Toppersnotes
Unleash the topper in you

UGC-NET

Computer Science & Application

NATIONAL TESTING AGENCY (NTA)

PAPER – 2 || VOLUME – 1

DISCRETE STRUCTURES AND OPTIMIZATION,
COMPUTER SYSTEM ARCHITECTURE,
PROGRAMMING LANGUAGES AND COMPUTER GRAPHICS

SIERRA INNOVATIONS

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Unit 1: DISCRETE STRUCTURES AND OPTIMIZATION

Unit 2: COMPUTER SYSTEM ARCHITECTURE

Unit 3: PROGRAMMING LANGUAGES AND COMPUTER GRAPHICS

Mathematical Logic

Propositional Equivalences –

logical exp are equivalent if they have same truth value in all cases.

there are three types of propositions –

1. Tautology (Always True) $p \vee \sim p$
2. Contradiction (Always False) $p \wedge - p$
3. Contigency (Not 1, not 2) $p \vee q$

<u>logical equivalence</u> –

PYQ → if $p \leftrightarrow q$ is tautology that p & q are logical equivalent.

Imp <u>Notation</u> – $p \equiv q$

~~or By using~~ truth table.

logical equivalence use only conjunction, disjunction & negation.

Normal Forms –

1. Conjunctive NF. POS (Product of Sum)
2. Disjunctive NF. SOP

1. <u>CNF</u> – obtained by operating AND among variables, or obtained by intersection among variable connected with unions.

   eg $(A \vee B) \wedge (A \vee C)$

2. <u>DNF</u> – OR operation

   & union of intersected variables

   $(A \wedge B) \cup (A \wedge C)$

Predicates & Quantifiers -

- Predicates is an expression of one or more variables defined on some specific domain.

- Predicate with variables = proposition

eg let $E(x,y)$ denote $x = y$

let $X(a,b,c)$ denote $a+b+c=0$

Well formed formula (WFF) @ previous notes

- Quantifiers - variables of predicate is quantified by it.

1) Universal Quantifier. symbol $\forall$
2) Existential Quantifier. symbol $\exists$

1) UQ stmt within its scope are true for every value of specific variable.

2) EQ stmt within its scope are true for some values of specific variable.

Nested Quantifiers -

appear within scope of another quantifier.

Rules of inference – or transformation rule
अनुमान

a logical form consisting of a function which takes premises, analyzes their syntex & return conclusion.

1) Modus Ponens – or law of detachment
Modus Tallens
Disjunction additions
Conjuctive specification
Conjuctive addition
Dis... syllogism
Hypothetic syllogism
Proof by division into cases
Rule of contradiction

Sets & Relations

Set Operations –
include Union ($\cup$), Intersection ($\cap$), Disjoint
Set difference ($-$), Complement (a')

Properties of union & intersection of sets;

1) Associative Property –
$$A \cup (B \cup C) = (A \cup B) \cup C$$

2) Commutative Property –
$$A \cup B = B \cup A \quad \text{or} \quad A \cap B = B \cap A$$

3) Identity Property for union –
$$A \cup \phi = A$$

4) Intersection Property for Empty set –
$$A \cap \phi = \phi$$

5) Distributive Property –
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup B)$$

Relations - can be represented using direct graph

- No of vertices in a graph is equal to no of element in the set

Types
- Empty Relation (ϕ)
- Full Relation ($X \times Y$)
- Identity Relation $((x, x) \mid x \in X)$
- Reflective $\forall a \in A \quad a = a$
- Irreflexive $a \in A$
- Symmetric Relation $a = b$, $b = a$
- Anti - Symmetric Relation
- Transitive Relation $a = b$ & $b = c$ ie $a = c$
- Equivalence Relation

Equivalence Relation -
- is a binary relation ie. reflexive, symmetric & transitive.
- Two elements of given set are equivalent to each other if & only if they belong to same equivalence class.

Partially ordering - (PO)
- a relation R on set A is called PO if it is ~~reflexive~~, anti - symmetric & ~~transitive~~.
- A set A together with a partial ordering R is called a partialy ordered set or poset.

Reflexive Relation - (Closure) (ie. Relation with itself)
eg (1,1), (2,2) should be there.

Symmetric Relation - ie if (2,3) is there, (3,2) also needed. | eg (4,2) so (2,4) also needed.

Counting, Mathematical Introduction

Basics of counting

it has two basic rules

1) Sum Rule (Disjunction Rule)
2) Product Rule (Sequential Rule)

Sum Rule - A & B are disjoint ie $A \cap B = \phi$ then

$$|A \cup B| = |A| + |B|$$

Product Rule -

$$|A \times B| = |A| \times |B|$$

$|A|$ will denote no. of elements in an empty set.

Both rules are used to decompose difficult counting problems into simple one.

Pigeonhole Principle -

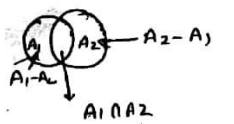
there are 10 pigeons & 9 pigeon holes when pigeons fly to home then one of them have to stay 2 in one hole.

Permutation & Combination - Done

Inclusion - Exclusion Principle -

states that A_1 & A_2 - finite sets ie subset of universalse

so, $(A_1 - A_2)$, $(A_2 - A_1)$ & $(A_1 \cap A_2)$ are disjoint



$$\text{so, } |(A_1 - A_2) \cup (A_2 - A_1) \cup (A_1 \cap A_2)| = |A_1| - |A_1 \cap A_2| + |A_2| - |A_1 \cap A_2| + |A_1 \cap A_2|$$

ie union of set is given by,

sum of sizes of all single set - sum of all 2 set + sum of all 3 set - sum of all 4 set & so on

Permutation and Combination

* in combination, order doesn't matter
* in permutation, order does matter
* a permutation is an ordered combination
* There are two types of permutation :

    (i) Repetition is allowed

    (ii) Non Repetition

⇒ Permutation with Repetition –

– when a thing has n different types, we have n choices each time

eg¹ choosing 3 things

$= \boxed{n \times n \times n}$

eg² we have 10 numbers & we have to select only three of them.

$n \times n \times \_\_\_ (r \text{ times}) = \boxed{n^r}$

$= 10 \times 10 \times \text{---} \text{(3 times)}$

$= 10^3 = 1000$ permutations

⇒ Without Repetition –

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

eg What is the permutation of 4?

sol $4 \times 4 \times 4 \times 4 = \boxed{256}$

Q How many three letter words with or without meaning can be formed out of the letters of the word SWING when repetition of letters is not allowed?

sol here $n = 5$ (∵ SWING has 5 letters)

we have to frame 3 letter words (r)

so Permitation $P(n, r) = \frac{5!}{(5-3)!}$

$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \boxed{60}$

Q How many 3 letter words with or without meaning can be formed out of letters of word SMOKE when $\boxed{\text{repetition is allowed}}$?

sol SMOKE has 5 alphabets

so $n = 5$

& we have to arrang in 3 form

Permutation (when repetition is allowed)

$$5^3 = 5 \times 5 \times 5 = \boxed{125}$$

Q In how many ways 6 children can be arranged in a line, such that

(i) Two particular children of them are always together

(ii) Two particular children of them are never together sol (i) 2 students need to be together, here we can consider them 1.

Thus the remaining 7 gives the arrangement in 5! ways

ie $5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ —— (1)

also, two children in a line can be arranged in 2! ways —— (2)

Hence, the total no. of arrangements

$120 \times 2 = \boxed{240 \text{ ways}}$

(ii) Total no. of arrangements of 6 children will be 6!

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$ ways —— (1)

two children together can be arranged in 240 ways —— (2)

∴ Two particular children are never together will be

$720 - 240 = \boxed{480 \text{ ways}}$

Q It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Sol 5 men and 4 women

ie. total 9 positions

Four places can be occupied by 4 women in $P(4,4)$ ways $= 4!$

$= 4 \times 3 \times 2 \times 1$

$= 24$ ways

Remaining 5 positions can be occupied by 5 men ie $5!$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ ways

$\therefore$ Total no. of ways of seating arrangements

$= 24 \times 120$

$= \boxed{2880 \text{ ways}}$

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Imp Permutation Formulas

$$1! = 1$$

$$0! = 1$$

Q Find the number of words, that can be formed with letters of the word INDIA

Sol INDIA = 5 words

'I' comes twice

when a letter comes more than once in a word, we divide the factorial of the no. of all letters in the word by the number of occurrences of each letter.

$$\therefore \text{INDIA} = \frac{5!}{2!}$$

$$= \frac{5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}}$$

$$= \boxed{60}$$

Q Find the no. of words, with or without meaning, that can be formed with the letters of the word SWIMMING?

Sol SWIMMING = 8 ~~words~~ letters

here, I comes 2 times

& M comes 2 times

∴ no. of words formed

$$= \frac{8!}{2! \times 2!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times \overset{2}{\cancel{4}} \times 3 \times \cancel{2 \times 1}}{(\cancel{2} \times 1) \times \cancel{(2 \times 1)}}$$

$$= 8 \times 7 \times 6 \times 5 \times 2 \times 3$$

$$= \boxed{10080}$$

Q Find the no. of different words that can be formed with the letters of word "BUTTER" so that the vowels are always together.

Sol BUTTER contains 6 letters

U, E should always come together

so BTTR(UE)

so in total we have 5 words

ie. B, T, T, R, UE

ie. $\frac{5!}{2!}$ $\boxed{2! \because \text{T is twice}}$

$= \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}} = \boxed{60}$

No. of way U & E are arranged $= 2!$

ie $\boxed{2}$

Total no. of permutations possible

$= 60 \times 2 = \boxed{120 \text{ ways}}$