Toppersnotes

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IES/GATE

CIVIL ENGINEERING

VOLUME – VIII

Steel

SIERRA INNOVATIONS

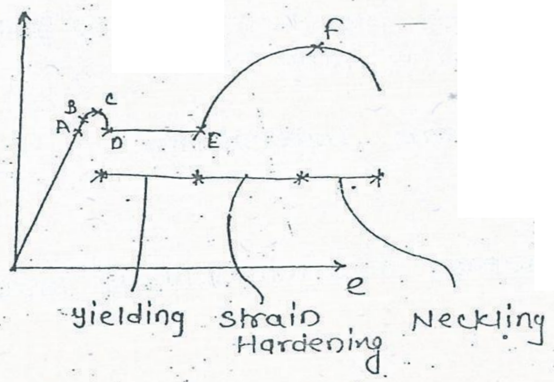
Index

Steel

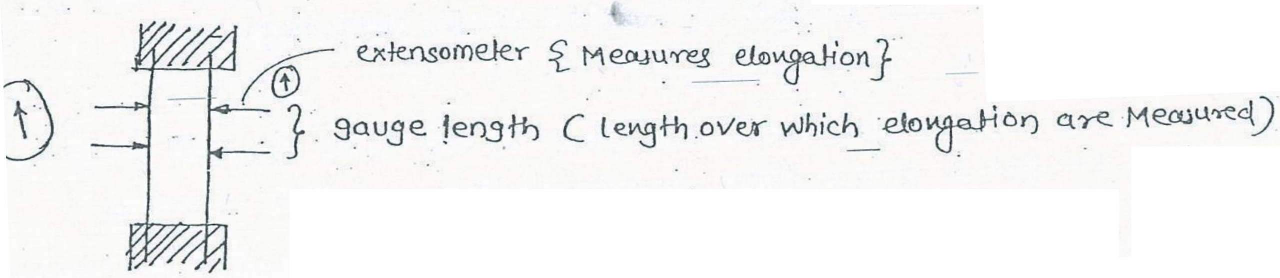
Chapter 1 - Plastic Analysis Of BEAM

CONCEPTS (1)

(1)



Actual stress strain curve



* stress controlled Test

$$F = \frac{P}{A}$$

$$e = \frac{\delta l}{l}$$

ed stress strain curve

A - Proportionality limit :

limit upto which stresses proportion to strain

B - elastic limit :

limit upto which The Material comes Back to its original position After Removal of The loads.

* The Above Two limits are Independant

* YIELDING

It is The Increase in strain without Increase in The stress. So W

* STRAIN HARDENING

AFTER The Material has yielded, It becomes stronger and Harder. This process is called strain Hardening.

* Necking

It is The local Reduction In the Area of cross section It takes place After ultimate stress point is crossed.

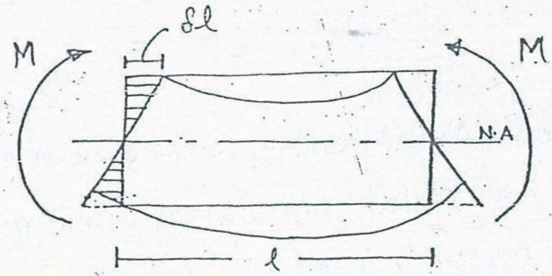
* In plastic Analysis, The effect of strain Hardening is Neglected (Because High Deformations are Not Acceptable in structural Design). So plastic Analysis is Based on Idealised stress strain curve.

* Stress strain curve is drawn by conducting strain controlled Test on Mild_steel Bar.

Concept ② Assumptions in Plastic Analysis of Beam.

① Plane section Remain plane After Bending Also. This Assumption is Called Bernoulli's Assumption (It implies that strain varies linearly over the depth of the cross section)

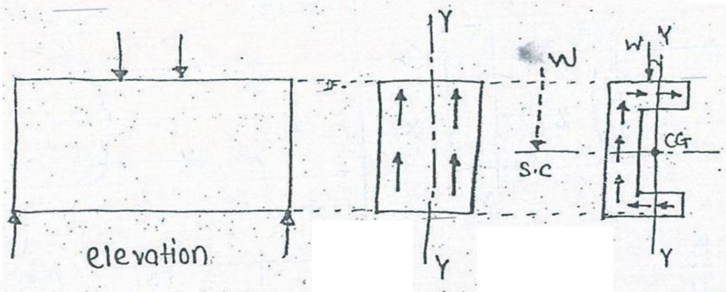
ex



$e = \frac{\delta l}{l}$ since δl varies linearly over the depth, strain also varies over the depth.

The above Assumption is valid upto collapse loads.

② The cross section must be symmetrical with Respect to plane of loading. (otherwise if the c/s is Not symmetrical with Respect to plane of loading, Twisting Moment are developed in the Beam and flexure formula cannot be Applied Directly).



S.C – Shearcentre

Note

① Like force is a vector, moment is also a vector. It has magnitude and direction. The direction of the movement is found from Right hand screw Rule. {i.e Nut and Bolt system. i.e whatever Direction the Nut Advances that is the direction of the moment)

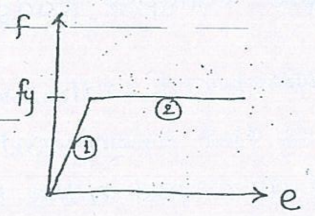
② Twisting Moment: If any moment acts along the longitudinal Axis of a Member, then it will Twis the Member. so the Moment is Called Twisting Moment.

③ Bending Moment IF any Movement acts perpandicular to longitudinal Axis of a Member, Then it will Bend The Member. so The Moment is called Bending Moment.

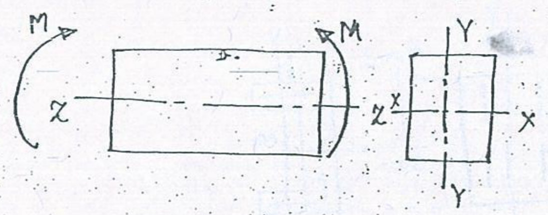
④ A Moment can be a Bending Moment ~~or~~ a Twisting Moment depending on it's Direction.

③ The effect of Axial force and shear force are Neglected. i.e Axial and shear Deformation are Neglected in plastic Analysis of Beam.

④ The stress strain curve is Assumed to be bi linear { i.e it consist of 2 straight lines



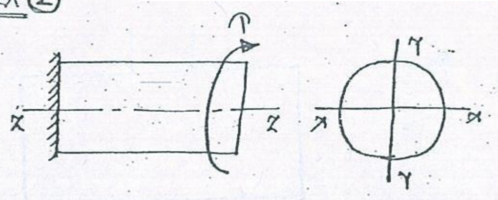
ex. ①



$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

Since 'm' is acting along x-axis M.I is also taken along x-axis

ex ②

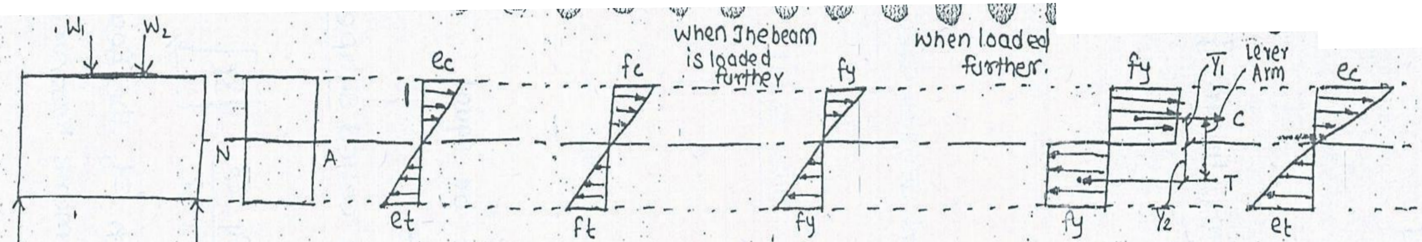


$$\frac{T}{J} = \frac{f_s}{r} = \frac{C\theta}{l}$$

Since 'T' is acting along z axis M.I is also taken along z axis

$J = I_{zz}$ = Polar M.I

Concept ③ PLASTIC MOMENT OF A SECTION



elevation

c/s

Bending strain Distribution { Linear Because of Bernoullis Assumption }

Bending stress Distribution. { Linear Because of Hook's law } | Safe moment (m)

Bending stress at yielding yield moment (my)

Bending stress Distribution at Plastic state Plastic moment mp

Bending Strain Distribution at collapse condition Plastic state

Note

① Safe moment (or) Working Moment :-

$$m = f \cdot Z$$

f - permissible Bending stress

Z - section Modulus { also called flexural strength parameter } Because for a given Material, Z decides The strength of The Beam }

The above formula $\boxed{m = f \cdot Z}$ is Applicable if The stress varies linearly.

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Note ....

② yield moment

$$\boxed{M_y = f \cdot Z}$$

{We can use $f_y \cdot Z$ Because stress is varying linearly}

③ Plastic moment (M_p)

M_p = C × lever Arm = T × lever Arm

{We can not write $M_p = f_y \cdot Z$ because stress in not varrying linearly}

$$M_p = \left(f_y \times \frac{A}{2}\right) \times (\bar{y}_1 + \bar{y}_2)$$

$$= f_y \times \left[\frac{A}{2}(\bar{y}_1 + \bar{y}_2)\right] = \boxed{f_y \cdot Z_p}$$

where, Z_p = plastic Modulus $= \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$

where, $\bar{y}_1, \bar{y}_2$ = centroidal distances of comp^n area and Tension area from N.A.

④ At fully plastic state, NA cuts The entire Area into Two equal Areas.

(Because from equilibrium Consideration)

$\Sigma X = 0 \implies +C - T = 0$

[$\xrightarrow{}$ +ve $\xleftarrow{}$ −ve] $\boxed{C = T}$

Since f_y is same Throughout The Depth, areas must be equal to make C = T.

⑤ The Ratio of plastic Moment and yield moment is called shape factor

$$\text{Shape factor} = \frac{\text{Plastic moment}}{\text{Yield moment}} = \frac{M_p}{M_y} = \frac{f_y \cdot Z_p}{f_y \cdot Z} = \boxed{\frac{Z_p}{Z}}$$

— ~~Shear~~ Shape factor Represent The Reserved strength of The Beam Section Beyond yielding. More Shape Factor implies more Reserved strength Beyond yielding.

Note

⑥ Load factor = (L.F) = $\frac{\text{ultimate load}}{\text{working load}}$ (R.C.C.) = $\frac{\text{Plastic moment}}{\text{working moment}}$ (steel)

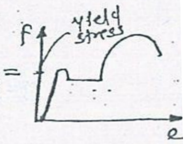
$$L.F = \frac{M_P}{M} = \frac{f_y \cdot Z_P}{f \cdot Z} = \left(\frac{f_y}{f}\right)\left(\frac{Z_P}{Z}\right)$$

F.O.S   S.F

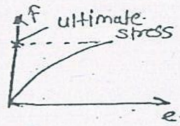
$\boxed{L.F = F.O.S \times S.F}$ {This formula is applicable to ductile Material like Mild steel only}

⑦ Factor of safety (F.O.S)

ⓐ F.O.S for Ductile Materials like Mild steel = $\frac{\text{Yield stress}}{\text{Working stress}}$ =



ⓑ F.O.S for brittle Materials like concrete = $\frac{\text{ultimate stress}}{\text{working stress}}$ =



*ⓒ Margin of safety {defined only for Brittle Material}

$$\boxed{M.O.S = \frac{\text{ultimate stress}}{\text{working stress}} - 1}$$

cancelling the areas

$$\boxed{M.O.S = \frac{\text{ultimate load}}{\text{working load}} - 1}$$

⑧ Moment curvature relationship

ⓐ Flexural formula = $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$

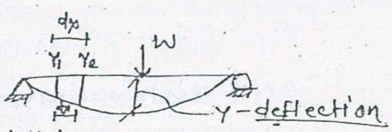
where, M - moment of Resistance

R - Radius of curvature of Bent up Beam

$\frac{1}{R}$ - $\boxed{\text{curvature} \propto M}$

In Deflection

$\theta = \frac{y_2 - y_1}{dx}$



y - Deflection - Duration of the beam from initial configuration

$\frac{dy}{dx}$ = Slope = θ ⇒ Rate of change of deflection along length of Beam.

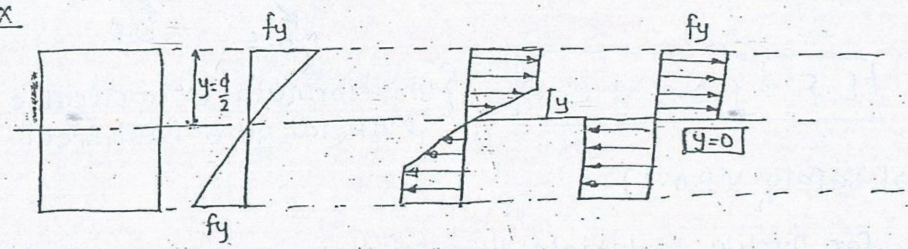
$\frac{d^2y}{dx^2} = \frac{d\theta}{dx} = \frac{1}{R}$ = curvature = Rate of change of slope along length of Beam

(b) At yielding, $\boxed{\frac{1}{R} = \frac{M_y}{EI}}$

(c) At fully plastic state curvature "Becomes Infinity" {It means rate of change of slope is very highly}

(1M)

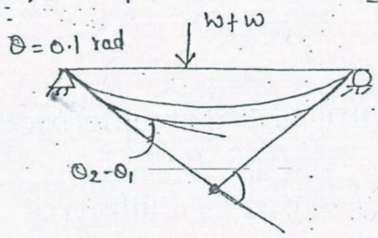
ex



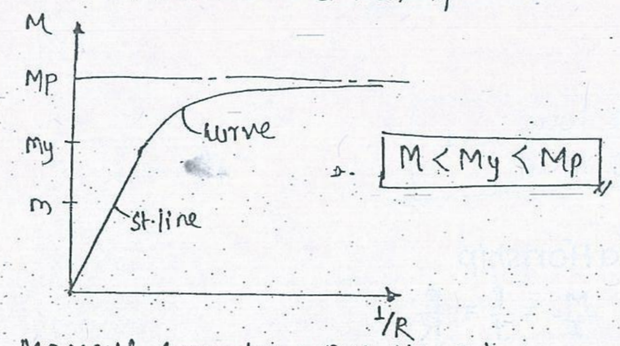
y = Distance from N·A to extreme fibre where stress varries linearly

$\frac{F}{y} = \frac{E}{R}$ $\quad \frac{1}{R} = \frac{F}{E \times y} = \frac{f}{E \times 0} = \infty$

At fully plastic state $\boxed{y=0}$



(d) Moment curvature relationship



$\boxed{M < M_y < M_p}$

Moment curvature Relationship
{Based on idealised stress strain curve}

(g) Plastic Moment capacity of a section = $M_p = f_y \cdot \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$

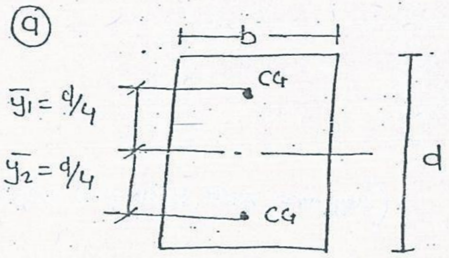
M_p for a given a Material depends on the Area of the cross section and distribution of the area. Distribute the Area such that $\bar{y}_1, \bar{y}_2$ are Max. so that Z_p will be Maximum.

Note :- I section is the Most efficient c/s for a given Area Because $\bar{y}_1, \bar{y}_2$ are Max^m.

(10)

Concept (4) SHAPE FACTOR FOR DIFFERENT C/S

(a)



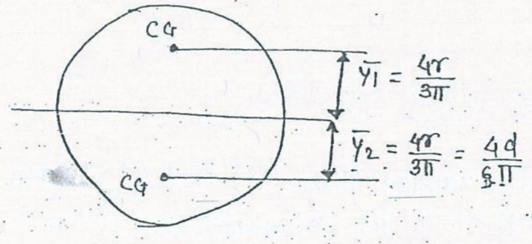
$$S.F = \frac{Z_p}{Z} = \frac{A/2\,(\bar{y}_1+\bar{y}_2)}{\left(\frac{bd^2}{6}\right)} = \frac{\frac{bd}{2}\left(\frac{d}{4}+\frac{d}{4}\right)}{\left(\frac{bd^2}{6}\right)}$$

$$\boxed{S.F = 1.5}$$

$$\boxed{Z_p = \frac{bd^2}{4}} \qquad \boxed{Z = \frac{bd^2}{6}}$$

Note :- Shape factor 1.5 Means There is 50% of Reserved strength beyond yielding.

(b)



$$S.f = \frac{Z_p}{Z} = \frac{A/2\,(\bar{y}_1+\bar{y}_2)}{\left(\frac{\pi d^3}{32}\right)}$$

$$= \frac{\frac{\pi d^2/4}{2}\left(\frac{4d}{6\pi}+\frac{4d}{6\pi}\right)}{\left(\frac{\pi d^3}{32}\right)}$$

$$= \frac{d^3/6}{\left(\frac{\pi d^3}{32}\right)} = \frac{32}{6\pi}$$

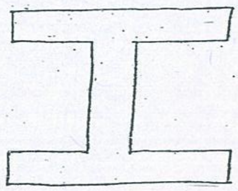
$$\boxed{S.F = 1.7}$$

$$\boxed{Z = \frac{I}{y} = \frac{\pi d^3}{32}}$$

$$\boxed{Z_p = \frac{d^3}{6}}$$

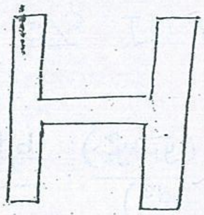
Here reserve strength does not imply most efficient c/s.
Most efficient c/s depends on $M_p = f_y \cdot \frac{A}{2}(\bar{y}_1+\bar{y}_2)$

(3)



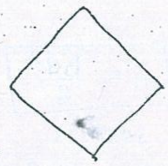
$\boxed{S.F = 1.12 \text{ to } 1.14}$

(4)



$\boxed{S.F = 1.5}$

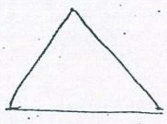
(5)



$\boxed{S.F = 2.0}$

(square with diagonal horizontal)

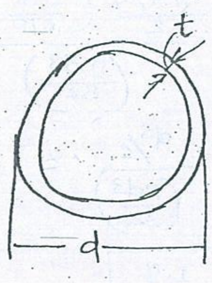
(6)



$\boxed{S.F = 2.34}$

(10 Marks)

(7) S.F for a Thin Hollow Circular Section



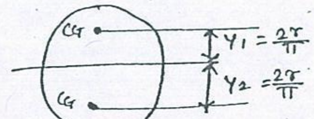
Note :- * Thin Implies That t^2 and t^3 Terms are Neglected in Calculations. Area is Treated as line Area

$$S.F = \frac{Z_p}{Z}$$

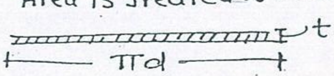
$$Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$Z_p = \frac{\pi d t}{2}\left(\frac{d}{\pi} + \frac{d}{\pi}\right)$$

$$\boxed{Z_p = d^2 t}$$



Area is Treated as line Area

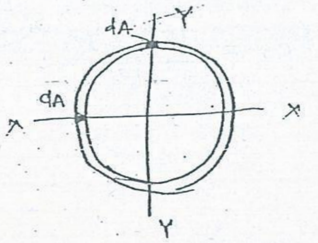


$\boxed{A = \pi d t}$

$$Z = \frac{I_{xx}}{y}$$

$$Z = \frac{\pi d^3 t/8}{d/2}$$

$$\boxed{Z = \frac{\pi d^2 t}{4}}$$



$$I_{zz} = \int r^2 dA = r^2 \int dA$$

$$\boxed{I_{zz} = A r^2}$$

$$I_{zz} = \pi d t \times \left(\frac{d}{2}\right)^2 = \boxed{\frac{\pi d^3 t}{4}}$$

from $\perp^r$ Axis Theorm

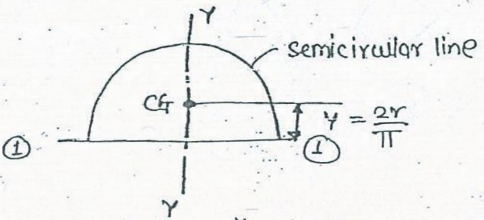
$$I_{zz} = I_{xx} + I_{yy} \; (= I_{xx})$$

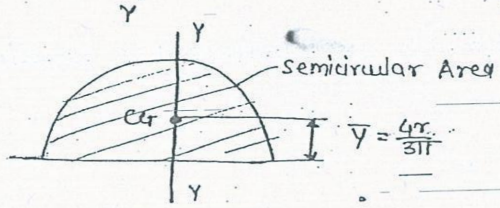
$$I_{xx} = \frac{I_{zz}}{2} = \frac{\pi d^3 t/4}{2}$$

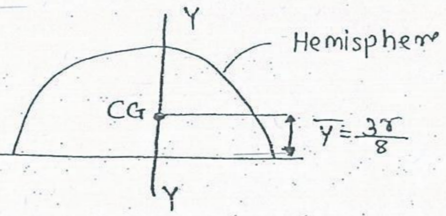
$$\boxed{I_{xx} = \frac{\pi d^3 t}{8}}$$

Note

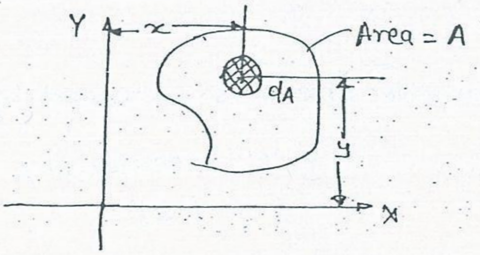
(1)







(2) Area Moment of Inertia (or) Second Moment of Area.



$y\,dA$ is called first moment of Area

$y^2 dA$ is called moment of Area or MI

$$I_{xx} = \int y^2 dA, \quad I_{yy} = \int x^2 dA, \quad I_{zz} = \int r^2 dA$$

- I is only a mathematical term But when it is combined with a Material property like young's Modulus, It Represents a measure of Resistance to rotation ~~ore~~ (or) Buckling   EI = Flexural Rigidity – $M \times D^2$

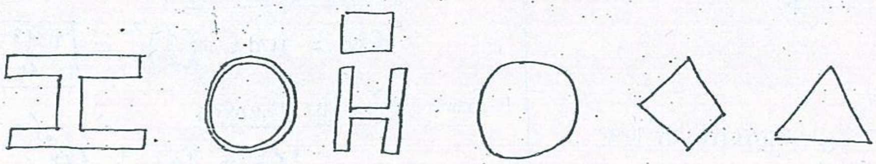
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$ Buckling

CJ = Torsional rigidity $= T$

$$= \frac{T \times \cancel{l}^{1}}{\cancel{\theta}^{1}}$$

$$= T$$

Note: ~~Ascending~~ Increasing order of S.F



1.12	1.27	1.5	1.7	2.0	2.34
Min^m					Max^m

Q1 For A Rectangular section S.F is 1.5. permissible Bending stress is $0.66\,f_y$. Load factor is .

$\Rightarrow$

Load factor = F.O.S × S.F

$$= \left(\frac{f_y}{F}\right) \times 1.5 \qquad \boxed{F = 0.66\,f_y}$$

$$= \left(\frac{f_y}{0.66\,f_y}\right) \times 1.5$$

$$= \left(\frac{f_y}{\frac{2}{3} f_y}\right) \times 1.5$$

$$\boxed{L.F = 2.25}$$

Q2 For I section S.F = 1.12
FOS in Bending = 1.5. If Allowable stress is increased by 20%.
Then load factor is

$\Rightarrow$

If 'F' is Increased by 20%.
$\therefore F = 1.2F$

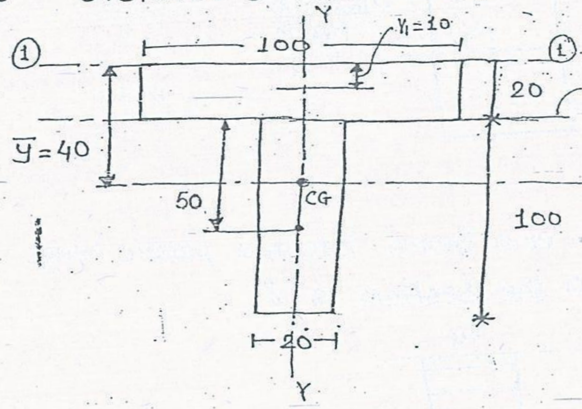
$$L.F = F.O.S \times S.F$$

$$= \left(\frac{f_y}{F}\right) \times 1.12$$

$$= \frac{1.5}{1.2\,f_y} \times 1.12$$

$$= \frac{1.5}{1.2} \times 1.12 = \boxed{1.4}$$

Q3 Distance Between elastic and plastic N.A is ?



(1) Elastic N.A — From Varignon's Theorm { i.e Moment of an area about any Axis = Moment summation of component Area }.

Note

Since Area is symmetrical about y axis, CG lies on y axis. To locate CG on Y-Axis take Axis of Referance $\perp^{er}$ to Y axis as shown in fig.

$$A \times \bar{y} = a_1 y_1 + a_2 y_2 + \ldots$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{[(100 \times 20) \times 10] + [(100 \times 20) \times (50 + 20)]}{(100 \times 20) + (100 \times 20)}$$

$= 40$ mm from (1)-(1)

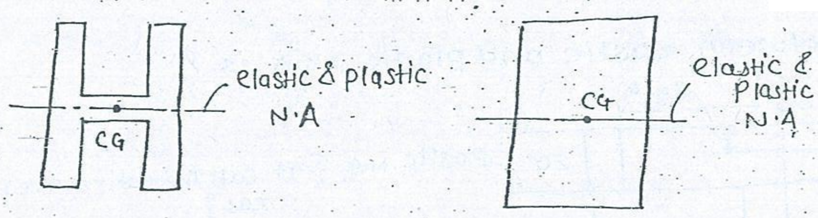
(2) Plastic N.A — since 2 Rectangular Areas are equal, Plastic N.A passes through the junction of 2 Rectangles i.e 20 mm from (1)-(1)

* Distance Bet^n elastic & plastic N.A

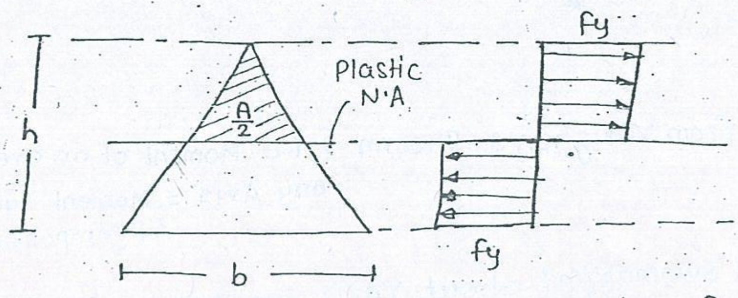
$= 40 - 20$

$= 20$ mm

Note:- If C/s is symmetrical about X-axis, then elastic and plastic NA always coincide



Q4 When a Triangular section of a Beam becomes plastic hinge Then compressive force acting on The section is ?

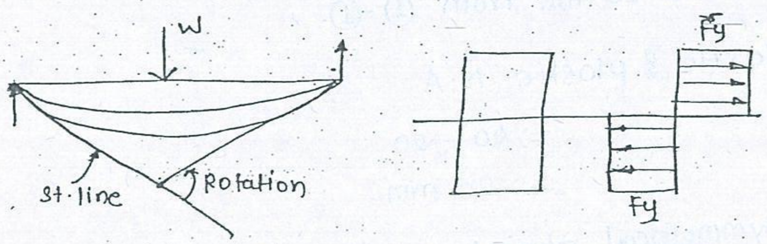


$$C = fy \cdot \frac{A}{2} = fy \times \frac{1}{2} \times \left(\frac{1}{2} \times b \times h\right)$$

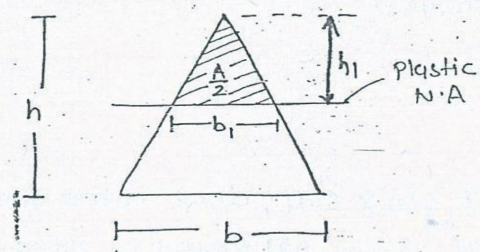
$$\boxed{C = \frac{bh}{4} \cdot fy}$$

Note ① Whatever may be The shape of c/s, stress Distribution is Rectangular at plastic state

② Plastic hinge – If c/s of The Beam is subjected to fy Throughout its Depth, Then It Cannot Take any further loading. If any Additional load is Applied Then The Beam Rotates at That section. Since The Beam Rotates Due to moment (m_p). We say That a Plastic hinge is formed at That section.



Q 5 In the above problem the depth plastic N.A from top fibre is ?



No of unknowns = 2 (b_1, h_1)

so, we require 2 equations to find them

(1) Plastic N.A cuts the entire area into 2 equal Areas.

Comp^n Area = $\frac{1}{2}$ × Total Area

$$\frac{1}{2} \times b_1 \times h_1 = \frac{1}{2} \times \left(\frac{1}{2} \times b \times h\right)$$

$$\boxed{b_1 = \frac{bh}{2h_1}} \text{ —— (1)}$$

(2) from similar Triangles concept (or from linear Interpolation)

$$\frac{h_1}{b_1} = \frac{h}{b} \qquad \boxed{b_1 = \frac{bh_1}{h}} \text{ —— (2)}$$

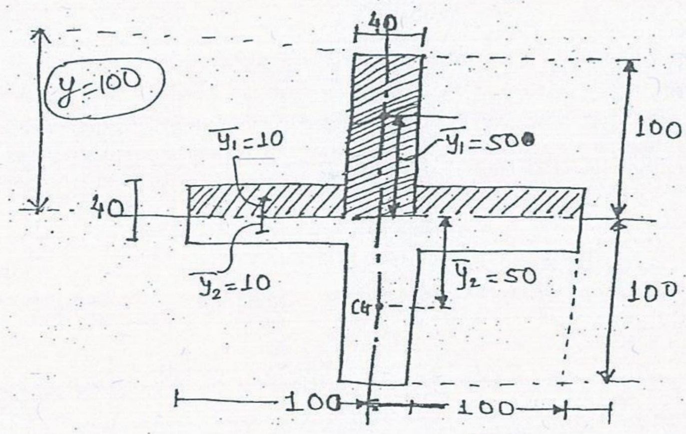
(2) in 1

$$\frac{bh_1}{h} = \frac{b.h}{2h_1} \Longrightarrow \boxed{h_1 = \frac{h}{\sqrt{2}}} \text{ —— (A)}$$

(A) in (2)

$$\boxed{b_1 = \frac{b}{\sqrt{2}}} \text{ —— (B)}$$

Q6 For the c/s shown in fig. S.F is ?



$$S.F = \frac{Z_P}{Z}$$

$$Z_P = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$= \left[(100 \times 40)(50+50)\right] + \left[(160 \times 20) \times (10+10)\right]$$

for vertical sect^n — For Horizontal sect^n

$$\boxed{Z_P = 464000 \text{ mm}^3}$$

$$Z_{xx} = \frac{I_{xx} \text{ of entire c/s}}{y \ (=100)}$$

$$= \frac{\left[40 \times \frac{200^3}{12}\right] + \left[160 \times \frac{40^3}{12}\right]}{100}$$