



# SSC - CGL

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**COMBINED GRADUATE LEVEL**

**STAFF SELECTION COMMISSION**

**VOLUME - VI**

# Advance Maths



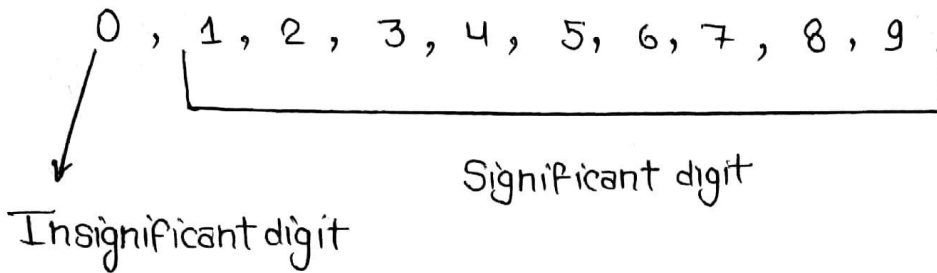
## Contents

<b>1. Number System</b>	<b>1</b>
<b>2. Surds &amp; Indices</b>	<b>23</b>
<b>3. HCF &amp; LCM</b>	<b>44</b>
<b>4. Algebra</b>	<b>55</b>
<b>5. Trigonometry</b>	<b>85</b>
<b>6. Height &amp; Distance</b>	<b>107</b>
<b>7. Geometry</b>	<b>112</b>
<b>8. Mensuration</b>	<b>155</b>
<b>9. Data Interpretation</b>	<b>194</b>
<b>10. Coordinate Geometry</b>	<b>213</b>



## NUMBER SERIES

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Natural No-

$$N = [1, 2, 3, 4, 5, \dots, \infty]$$

Whole No-

$$W = [0, 1, 2, 3, 4, \dots, \infty]$$

Integer No-

$$I = -\infty \text{ to } +\infty$$

$$= [-\infty, \dots, -3, -2, -1, 0, 1, 2, 3, \dots, \infty]$$

Even Number-

The number which is divisible by 2 is called even no.

or

Even number Contains unit digit 0, 2, 4, 6, 8

eg:- 2, 4, 6, ..., 1972, 2008

Odd Number-

The number which are not divisible by 2.

or

Odd no's Contain Unit digit 1, 3, 5, 7, 9.

eg:- 1, 3, 5, 7, ..., 1983, 2005, 2115.

Note:- 0 is neither Odd or even .

### Prime Number:-

The number which is not divisible by any other number except 1 and onself.

- 2 is the Only even prime number .

- First Prime number - 2

- Prime number from 1 to 100 = 25

Prime number from 1 to 75 = 21

Prime number from 1 to 50 = 15

Prime number from 1 to 25 = 9

Que:- How many Prime numbers are there from 1 to 105?

Soln:- From 1 to 100 , Prime number = 25

From 101 to 105 , Prime numbers = 101, 103

Total Prime numbers =  $25 + 2 = 27$

### Composite Number

The number which can be divided by any other number is called Composite number.

- 0 and 1 are neither Prime number nor Composite number .

### Co-Prime Number

The number which are not divisible by each other.

eg:- • (3, 4)

• (3, 4, 6)

- HCF of Co-Prime numbers be always 1.

## Rational Number -

- 0 is a rational Number.
- $\frac{a}{b}$ ,  $b \neq 0$  a and b both are integers.
- All the numbers which can be written in the form of  $\frac{a}{b}$  is rational number but only condition is  $b \neq 0$

eg:  $\frac{3}{1}$ ,  $\frac{0}{1}$ ,  $-1$

- All the natural no, whole no, Integer no's are rational no's but vice-versa is not true.
- If no. of decimal digit are finite in any number then the no will be rational.

$$\begin{array}{r} 3.5 \\ \rightarrow \frac{35}{10} \end{array}$$

$$\begin{array}{r} 8.73 \\ \rightarrow \frac{873}{100} \end{array}$$

- $0.44444 \dots \infty = .\overline{4} = \frac{4}{9}$
- $0.838383 \dots \infty = .\overline{83} = \frac{83}{99}$

If repetition occurs in finite way after the decimal then they can be called rational numbers.

- $2\sqrt{25}$ ,  $3\sqrt{27}$ ,  $6\sqrt{64}$

All the no's of which we can find out complete square root is called rational no.

## Irrational no's -

All the no's which can't be written in the form of rational no is called irrational no's.

- $0.123486784 \dots \infty$
- $\sqrt{2}$  ,  $\sqrt{3}$
- $\pi$  - Irrational no.
- $\frac{22}{7}$  - rational no.

## Rules of divisibility -

- • If Unit digit is 0 then divisible by 5 → eg: 2870
- If last two digit are 00 then divisible by 25. eg - 16900
- If last three digits are 000 then divisible by 125.

## → Rule of 3 and 9:-

The sum of digit should be divisible by 3 and 9.

eg: 7832211

$$7+8+3+2+2+1+1 = 24$$

divisible by Only 3.

eg: 7833213

$$7+8+3+3+2+1+3 = 27$$

divisible by both 3 and 9.

## Rule of 6:-

Rule of 2 and 3 both are applied

eg: 23412

$$2+3+4+1+2 = 12 \rightarrow \text{divisible by 3.}$$

$$23412 \underline{2} \rightarrow \text{divisible by 2.}$$

eg:  $72 * 72$ , if given number is divisible by 9 then find the possible value of \*?

Soln:- min Possible values are Only 0 and 9.

eg: What is the minimum number found by Only 0 and 1 which is completely divisible by 225?

Soln:-  $225 = 25 \times 9$

Rule of 25 and 9 should be applied.

$$= 11111111100$$

## → Rule of 2, 4, 8:-

- If unit digit is divisible by 2 then number be divisible by 2. eg:- 2874
- If last two digit are divisible by 4 then number be divisible by 4. eg:- 16924
- If last three digits are divisible by 8 then number be divisible by 8. eg - 176864



## Rule of 11.

Difference of even place and Odd Place should be divisible by 11.

eg: If 738A6A is completely divisible by 11 then And the value of A ?

Soln:- even place of number - Odd Place of Number

$$(A + A + 3) - (6 + 8 + 7) = 0$$

$$2A - 18 = 0$$

$$A = 9$$

eg: Is 361658 divisible by 11 ?

Soln:-  $(8 + 6 + 6) - (5 + 1 + 3)$   
 $20 - 9 = 11$

11 is divisible by 11 so yes, it is divisible.

## Test of Unit digit:-

### Multiplication:

eg:

783	x	694	x	897	x	386
↓		↓		↓		↓
3	x	4	x	7	x	6
		1②		2x7=1④		4x6=2④

So, Unit digit is 4.

eg:  $879 \times 637 \times 583 \times 985$

- (If at least One '5' x all odds = Unit digit = 5)

$$\begin{array}{cccc}
 879 & \times & 637 & \times & 583 & \times & 985 \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 9 & & 7 & & 3 & & 5 \\
 \text{Odd} & & \text{Odd} & & \text{Odd} & & 
 \end{array}$$

answer is 5 -

eg:  $878 \times 679 \times 383 \times 582 \times 685$

- (If at least One '5' x at least One even = Unit digit = 0)

## Powers

- Whatever Power is Placed On 0, 1, 5, 6, Unit digit Will be Same 0, 1, 5, 6.

eg:  $6156^{273} \rightarrow \text{Unit digit} = 6$       eg:  $275^{2551} \rightarrow \text{Unit digit} = 5$

- $9^1 = 9$  ,  $9^2 = 81$   
 $9^3 = 729$  ,  $9^4 = 6561$

$9^{\text{odd}}$  then Unit digit  $\rightarrow 9$

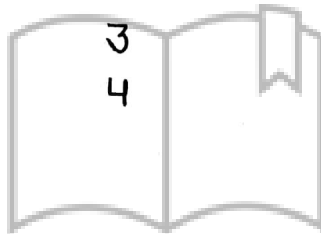
$9^{\text{even}}$  then Unit digit  $\rightarrow 1$

•  $\therefore$  IF  $x^n$  and  $n > 4$

then do  $\frac{n}{4}$ , means divide to last two digit of  $n$  by 4.

IF Remainder                      Power

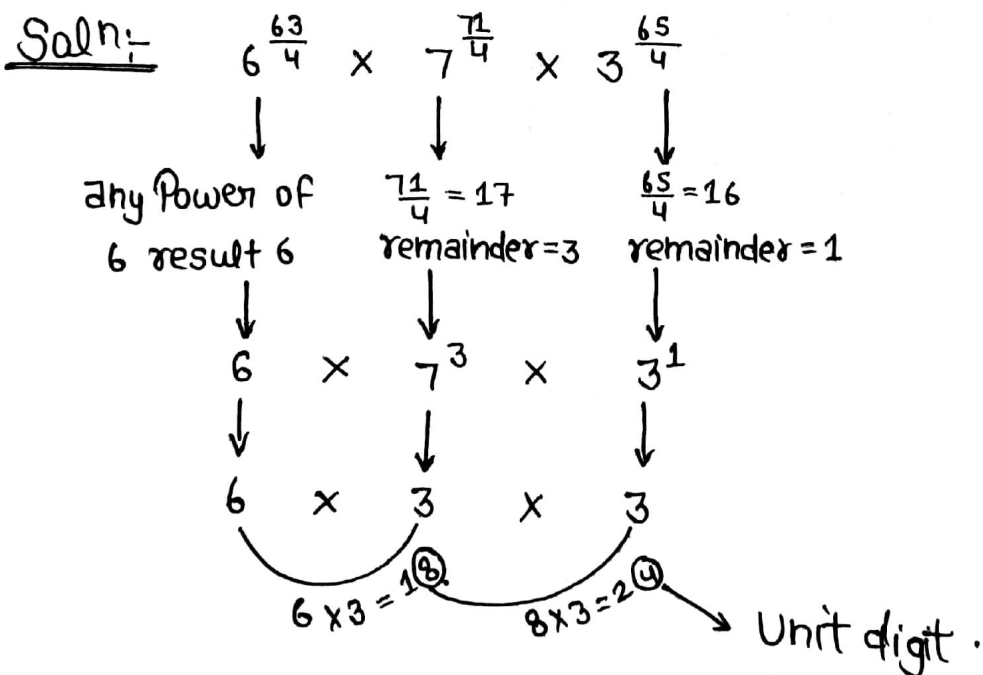
1	1
2	2
3	3
0	4



eg:  $(122)^{173}$

Soln:-  $(122)^{\frac{173}{4}}$  here remainder = 1 then Power 1  
 $(122)^1 = 2^1 = \underline{2} = \text{Unit digit}$

eg:  $6^{63} \times 7^{71} \times 3^{65}$



eg:-  $124^{372} + 124^{373}$ , what would be Unit digit of given data?

Soln =  $124^{372} + 124^{373}$   
 $= 124^{\frac{72}{4}} + 124^{73/4}$   
 $\downarrow \qquad \qquad \downarrow$   
 $4^4 + 4^1$   
 $\downarrow \qquad \qquad \downarrow$   
 $6 + 4 = 10 \rightarrow \text{Unit digit} = 0$

eg:-  $3^{108} - 8^{34}$ , Unit digit = ?

Soln:-  $3^{108} - 8^{34}$   
 $\downarrow \qquad \qquad \downarrow$   
 $3^4 = 81 \quad 8^2 = 64$   
 $\downarrow \qquad \qquad \swarrow \quad \nwarrow$   
 $1 \quad - \quad 4$   
 $= 11 - 4 = 7 \rightarrow \text{Unit digit}$   
 $\downarrow$   
 (take Carry)

eg:- Number at Zero's:

$5 \times 2 = 10$

$5 \times 2 \times 5 \times 2 = 150$

$5 \times 2 \times 5 \times 2 \times 5 \times 2 = 1000 \dots \dots \text{etc.}$

eg: Find the number of zero's in the end of given multiple  $1 \times 2 \times 3 \times 4 \times \dots \times 1000$

Soln:- First find the number of factors of 5 because no. of '2' are a lot

$$\text{Factors of 5} = \frac{1000}{5} \text{ (last number)} = 200$$

$$\begin{array}{r} 200 \\ \underline{\quad 5} \\ 40 \\ \underline{\quad 5} \\ 8 \\ \underline{\quad 5} \\ 1 \end{array} = 40$$

$$\begin{array}{r} 40 \\ \underline{\quad 5} \\ 8 \\ \underline{\quad 5} \\ 1 \end{array} = 8$$

$$\begin{array}{r} 8 \\ \underline{\quad 5} \\ 1 \end{array} = 1$$

$$\text{Factors of 5} = 249$$

Factors of 2 = more (than 249 (Can understand by given data)

then these are 249 zero's.

eg: Find the numbers of zero's in the end given multiplication?

$$1 \times 2 \times 3 \times 4 \times \dots \times 100$$

Soln:- Factors of 5 =  $\frac{100}{5}$  (last no) = 20

$$\frac{20}{5} = 4$$

$$\text{Factors of 5} \rightarrow 24$$

$$\text{Factors of 2} \rightarrow \text{more than 24}$$

24 Pairs

So Zero's are 24.

Eg: Find the number of Zero's in the End of given multiplication?

$$1 \times 3 \times 5 \times 7 \times 9 \times \dots \times 99$$

Soln:- Factor of 5 = 12

Factor of 2 = 0 (there is no even no so there is no pair)

So no. of Zeros are '0'

⇒ No of Completely divisible-

Que:- How many natural numbers between 3 to 200 Completely divisible by 7.

Soln:-  $\frac{3}{7} \quad \frac{200}{7}$

Quotient 0 28

$$28 - 0 = 28 \checkmark$$

∴ Note — If the first number, divisor the given number Completely then odd '1' in difference.

Que:- How many three digit numbers are there Completely divisible by 6?

Soln:-  $\frac{100}{6} \quad \frac{999}{6}$

16 166

$$166 - 16 = 150$$

**Que:-** Numbers of numbers below 1000 which are divisible by 10 and 13 both?

Soln:- LCM of (10, 13) = 130

$$\frac{0}{130} \qquad \frac{999}{130}$$

$$0 \qquad 7$$

$7 - 0 = 7$

⇒

Dividen	Divisor	Remainder
998	97	1
99	99	1
998	88	1
98	80	79
1073		
89		
987		
79		

If difference of base of dividend and divisor is 1 then.

(a) When Power is even then remainder = 1

(b) When Power is Odd then remove the Power and divide.

**Que:-** What is the remainder after dividing  $5^{111}$  by 126?

Soln:-  $5^{111} \div 126$

$$\downarrow$$

$$(5^3)^{37} = 125^{37} \div 126$$

Remainder = 125

Q → What is the remainder after dividing  $(9^{19} + 6)$  by 8?

Soln:-

$$\begin{aligned}
 &= (9^{19} + 6) \div 8 \\
 &= \left( \frac{9^{19}}{8} + \frac{6}{8} \right) \\
 &= 1 + 6 = 7
 \end{aligned}$$

Que:- What is the remainder after dividing  $2^{33}$  by 10?

Soln:-  $= 2^{33} \div 10$

$$\downarrow \frac{33}{4}$$

$$\downarrow 1$$

$2 \div 10$  then remainder = 2

⇒ Divisor  $\overline{) Dividend}$  Quotient      Divident = divisor = divisor  $\times$  quotient + remainder

eg: Divide 27 by 6?

Soln:-

$$\begin{array}{r}
 \text{Divisor} \rightarrow 6 \overline{) 27} \leftarrow \text{dividend} \\
 \underline{24} \\
 3 \text{ - remainder}
 \end{array}$$

(4 - Quotient)

Que:- When a number is divided by 3- remainder is 1. When the quotient of it divided by 2 then remainder is 1. If the Initial number is divided by 6 then what is the remainder.

Soln:-

$$\begin{array}{r}
 \text{Divisor} \quad 3 \times 3 = 9 \\
 + 3 \quad \times 2 \\
 \hline
 \text{Remainder} \quad \underline{1} \quad \downarrow + \\
 9 + 1 = 10 \quad \underline{3}
 \end{array}$$

Initial number = 10  
then  $10 \div 6 = 4$



Que:- In a divide, divisor is 10 times of it's quotient  
 5 times of its remainder. If remainder is 46 then  
 what is the dividend?

Soln:- Remainder = 46  
 Divisor =  $46 \times 5 = 230$   
 Quotient =  $\frac{230}{10} = 23$

$$\begin{aligned} \text{Dividend} &= \text{divisor} \times \text{Quotient} + \text{Remainder} \\ &= 230 \times 23 + 46 \\ &= 5336 \end{aligned}$$

Que:- If any number is successively dividend by 4, 5, 6 then  
 remainders are 2, 3, 4 respectively. What is this smallest  
 numbers?

Soln:- Method-1

$53 \times 4 = 212$

Divisor →	4	}	5	6
	+	x	+	
	↓	x	↓	↓
	2	x	3	4/10
	←		←	
	$12 + 2 = 24$		$50 + 3 = 53$	
	$\times$			
	$= 214$			

∴ Add Vertically then  
 multiply the result  
 in cross, again add  
 vertically and multiply  
 in cross, So on.

Method-2

Do any options

## Arithmetic Progression (AP):

2, 5, 8, 11, 14, .....

1, 2, 3, 4, 5, .....

$a, a+d, a+2d, a+3d, \dots, a+(n-1)d$

$a$  = first term,  $d$  = Second term - first term

$l$  = last term,  $n$  = number of terms.

$$\text{Last term} = T_n = a + (n-1)d$$

$$\begin{aligned} \text{Sum of } n \text{ terms (S}_n) &= \frac{n}{2} (a+l) \quad \text{or} \\ &= \frac{n}{2} [2a + (n-1)d] \end{aligned}$$

$$= \frac{(a+l)(l+d-a)}{2d} \quad \text{or}$$

$\therefore$  In arithmetic Progression, difference between every two consecutive digits would be same.

Que:- 4, 9, 14, 19, ..... 109, find the number of terms in given equation?

Soln:-

$$a = 4, d = 9 - 4 = 5$$

$$a + (n-1)d = T_n$$

$$4 + (n-1)5 = 109$$

$$(n-1) = \frac{105}{5} = 21$$

$$n = 22$$