Toppersnotes
Unleash the topper in you

# SSC - CHSL

COMBINED HIGHER SECONDARY LEVEL

STAFF SELECTION COMMISSION

VOLUME – VI

# Advance Maths

SIERRA INNOVATIONS

# Contents

# ALGEBRA

These are second formula's which are often used in this topic.

1- $(a+b)^2 = a^2+b^2+2ab$

2- $(a+b)^2 = a^2+b^2+2ab$

3- $(a^2-b^2) = (a+b)(a-b)$

4- $(a+b)^2 = (a-b)^2+4ab$

5- $(a-b)^2 = (a+b)^2-4ab$

6- $(a+b)^3 = a^3+3a^2b+3ab^2+b^3 = a^3+b^3+3ab(a+b)$

7- $(a+b)^3 = a^3-b^3-3ab(a-b) = a^3-b^3-3a^2b+3ab^2$

8- $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

9- $(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

10- $(a-b)^4 = a^4-4a^3b+6a^2b^2+4ab^3+b^4$

11- $(a-b)^5 = a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5$

12- $a^3+b^3 = (a+b)(a^2+b^2+ab)$

13- $a^3-b^3 = (a-b)(a^2+b^2+ab)$

14- $a^3+b^3 = (a+b)^3(a^2+b^2+ab)$

15- $a^3-b^3 = (a-b)^3+3ab(a-b)$

16- $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

17- $a^2+b^2+c^2-ab-bc-ca = \frac{1}{2}\{2a^2+2b^2+2c^2-2ab-2bc-2ca\}$

$= \frac{1}{2}\{a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ac\}$

$= \frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}$

18- $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

19- If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$

20- $a^2 + b^2 = (a+b)^2 - 2ab$

21- $a^2 + b^2 = (a-b)^2 + 2ab$

22- $(a+b+c)^2 = (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)$

23- $(a+b+c)^3 = a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a)$

24- $a^4 + b^4 + a^2b^2 = (a^2 + b^2 - ab)(a^2 + b^2 + ab)$

25- $(a+b)^2 - (a-b)^2 = 4ab$

26- $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

**Que:-** If $x+\frac{1}{x}=2$ then what is the value of $x^{99}+\frac{1}{x^{197}}$ ?

**Soln:-** In such questions, we put possible value of $x$ in eqn here possible value is $x = 1$

So answer $(1)^{99}+\frac{1}{(1)^{199}} = 1+1 = 2$

**Que:-** If $x+\frac{1}{x}=-2$, what is the value of $x^{99}+\frac{1}{x^{511}} = ?$

**Soln:-**

$x = -1$

$(-1)^{99}+\left(\frac{1}{(-1)^{511}}\right) = -2$

**Que:-** $\frac{5x-3}{x}+\frac{5y-3}{y}+\frac{5z-3}{z}=0$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=?$

**Soln:-** **Basic:-**

$= \frac{5x}{x}-\frac{3}{x}+\frac{5\cancel{y}}{\cancel{y}}-\frac{3}{y}+\frac{5\cancel{z}}{z}-\frac{3}{z}=0$

$= 15-\frac{3}{x}-\frac{3}{y}-\frac{3}{z}=0$

$= 15 = 3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=5$

**Trick:**

$= \frac{5x-3}{x}+\frac{5y-3}{y}+\frac{5z-3}{y}$, $= \frac{5+5+5}{3}=5$

$\therefore$ Do the sum of common multibles of albhabet and divide by constant value.

Que:- $\frac{9x-5}{x}+\frac{9y-5}{y}+\frac{9z-5}{z}=0,\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=?$

**Soln:-**

$$=\frac{9x-5}{x}+\frac{9y-5}{y}+\frac{9z-5}{z}$$

$$=\frac{9+9+9}{5}=\frac{27}{5}$$

Que:- $2a+3b=4,\ 8a^3+27b^3+72ab=?$

**Soln:- Basic:**

$$2a+3b=4 \quad \text{—— (1)}$$

Cube to both Sides of eqn

$$(2a+3b)^3=(4)^3$$

$$(2a)^3+(3b)^3+3\times2\times3b\,(2a+3b)=64$$

$$8a^3+27b^3+18ab\times4=64$$

$$8a^3+27b^3+72ab=64$$

**Trick:**

Assume the Value of a and b which Satisfies the eqn (1) and Calculation of that eqn should be easy.

$$2a+3b=4$$

$$\downarrow \qquad \downarrow$$

$$a=2 \quad b=0$$

$$2\times2+3\times0=4$$

$$4=4$$

then,

$$=8\times(2)^3+27\times(0)^3+72\times2\times0$$

$$=64$$

Que:- $a^2+b^2-c^2=0$

$$\frac{a^6+b^6-c^6-}{a^2 \cdot b^2 \cdot c^2} = ?$$

Soln:- Trick:

Assume the Value of A, B and C

$a=1$ , $b=1$ , $C=\sqrt{2}$

Put these Values in eqn

$$= \frac{a^6+b^6-c^6}{a^2-b^2-c^2}$$

$$= \frac{(1)^6+(1)^6-(\sqrt{2})^6}{(1)^2-(1)^2-(\sqrt{2})^2} = \frac{1+1-8}{1 \cdot 1 \cdot 2} = \frac{6}{2} = -3$$

Basic:

$a^2+b^2-c^2-0$

$a^2+b^2 = c^2$ ——— ①

Cube to the both Sides of eqn

$(a^2+b^2)^3 = (c^2)^3$

$\because$ { $a^2+b^2=c^2$ Given in eqn ①

$a^6+b^6+3a^2b^2 \times (ca^2+b^2) = c^6$

$a^6+b^6+3a^2b^2 \times c^2 = c^6$

$a^6+b^6-c^6 = -3a^2b^2c^2$

Put the Value in eqn

$$\frac{-3a^2b^2c^2}{a^2b^2c^2} = -3$$

Que:- $a+b+c=0$ $\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right)\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right) = ?$

**Soln:-** $a+b+c=0$

$a+b=-c\ ,\ b+c=-a\ ,\ c+a=-b$

Put the value in eq$^n$

$$=\left(\frac{-c}{c}+\frac{(-a)}{a}+\frac{-b}{b}\right)\left(\frac{a}{-a}+\frac{b}{-b}+\frac{c}{c}\right)$$

$$=-3\times-3=9$$

**Que:-** $\frac{a^2-b^c}{a^2+b^c}+\frac{b^2-ac}{b^2+ac}+\frac{c^2-ab}{c^2+ab}=1$

then what is the value of $\frac{a2}{a^2+bc}+\frac{b^2}{b^2+ac}+\frac{c^2}{c^2+ab}=?$

**Soln:-** **Basic**

$$\frac{a^2-bc}{a^2+bc}+\frac{b^2-ac}{b^2+ac}+\frac{c^2-ab}{c^2+ab}=1$$

Add 3 in both sides,

$$\frac{a^2-bc}{a^2+bc}+1+\frac{b^2-ac+1}{b^2+ac}+\frac{c^2-ab}{c^2+ab}+1=1+3$$

$$\frac{a^2-\cancel{bc}+a^2+bc}{a^2+bc}+\frac{b^2-ac+b^2+ac}{b^2+ac}+\frac{c^2-ab+c^2+ab}{c^2+ab}=4$$

$$\frac{2a^2}{a^2+bc}+\frac{2b^2}{b^2+ac}+\frac{2c^2}{c^2+ab}=4$$

$$\frac{a^2}{a^2+b^c}+\frac{b^2}{b^2+ac}+\frac{c^2}{C2+ab}=2$$

**Trick:-**

$$\frac{a^2-b^c}{a^2+bc}+\frac{b^2-ac}{b^2+ac}+\frac{c^2-ab}{c^2+ab}=1$$

Here in Such questions, We Consider that $\frac{1}{3}$ is the Value of each part of eq$^n$ (L.H.S) and all of three makes 1 as where Sum:

$$\frac{a^2-bc}{a^2+bc}=\frac{1}{3} \quad \frac{b^2-ac}{b^2+ac}=\frac{1}{3}, \quad \frac{c^2-ab}{c^2+ab}=\frac{1}{3}$$

$$3a^2-3bc=a^2+bc, \quad 3b^2-3ac=b^2+ac, \quad 3c^2-3ab=c^2+ab$$

$$2a^2=4bc, \quad b^2=2ca, \quad c^2=2ab$$

$$a^2=2bc$$

So Put the Values in equation

$$\frac{2bc-bc}{2bc+bc}+\frac{2ac-ac}{2ac+ac}+\frac{2ab-ab}{2ab+ab}$$

$$=\frac{2bc}{3bc}+\frac{2ac}{3ac}+\frac{2ab}{3ab}$$

$$=\frac{6}{3}=2$$

Que:- $(a^2+b^2+c^2)=(ab+bc+ca)$

Find $\frac{a+b}{c}=?$

**Soln:-** **Basic -** multiply the eq$^n$ by 2 in both Sides

$$2(a^2+b^2+c^2)=2ab+2bc+2ca$$

$$a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ca=0$$

$$(a-b)^2+(b-c)^2+(c-a)^2=0$$

As We Studied before, each Part has equal Value of eq$^n$

So One Part is equal to Zero.

$(a-b)^2 = 0$ | $(b-c)^2 = 0$ | $(c-a)^2 = 0$

$a-b = 0$ | $b = c$ | $c = a$

$a = b$

then $a = b = c$

$= \frac{a+b}{c}$

$= \frac{c+c}{c} = 2$

Trick

In this type of questions, We assume the Value or

for easier you Can assume $a = b = c = 1$

$= \frac{a+b}{c}$

$= \frac{1+1}{1} = 2$

Que:- $(a^2+b^2+c^2) 2\, ab+bc+ca$

$\frac{4+5y-3z}{z} = ?$

Soln:- Assume the Value

$a = b = c = 1$

So, $\frac{4\times1+5\times1-3\times1}{1} = 6$

Que:- $a+\frac{1}{b} = 1$, $b+\frac{1}{c} = 1$, $a, b, c \neq 0$

then, find the Value of $a \cdot b \cdot c$ ?

Ans:- **Basic**

$a + \frac{1}{b} = 1$

$\frac{ab+1}{b} = 1$

$ab+1 = b$

multiply the eq$^n$ in both Sides by c

$abc + c = bc$

$abc = bc - c$

$abc = c(b-1)$

$abc = -1$

$b + \frac{1}{c} = 1$

$bc + 1 = c$

$bc - c = -1$

$(b-1) = -1$

$c + \frac{1}{a} = 1$

$ac + 1 = a$

**Trick:-** Assume the Values,

$a = 2, \quad b = -1, \quad c = \frac{1}{2}$

Put in eq$^n$ = abc

$= 2 \times -1 \times \frac{1}{2} = -1$

Que:- $x^2 = y+z, \; y^2 = z+x, \; z^2 = x+y$

Find the Value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$ ?

Sol$^n$:- **Trick** Assume the Value of x, y and z

$x = y = z = 2$

$= \frac{1}{2+1} + \frac{1}{2+1} + \frac{1}{2+1} = \frac{3}{3} = 1$

**Basic** $= \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

multiply the numerator and denominator

$$= \frac{x}{x^2+x} + \frac{y}{y^2+y} + \frac{z}{z^2+z}$$

$$= \frac{x}{y+z+x} + \frac{y}{z+x+y} + \frac{z}{x+y+z}$$

$$= \frac{x+y+z}{x+y+z} = 1$$

Que:- $a^3b = abc = 150$, find Value of c?

Soln:- IF $\cancel{a}^3 \cancel{b} = abc$

$a^2 = c$

now assume the Value a and C and Put in eqn

$a = c = 1$

$abc = 180$

$1 \times b \times 1 = 180$

$\boxed{b=180}$, $\boxed{C=1}$, $\boxed{a=1}$

∵ We assume the Value which Satisfies the eqn after Putting The Values means L.H.S and R.H.S should be equal.

Que:- $\frac{a}{a+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$

$$\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} = ?$$

Soln:- $\frac{a}{b+c} = \frac{1}{3}$, $3a = b+c$ ——— ①

$\frac{b}{c+a} = \frac{1}{3}$, $3b = c+a$ ——— ②

$\frac{c}{a+b} = \frac{1}{3}$, $3c = a+b$ ——— ③

Sum to eq$^n$ ①, ② and ③

$$3(a+b+c) = 2a + 2b + 2c$$

$$a+b+c = 0$$

and Put the Value from eq$^n$ ①, ② and ③

$$= \frac{a^{\cancel{2}}}{3\cancel{a}} + \frac{b^2}{3b} + \frac{c^2}{3\cancel{c}}$$

$$= \frac{(a+b+c)}{3} = \frac{0}{3} = 0$$

Que:- $a^2+b^2+c^2 = 2(a-b-c) - 3$

$2a - 3b + 4c = ?$

Sol$^n$:- Basic:

$$a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$$

$$(a-1)^2 + (b+1)^2 + (c+1)^2 = 0$$

$$a = 1, \quad b = -1, \quad c = -1$$

Put in eq$^n$

$$= 2\times 1 - 3\times -1 + 4\times -1$$

$$= 2+3-4 = 1$$

Trick:- Assume the Value that Satisfy above eq$^n$

$$a = 1, \; b = -1, \; c = -1$$

$$(1)^2 + (-1)^2 + (-1)^2 = 2(1+1+1) - 3$$

$$3 = 3$$

So,

$$2\times 1 - 3\times -1 + 4\times -1 = 1$$

## Formula:

$$x^2+y^2+ax+by+c=0$$

then,

$$\boxed{x = \frac{-\text{Cofficient of } x}{2\times \text{Cofficient of } x^2}}$$

$$\boxed{y = \frac{-\text{Coefficient of } y}{2\times \text{Coefficient of } y^2}}$$

$$\boxed{z = \frac{-\text{Coefficient of } z}{2\times \text{Coefficient of } z^2}}$$

So,

$$a^2+b^2+c^2-2a+2b+2c-3=0$$

$$a = \frac{+2}{2\times1} = 1$$

$$b = \frac{-2}{2\times1} = -1, \quad c = \frac{-2}{2\times1} = -1$$

Put the Values in eq$^n$

$$= 2\times1-3\times-1+4\times-1$$
$$= 1$$

Que:- $x^2+y^2-6x-8x+25=0$

$$x-y = ?$$

Sol$^n$:- Formula:

$$x = \frac{-\times-6}{2\times1} = 3$$

$$y = \frac{-\times-8}{2\times1} = 4$$

$$x-y = 3-4 = -1$$

# Practise Yourself :-

Que.1 ⇒ $a + \frac{1}{a+2} = 0$, $(a+2)^3 + \frac{1}{(a+2)^3} = ?$

Que.2 ⇒ $p - 2q = 4$, $p^3 - 8q^3 - 24pq - 64 = ?$

Que.3 ⇒ $\frac{6x-2}{x} + \frac{6y-2}{y} + \frac{6z-2}{z} = 0$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = ?$

Que.4 ⇒ $x + \frac{1}{y} = 1$, $y + \frac{1}{z} = 1$, $z + \frac{1}{x} = ?$

Que.5 ⇒ $x^3 y = xyz = 150$, find the value of $y$ ?

Que.6 ⇒ $x^2 + y^2 - 10x + 12y + 61 = 0$, $2x + 3y = ?$

Que.7 ⇒ $16x^2 + 4y^2 - 40x + 12y + 34 = 0$, $x - y = ?$

Ans.1 ⇒ 2

Ans.2 ⇒ 0

Ans.3 ⇒ 3

Ans.4 ⇒ 01

Ans.5 ⇒ 150

Ans.6 ⇒ 8

Ans.7 ⇒ $11/4$

⇒ These are two Concepts on the basis of that SSC of asks the questions.

- $\boxed{x+\frac{1}{x}=a \longrightarrow x^2+\frac{1}{x^2}=a^2-2}$

$$\left(x+\frac{1}{x}\right)^2=a^2$$

$$x^2+\frac{1}{x^2}=a^2-2$$

- $\boxed{x+\frac{1}{x}=a \longrightarrow x^2+\frac{1}{x^2}=a^2+2}$

- $\boxed{x+\frac{1}{x}=a \longrightarrow x^2=\frac{1}{x^2}=a\sqrt{a^2-4}}$

let $a=x$, $b=\frac{1}{x}$

then,

$(a+b)^2-(a-b)^2=4ab$ [Given in Formula Section - formula (25)]

$$\text{So, } \left(x+\frac{1}{x}\right)^2-\left(x-\frac{1}{x}\right)^2=4\times \cancel{x}\times\frac{1}{x}$$

$$\left(x-\frac{1}{x}\right)=\sqrt{\left(x+\frac{1}{x}\right)^2-4},\ \left(x+\frac{1}{x}\right)=\sqrt{\left(x=\frac{1}{x}\right)^2+4}$$

$$\left(x-\frac{1}{x}\right)=\sqrt{a^2-4}$$

$$\text{So, } x^2-\frac{1}{x^2}=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=a\sqrt{a^2-4}$$

- $\boxed{x-\frac{1}{x}=a,\ x^2-\frac{1}{x^2}=a\sqrt{a^2+4}}$

- $\boxed{x+\frac{1}{x}=a \longrightarrow x^3+\frac{1}{x^3}=a^3-3a}$

$$=\left(x+\frac{1}{x}\right)^3=a^3$$

$$=x^3+\frac{1}{x^3}+3* x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$=x^3+\frac{1}{x^3}+3a=a3$$

$$=x^3+\frac{1}{x^3}=a^3-3a$$

$$\boxed{x - \frac{1}{x} = a \longrightarrow x^3 - \frac{1}{x^3} = a^3 + 3a}$$

$$\left(x - \frac{1}{x}\right)^3 = a^3$$

$$x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right) = a^3$$

$$x^3 - \frac{1}{x^3} = a^3 + 3a$$

- $$\boxed{Cx + \frac{1}{dy} = a \longrightarrow (Cx)^3 + \frac{1}{(dy)^3} = a^3 - 3a \times \frac{c}{d}}$$

- When $\left(x + \frac{1}{x} = 1\right)$ is given, then $\boxed{x^3 = -1}$

$$\left(x + \frac{1}{x}\right)^3 = (1)^3$$

$$= x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 1$$

$$= x^3 + \frac{1}{x^3} = 1 - 3$$

$$= \underset{-1}{\underset{\downarrow}{x^3}} + \underset{-1}{\underset{\downarrow}{\frac{1}{x^3}}} = -2$$

- When $\left(x + \frac{1}{x} = 1\right)$ is given, then $\boxed{x^3 = 1}$

- $x + \frac{1}{x} = \sqrt{3}$ , $\boxed{x^6 = -1}$

$$= \left(x + \frac{1}{x}\right)^3 = (\sqrt{3})^3$$

$$= x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 3$$

$$x^3 + \frac{1}{x^3} = 3 - 3 = 0$$

$$\frac{x^6 + 1}{x^3} = 1$$

$$x^6 = -1$$

- $x^n + \frac{1}{x^n} = 1$ , then $x + \frac{1}{x} = -1$
- $x + \frac{1}{x} = a$ , $x^4 + \frac{1}{x^4} = \left[(a^2-2)^2 - 2\right]$

$$\left[\left(x+\frac{1}{x}\right)^2\right] = (a^2)$$

$$\left[x^2 + \frac{1}{x^2} + 2\right] = a^2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (a^2-2)^2$$

$$x^4 + \frac{1}{x^4} = (a^2-2)^2 - 2$$

- $x^5 + \frac{1}{x^5} = \left(x^4 + \frac{1}{x^4}\right)\left(x + \frac{1}{x}\right) - \left(x^3 + \frac{1}{x^3}\right)$

  or $= \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$

- $x^6 + \frac{1}{x^6} = \left(x^4 + \frac{1}{x^4}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x^2 + \frac{1}{x^2}\right)$

∵ For any Power of $x$, We factorise the Power and multiply that and after that divide the difference of them.

$5 = (3)(2) - (1)$ , $6 = (4)(2) - (2)$
$(4)(1) - (3)$ , $7 = (5)(2) - (3)$
or $7 = (4)(3) - (1)$

Que:- $x + \frac{1}{x} = 4$ , $x^2 + \frac{1}{x^2} = ?$

Soln:- When, $x + \frac{1}{x} = a$ , then $x^2 + \frac{1}{x^2} = a^2 - 2$

So, $x^2 + \frac{1}{x^2} = (4)^2 - 2 = 14$