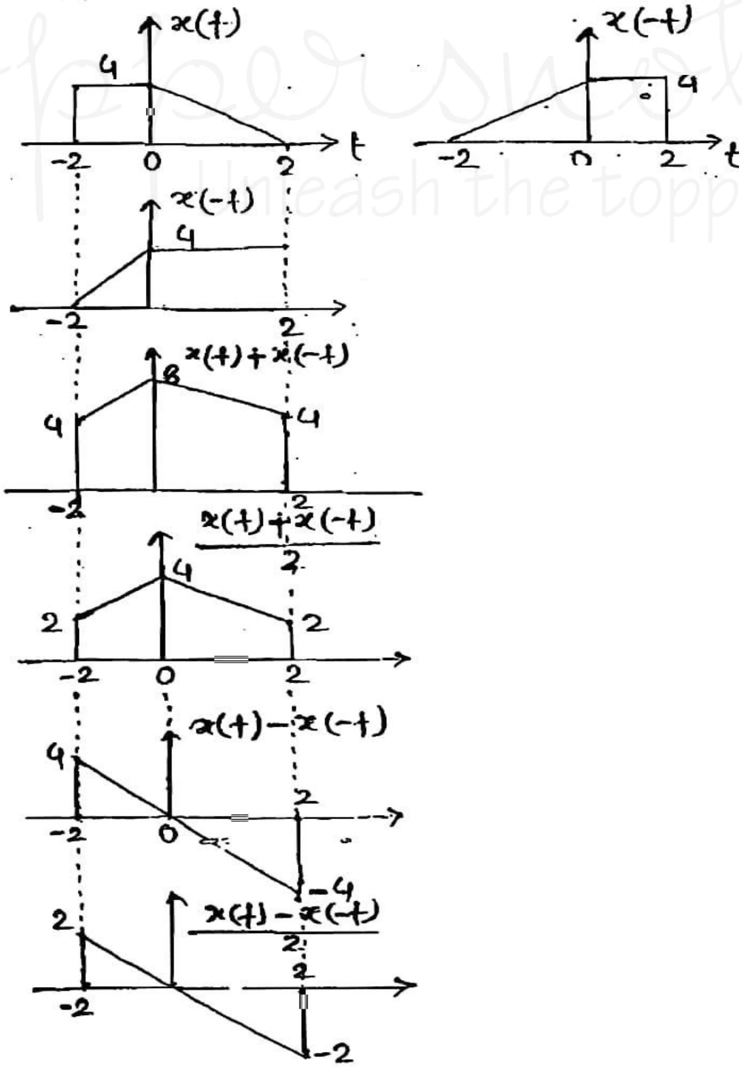
$$E \quad O \quad O \quad E \quad E$$

$$x_E(t) = 3 - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t \quad , \quad x_O(t) = \frac{-t^2}{\sin t} + \frac{\cos t}{t}$$

Que. → Draw $x_E(t)$ & $x_O(t)$ of:



Soln → for even part of x(t)



(2) Conjugate Symmetric (CS) & Conjugate antisymmetric (CAS) signal →

★ Conjugate symmetric (CS)

$$\boxed{x(t) = x^*(-t)}$$

$$x(t) = a(t) + jb(t) \text{ ——— (i)}$$

$$(t = -t)$$

$$x(-t) = a(-t) + jb(-t)$$

$$x^*(-t) = a(-t) - jb(-t) \text{ ——— (ii)}$$

From eq^n (i) & (ii)

$$a(t) = a(-t) \longrightarrow \text{Even}$$

$$b(t) = -b(-t) \longrightarrow \text{Odd}$$

Eg:- $x(t) = \underset{E}{\underset{\downarrow}{t^2}} + \underset{O}{\underset{\downarrow}{\sin t}}$

★ Conjugate antisymmetric (CAS)

$$\boxed{x(t) = -x^*(-t)}$$

$$x(t) = a(t) + j\,b(t)$$

$$a(t) = -a(-t) \longrightarrow \text{Odd}$$

$$b(t) = b(-t) \longrightarrow \text{Even}$$

Eg:- $x(t) = \underset{O}{\underset{\downarrow}{\sin t}} + j\underset{E}{\underset{\downarrow}{t^2}}$

(3) Halfwave Symmetric signal (HWS) →

For Half-wave symmetry (HWS)

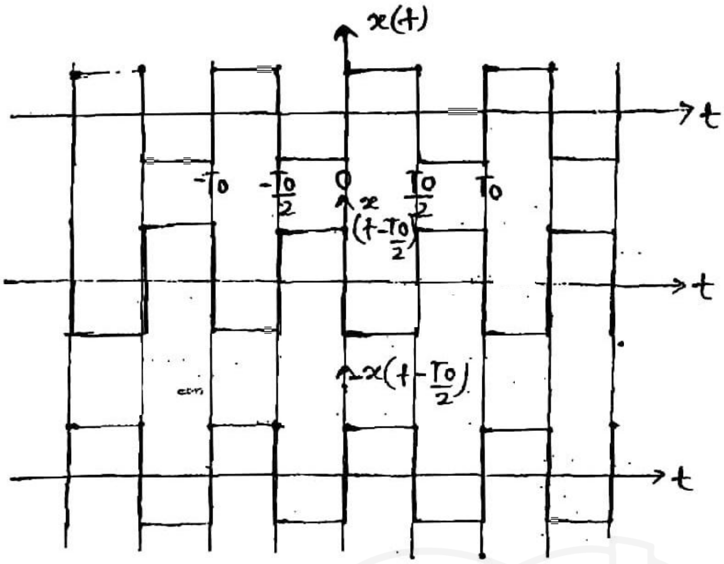
$$\boxed{x(t) = -x\left(t \pm \frac{T_0}{2}\right)}$$

time shifting

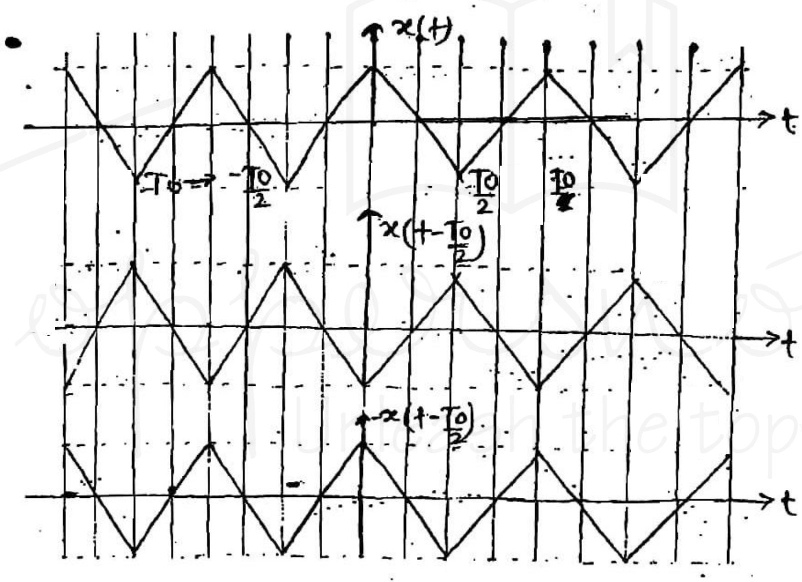
amp. reversal

Eg →

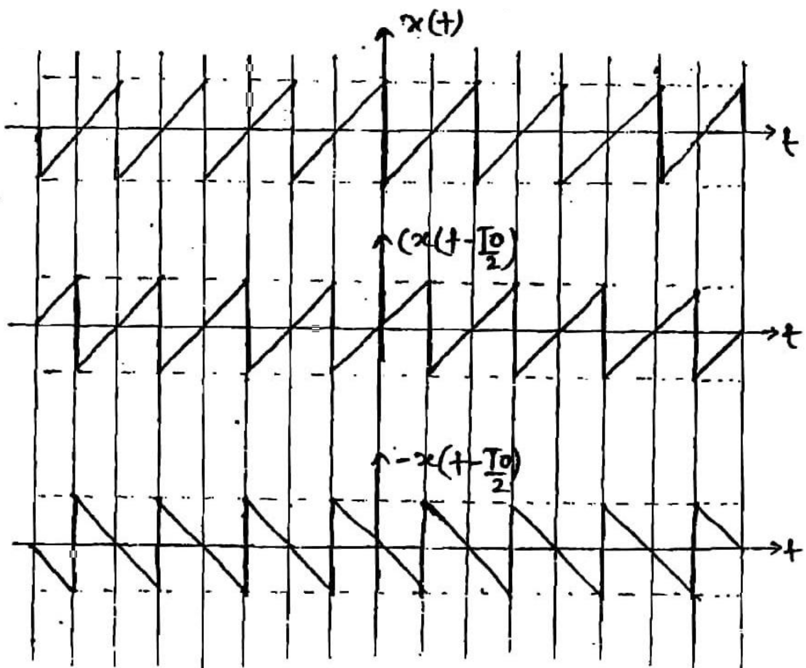
(1.)



(2.)



(3.)



so; sawtooth wave doesn't follow the HWS.

* The avg. value of a HWS is 0. but converse of this statement is not true.

(4.) Periodic & non-periodic signal →

Periodic → A signal repeats itself after some time period, the signal is said to be periodic.

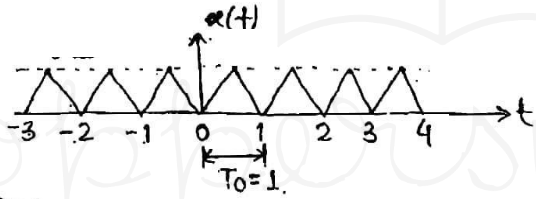
i.e. $$\boxed{x(t) = x(t \pm nT_0)}$$

where, n = an integer

T_0 = Fundamental time period. $\begin{Bmatrix} T_0 \neq 0 \\ T_0 \neq \infty \end{Bmatrix}$

FTP → t is the smallest, +ve & fixed value of the time for which signal is periodic.

eg →



FTP=1

Q → Find FTP of signal $x(t)$

$$x(t) = A_0 e^{j\omega_0 t}$$

Sol^n → Let 'T_0' be the FTP of the signal

i.e.

$$x(t) = x(t+T_0)$$

$$x(t+T_0) = A_0 e^{j\omega_0 (t+T_0)}$$

$$A_0 e^{j\omega_0 t} = A_0 e^{j\omega_0 (t+T_0)}$$

$$\cancel{A_0 e^{j\omega_0 t}} = \cancel{A_0 e^{j\omega_0 t}} e^{j\omega_0 T_0}$$

$$e^{j\omega_0 T_0} = 1 = e^{j2\pi K} \quad \text{(where K = an integer}$$

$$j\omega_0 T_0 = j2\pi K$$

$$\underset{\text{(smallest)}}{T_0} = \frac{2\pi \cancel{K}}{\omega_0} \quad 1 \text{ (least integer)}$$

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

Q.→ Find FTP of following signal →

(i) $x_1(t) = A_0 \sin(2\pi t)$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{2\pi} = 1$

(ii) $x_2(t) = A_0 \sin(2\pi t + 30^\circ)$

$\omega_0 = 2\pi$

$T_0 = 1$

(iii) $x_3(t) = -x_1(t)$

$= -A_0 \sin(2\pi t)$

$\omega_0 = 2\pi$, $T_0 = 1$

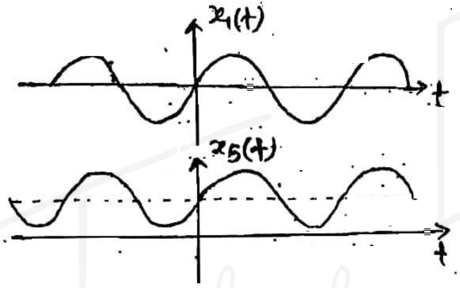
(iv) $x_4(t) = x_1(-t)$

$= -A_0 \sin 2\pi t$

$\omega_0 = 2\pi$, $T_0 = 2\pi$

(v) $x_5(t) = A_0 + x_1(t)$

$= A_0 + A_0 \sin(2\pi t)$



(vi) $x_6(t) = x_1(t - t_0)$

$= A_0 \sin[2\pi(t - t_0)]$

$\omega_0 = 2\pi$

$T_0 = 1$

* Time period of signal is unaffected by time shifting, time reversal, amp. reversal, amp. shifting & change in phase of signal.

(vii) $f(t) = \sin^2(4\pi t)$

$= \frac{1 - \cos 8\pi t}{2}$

$\omega_0 = 8\pi$

$T_0 = \frac{2\pi}{8\pi} = \frac{1}{4}$

* The sum of 2 (or) more than 2 periodic signal will be periodic if ratios of there fundamental time period (or) freq. are rational.

i.e. $x(t) = x_1(t) + x_2(t)$

$x_1(t) \to T_1, f_1, \omega_1$; $x_2(t) \to T_2, f_2, \omega_2$

→ $\frac{T_1}{T_2}$ (or) $\frac{\omega_1}{\omega_2}$ (or) $\frac{f_1}{f_2}$ (Rational no.)

→ $T_0 = LCM[T_1, T_2]$

→ $f_0 = HCF[f_1, f_2]$

Q.→ Find FTP of signal if it is periodic :-

(i) $x(t) = \sin 2t + \cos 3\pi t$

Sol^n→ $\omega_1 = 2$ $\quad \frac{\omega_1}{\omega_2} = \frac{2}{3\pi}$ (Irrational no)

$\omega_2 = 3\pi$

Hence it is non-periodic

(ii) $x(t) = \sin 2\pi t + \cos\sqrt{2}\pi t$

Sol^n→ $\omega_1 = 2\pi, \quad \omega_2 = \sqrt{2}\pi$

$$\frac{\omega_1}{\omega_2} = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2} \text{ (Irrational no.)}$$

Hence it is Non-periodic

(iii) $x(t) = \sin 4\pi t + \sin 7\pi t$

Sol^n→ $\omega_1 = 4\pi, \quad \omega_2 = 7\pi$

$$\frac{\omega_1}{\omega_2} = \frac{4\pi}{7\pi} = \frac{4}{7} \text{ (Rational no.)}$$

Hence it is periodic. Then calculate T_0.

1st method :-

$$\omega_0 = \cancel{2\pi}\ HCF[\omega_1, \omega_2] = HCF[4\pi, 7\pi]$$

$$\omega_0 = \pi$$

$$T_0 = \frac{2\pi}{\omega_0} = 2$$

*** $HCF\left[\frac{P_1}{q_1}, \frac{P_2}{q_2}\right] = \frac{HCF[P_1, P_2]}{LCM[q_1, q_2]}$ $\quad LCM\left[\frac{P_1}{q_1}, \frac{P_2}{q_2}\right] = \frac{LCM[P_1, P_2]}{HCF[q_1, q_2]}$

2nd method →

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi} = \frac{1}{2} \qquad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7\pi} = \frac{2}{7}$$

$$T_0 = LCM[T_1, T_2] = LCM\left[\frac{1}{2}, \frac{2}{7}\right]$$

$$= \frac{LCM[1,2]}{HCF[2,7]} = \frac{2}{1} = 2$$

* Area & avg. value of signal →

Area of $x(t)$:-

$$\text{Area} = \int_{-\infty}^{\infty} x(\tau)\,d\tau$$

Area of $x(t)$ over Range (t_1, t_2)

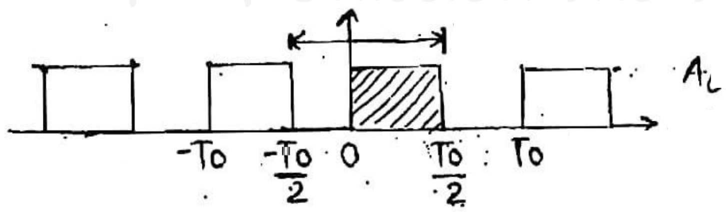
$$\text{Area} = \int_{t_1}^{t_2} x(\tau)\,d\tau$$

Avg. value of $x(t)$:

$$\text{Avg.} = \begin{cases} \dfrac{1}{T_0} \displaystyle\int_{-T_0/2}^{T_0/2} x(\tau)\,d\tau, & \text{For periodic sig.} \\ \displaystyle\lim_{T \to \infty} \dfrac{1}{T} \int_{-T/2}^{T/2} x(\tau)\,d\tau, & \text{for Non-periodic sig.} \end{cases}$$

Que.→ Find the avg. value of sig.

(i)



sol^n →

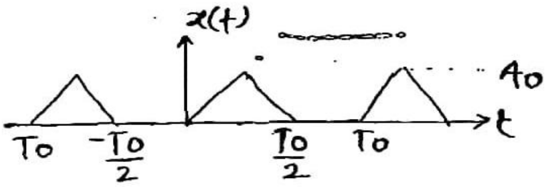
$$\text{avg.} = \frac{\displaystyle\int_{-T_0/2}^{T_0/2} x(\tau)\,d\tau}{T_0}$$

$$= \frac{\text{Area of } x(t) \text{ over '}T_0\text{'}}{T_0}$$

$$= \frac{A_0 \times \frac{T_0}{2}}{T_0}$$

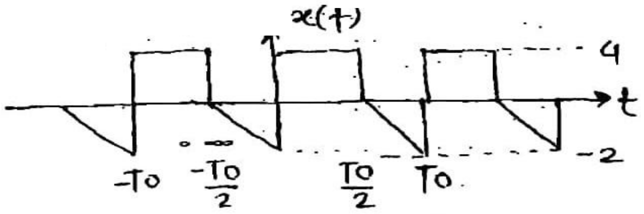
$$= \frac{A_0}{2}$$

(2.)



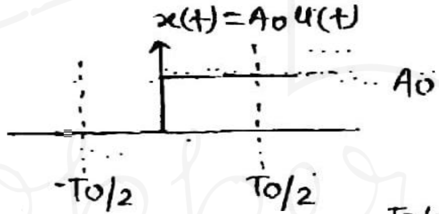
Sol^n → $\text{avg} = \dfrac{\text{Area over To}}{\text{To}} = \dfrac{1/2 \times A_0 \times \cancel{T_0}/2}{\cancel{T_0}} = \dfrac{A_0}{4}$

(3.)



Sol^n → $\text{Avg} = \dfrac{\text{Area over To}}{\text{To}} = \dfrac{-1/2 \times \frac{T_0}{2} \times 2 + 4 \times \frac{T_0}{2}}{T_0} = \dfrac{3}{2}$

(iv)



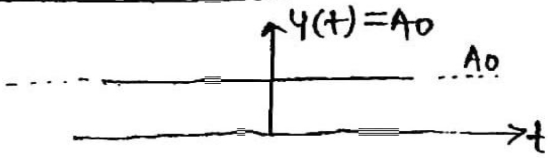
Sol^n → $\text{Avg} = \lim_{T_0 \to \infty} \dfrac{1}{T_0} \displaystyle\int_{-T_0/2}^{T_0/2} x(z)\,dz$

$= \lim_{T_0 \to \infty} \dfrac{1}{T_0} \displaystyle\int_{0}^{T_0/2} A_0\, dz$

$= \lim_{T_0 \to \infty} \dfrac{A_0 \times T_0/2}{T_0}$

$= \dfrac{A_0}{2}$

2nd method →



$\text{avg } y(t) = A_0$

$\text{avg } x(t) = \dfrac{\text{avg } y(t)}{2}$

$= \dfrac{A_0}{2}$

(5.) Energy & power signal →

* Energy of $x(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$\boxed{E = \int_{-\infty}^{\infty} |x(t)|^2 dt}$$

* Power of $x(t)$

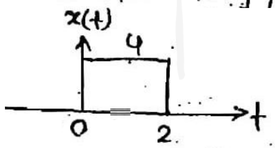
$$P = \begin{cases} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & \text{For periodic sig.} \\ \lim_{T_0\to\infty} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt; & \text{Non periodic sig.} \end{cases}$$

* For an energy sig., energy should be finite & power should be zero.
* Energy signals are absoulately integrable signal.

i.e. $\boxed{\int_{-\infty}^{\infty} |x(t)|\, dt < \infty}$

Q.→ Calculate energy of sig.

(i)



Sol^n→ $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

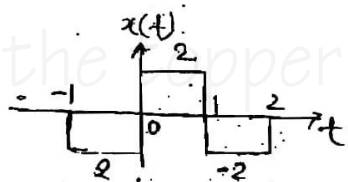
$= \int_0^2 4^2 dt = 32$

2nd method →

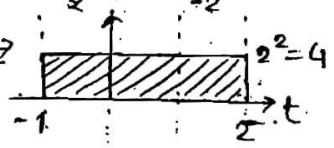
$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= 16 \times 2 = 32$

(ii)



Sol^n→

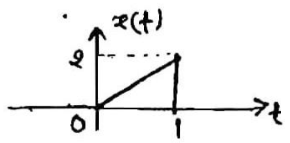


$E_{x(t)}$ = area of $|x(t)|^2$

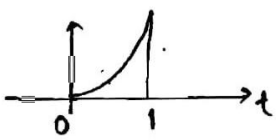
$= 4 \times 3$

$= 12$

(iii)



Sol^n→

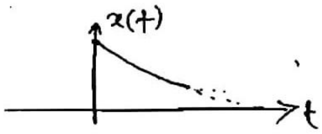


$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_0^1 (2t)^2 dt = \frac{4}{3}$

Q.→ Cal. area & energy of signal:-

(i) $x(t) = e^{-at}u(t)$, $a>0$



Sol^n→

$$\text{area} = \int_{-\infty}^{\infty} x(t)\,dt$$

$$= \int_{0}^{\infty} e^{-at}\,dt$$

$$= \left(\frac{e^{-at}}{-a}\right)_0^{\infty} = \frac{e^{-a\infty} - e^{0}}{-a}$$

$\because e^{-a\infty} = 0$, $a>0$ $(a=2)$

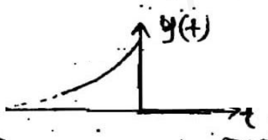
$e^{-2\infty} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$$= \frac{0-1}{-a} = \frac{1}{a}$$

$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

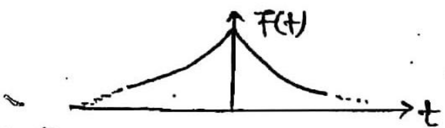
$$= \int_{0}^{\infty} e^{-2at}\,dt = \left(\frac{e^{-2at}}{-2a}\right)_0^{\infty} = \frac{e^{-2a\infty} - e^{0}}{-2a} = \frac{1}{2a}$$

(ii) $y(t) = x(-t) = e^{at}u(-t)$, $a>0$



Sol^n→ $\text{Area} = \frac{1}{a}$, $\text{Energy} = \frac{1}{2a}$

(iii) $f(t) = x(t) + y(t) = e^{-a|t|}$, $a>0$



Sol^n→

$f(t) = e^{-a|t|}$, $a>0$

$$= \begin{cases} e^{at}, & t<0 \\ e^{-at}, & t>0 \end{cases}$$

* $|t| = \begin{cases} -t, & t<0 \\ t, & t>0 \end{cases}$

$$\text{area} = \frac{1}{a} + \frac{1}{a} = \frac{2}{a}$$

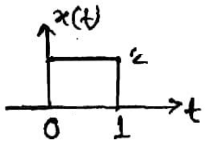
Energy = 1 . 1 . 1

Q.→ $x(t) \longrightarrow E$

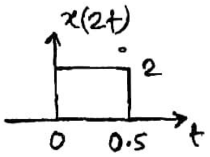
$x(2t) \longrightarrow ?$

(a) $\frac{E}{4}$ (b) $\frac{E}{2}$ (c) $2E$ (d) E

Soln→



$E \to 4$



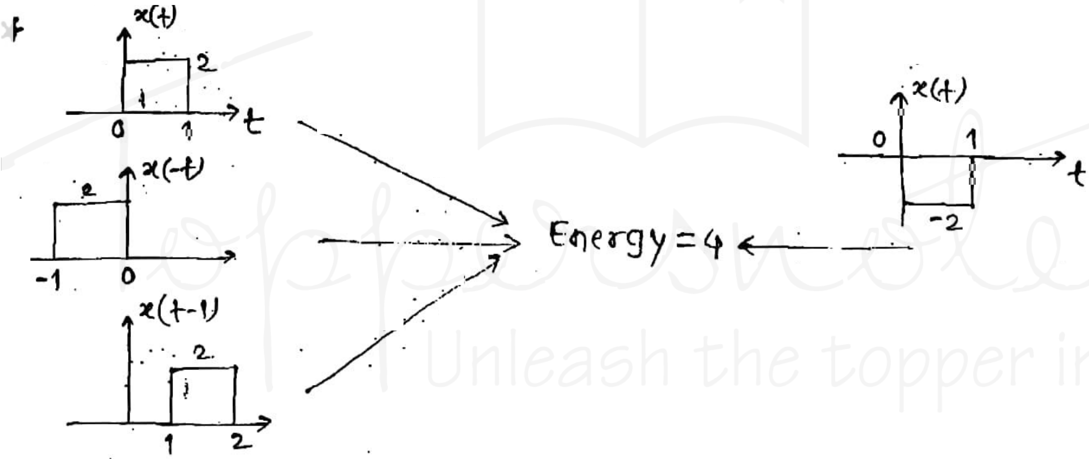
$E \to 2 = \frac{E}{2}$

$x(t) \longrightarrow E$

$x(2t) \longrightarrow \frac{E}{2}$

$x(-2t) \longrightarrow \frac{E}{2}$

*** $\boxed{x(at)_{a\neq 0} \longrightarrow \frac{E}{|a|}}$



* Energy of signal is independent of amp. reversal, time reversal, time shifting.

*** Power Signal →** * For this signal power should be finite & energy should be ∞.

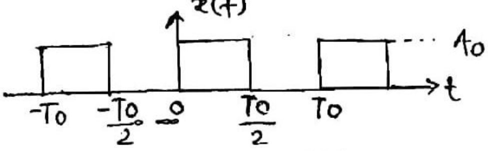
* Periodic power signals are absolutely integrable over their time period.

i.e. $\boxed{\int_{T_0} |x(t)|\,dt < \infty}$ periodic power sig.

$$P = \begin{cases} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2\,dt & ; \text{ for periodic signal.} \\ \lim\limits_{T_0\to\infty} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2\,dt & ; \text{ for Non-periodic} \end{cases}$$

Q→ Calculate power of signal :-

(i.)



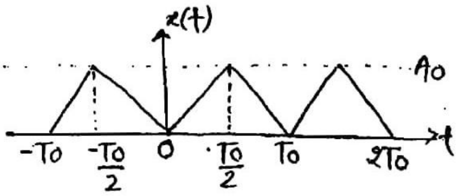
Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{0}^{T_0/2} A_0^2 dt$$

$$\boxed{P = \frac{A_0^2}{2}}$$

(ii)



Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} |x(t)|^2 dt$$

$$x(t) = mt = \left(\frac{2A_0}{T_0}\right)t \quad \left(\because m = \frac{A_0}{T_0/2}\right)$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} \left(\frac{2A_0}{T_0}\right)^2 t^2 dt$$

$$= \frac{8 \times A_0^2}{T_0^3}\int_{0}^{T_0/2} t^2 dt$$

$$= \frac{8A_0^2}{\cancel{T_0^3}} \times \frac{\cancel{T_0^3}}{\cancel{8} \times 3}$$

$$\boxed{P = \frac{A_0^2}{3}}$$

(iii) $x(t) = A_0 \sin\omega_0 t$

Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A_0 \sin\omega_0 t)^2 dt$$

$$P = \frac{A_0^2}{T_0}\int_{-T_0/2}^{T_0/2} \left(\frac{1-\cos 2\omega_0 t}{2}\right) dt$$

$$P = \frac{2A_0^2}{2T_0}\int_{0}^{T_0/2} (1-\cos 2\omega_0 t)\, dt$$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \left(\frac{\sin 2\omega_0 t}{2\omega_0}\right)\Bigg|_0^{\frac{T_0}{2}}\right]$$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \frac{\sin \cancel{2}\omega_0 \frac{T_0}{\cancel{2}}}{2\omega_0}\right]$$

$$(\because \omega_0 T_0 = 2\pi) \qquad = \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \frac{\cancel{\sin\omega_0 T_0}^{\,0}}{2\omega_0}\right]$$

$$= \frac{A_0^2}{T_0} \times \frac{T_0}{2}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

∵ RMS of the Given signal is $\frac{A_0}{\sqrt{2}}$

★★★ $\boxed{RMS^2 = \frac{A_0^2}{2} = P}$

* Power is also known as mean square value of signal.

Q.→ Calculate power of signal

(i.) $x_1(t) = A_0 \sin \omega_0 t$

(ii) $x_2(t) = x_1(t-t_0) = A_0 \sin[\omega_0(t-t_0)]$

(iii) $x_3(t) = x_1(2t) = A_0 \sin 2\omega_0 t$

(iv) $x_4(t) = A_0 \sin(\omega_0 t + \phi)$

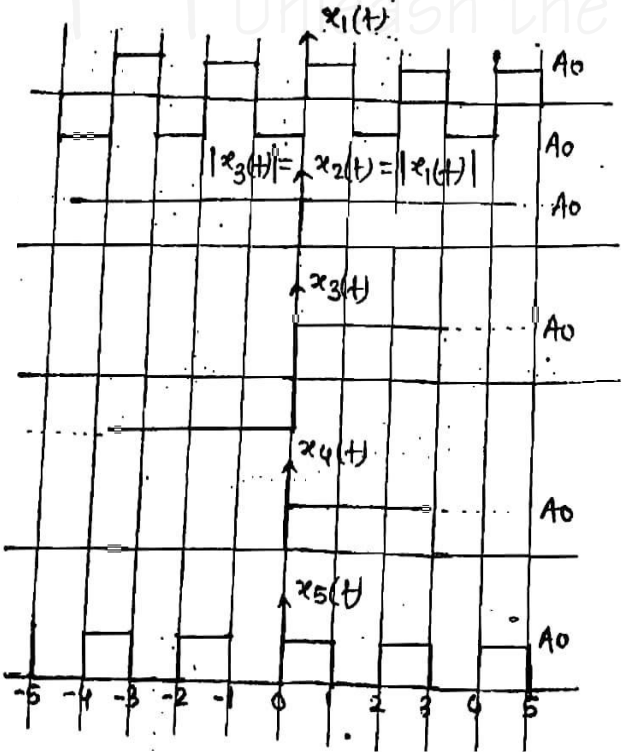
Soln→ For above all signals

$$Rms = \frac{A_0}{\sqrt{2}}$$

$$power = \frac{A_0^2}{2}$$

* Power calculation is independant of time shifting, time scaling, change in freq. (or) time period & change in phase of signals.

Q.→



Power	Avg.
$P_1 = A_0^2$	0
$P_2 = A_0^2$	A_0
$P_3 = A_0^2$	0
$P_4 = \frac{P_2}{2} = \frac{A_0^2}{2}$	$\frac{A_0}{2}$
$P_5 = \frac{P_2}{2} = \frac{A_0^2}{2}$	$\frac{A_0}{2}$