Toppersnotes
Unleash the topper in you

IES/GATE

CIVIL ENGINEERING

VOLUME – III

O.C.F., Foundation, PERT - CPM

SIERRA INNOVATIONS

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O.C.F., Foundation, PERT - CPM

Foundation Engineering

PERT – CPM

OPEN CHANNEL FLOW

Toppersnotes
Unleash the topper in you

Chapter 1 - Introduction

Pipe flow	Open channel flow
① Flow takes place due to difference in pressure	① Flow takes place due to gravity
② C/s is generally round	② It can be triangular, trapezoid, rectangular etc.
③ H.G.L lies above the top surface of the water	③ H.G.L coincides with the top surface of the water
④ Reynold's Number is used	④ Froude's number is used.

Classification of channels :-

① natural and artificial channels

eg. River — natural

Canal — artificial.

② Prismatic and non-prismatic channel

If the shape, c/s and bed slope is constant throughout then channel is prismatic otherwise non-prismatic.

③ Rigid & mobile boundary channels

⇒ Rigid boundary channel

① Boundaries are not deformable.

No Sedimentation and erosion takes place.

) 1° of freedom is there i.e. depth of flow. (only depth can change)

IOBILE BOUNDARY CHANNELS

) Boundaries are deformable

) Sedimentation and erosion takes place

3) 4° of freedom is there (all can change)

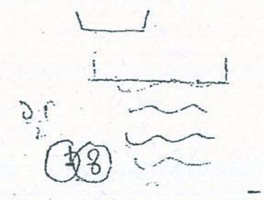
→ depth of flow ✓

→ width of the channel ✓

→ Bed slope ✓

→ Layout ✓

TYPES OF FLOW

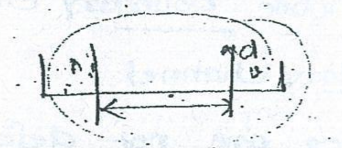


① Steady and Unsteady flow :-

If all the flow parameters such as depth of flow, Velocity, discharge etc. is constant w.r.t time then flow is steady otherwise unsteady.

② Uniform and non-uniform flow :-

If all the flow parameters w.r.t space is constant then flow is uniform otherwise non-uniform.



③ Laminar and turbulent flow :-

Pipe flow	O.C.F	
$Re < 2000$	$Re < 500$	Laminar
$2000 < Re < 4000$	$500 < Re < 2000$	Transition
$Re > 4000$	$Re > 2000$	Turbulent.

④ Critical, Sub-Critical and Supercritical flow :⇒

$$F_r = \sqrt{\frac{F_i}{F_g}}$$

$F_i = ma$ $\qquad$ $F_g = mg$

$F_i = \rho A V^2$ $\qquad$ $F_g = \rho A L g$

$$F_r = \sqrt{\frac{\rho A V^2}{\rho A L g}}$$

$$\boxed{F_r = \frac{V}{\sqrt{Lg}}} \quad \text{(pipe flow)}$$

$$\boxed{F_r = \frac{V}{\sqrt{gD}}} \quad \text{(OCF)}$$

$F_r = 1$ (critical flow)

$F_r > 1$ (Supercritical, Rapid, shooting & Torrential flow)

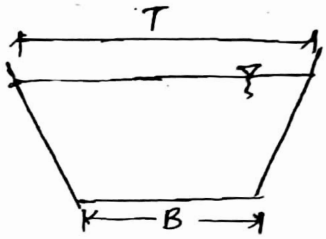
$F_r < 1$ (Subcritical, Tranquil & streaming flow)

→ Critical, Subcritical, Supercritical flow :-

Critical	Subcritical	Supercritical
→ $F_r = 1$	$F_r < 1$	$F_r > 1$

where, F_r = Froud's number

$$\boxed{F_r = \frac{V}{\sqrt{\frac{gA}{T}}}}$$



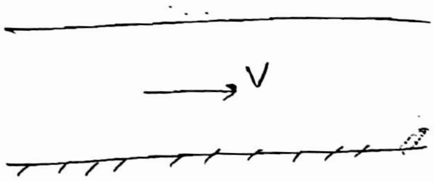
V → Avg. velocity of flow
A → Area of c/s of flow
T → Top width & g = acc. due to gravity.

Critical	Subcritical	Supercritical
→ $V = V_c = \sqrt{\frac{gA}{T}}$	$V < V_c$	$V > V_c$
→ $y = y_c$	$y > y_c$	$y < y_c$

→ Denominator of Froud's number represents a speed with which the disturbance created to flow travels in still water. i.e, "Celerity (C)".

(**) ∴ $\boxed{\text{Celerity } (C) = \sqrt{\frac{gA}{T}}}$

⇒



$$V_{wave/ground} = V_{wave/water} + V_{water/ground}$$
(US) (U/S) (U/S)

∴ $\boxed{V_{wave/ground}\ (US) = C - V}$ [∵ US → upstream]

→ At low flow velocity
$F_r < 1$
$C > V$
$C - V > 0$.

→ At low flow velocity, $F_r < 1$ and a small disturbance to the flow will cause disturbance wave which travels upstream with a velocity of $(C-V)$ w.r.t a stationary observer.

→ Due to the upstream movement of waves upstream conditions gets affected, In case of Subcritical flow, condition upstream is affected by condition at downstream. Hence downstream section is taken as "Control section".

At high flow velocity $F_r > 1$

$$V > C$$

$$C - V < 0$$

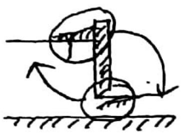
Thus upstream flow velocity of waves $(C-V)$ will become negative. i.e, the disturbance waves will not travel upstream, It will travel downstream with a velocity of $V-C$. Hence flow condition Down stream would be affected. Thus.

"Supercritical flow has upstream control".

Note:-

- Subcritical flow has downstream control while supercritical flow has upstream control.

  * Subcritical flow → Down stream Control
  * Supcritical flow → Up stream control



→ When $F_r = 1$, i.e, flow is critical, the disturbance velocity $C - V = 0$ i.e, the disturbance wave will not travel at all.

IE:- (Imp)

Q:- A wide channel is 1m deep and has a velocity of flow, $V = 2.13$ m/sec, if a disturbance is caused and elementary wave can travel upstream with a velocity of —?

a) 1 m/sec b) 2.13 m/sec c) 3.13 m/sec d) 5.26 m/sec

Sol:- $V = 2.13$ m/sec , wide channel (1m) → Rectangular channel

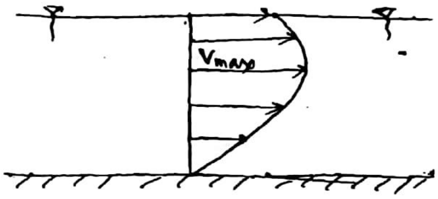
$$y = 1m$$

W.K.T, $C = \sqrt{\frac{gA}{T}}$

$\therefore$ celerity, $C = \sqrt{\frac{g \times yT}{T}} = \sqrt{gy} = \sqrt{9.81 \times 1} = 3.13$ m/sec

$\because V_{wave/ground} = C - V$
$= 3.13 - 2.13$
$= 1 \text{ m/sec}$

→ **Velocity Distribution :-**

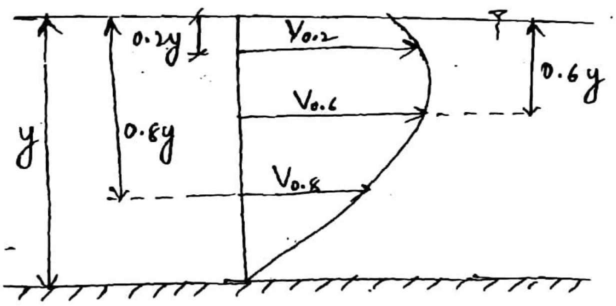


→ Reduction (or) Dip in the velocity is because of secondary currents, which is a function of aspect ratio.

$$\because \boxed{\text{Aspect ratio} = \frac{\text{Depth}}{\text{width}}}$$

Note ;

→ if Aspect ratio is large depth at which max. velocity will occur is deeper.

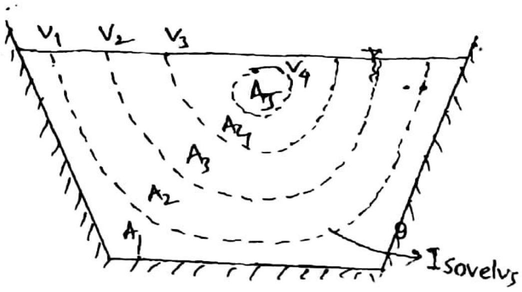


$$\boxed{\begin{array}{c} V_{avg} = \dfrac{V_{0.2} + V_{0.8}}{2} \\ \text{(or)} \\ V_{avg} = V_{0.6} \text{ (less reliable)} \end{array}}$$

V_{avg} → Average Velocities.

→ **Isovels :-**

→ Contour of equal velocity is called "Isovels".

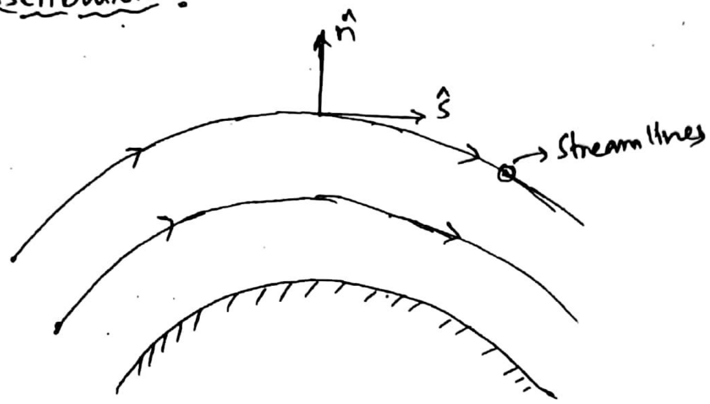


→ The avg. velocity is also sometimes calculated as,

$$\boxed{V_{avg} = K V_{surface}}, \quad [0.8 < K < 0.95]$$

Note:-
→ Psovelns is the better approach

→ Pressure Distribution :-



By Euler's formula ;

$$-\frac{\partial}{\partial n}(P + \gamma z) = \rho \cdot a_n$$

$a_n \rightarrow$ Normal acceleration

$$a_n = \frac{v^2}{r}, \quad \gamma = \rho g$$

$P \rightarrow$ Gauge pressure, z = Datum (elevation)

→ if stream lines are straight lines then the normal acceleration (a_n) would be zero, because, $r \rightarrow \infty \Rightarrow a_n \rightarrow 0$:

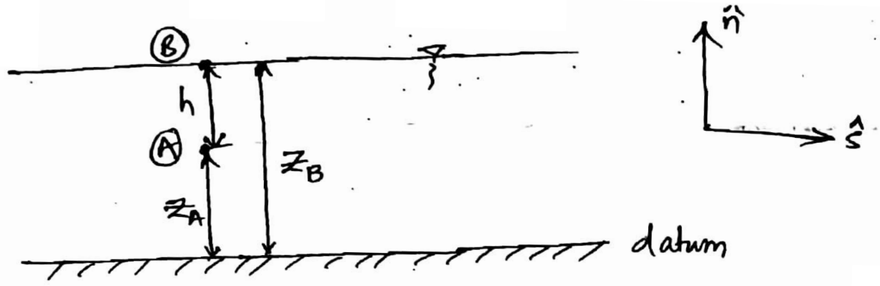
$$\rightarrow \quad \frac{-\partial(P + \gamma z)}{\partial n} = 0$$

$$P + \gamma z = \text{constant}$$

$$\Rightarrow \text{Piezometric head} = \left(\frac{P}{\gamma} + z\right) = \text{constant}$$

→ Hence the Piezometric head will become constant i.e, Pressure distribution is "Hydro static".

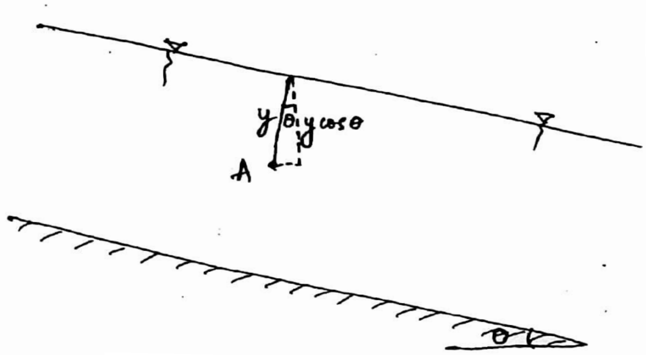
Proof :-



$\Rightarrow P_A + \gamma_w Z_A = \cancel{P_B}^{\,0} + \gamma_w Z_B$ [Here, $P_B = 0$ (atmospheric pressure)].

$\Rightarrow P_A = \gamma_w (Z_B - Z_A) = \gamma_w h.$

⇒ Non-Curvilinear flow :-



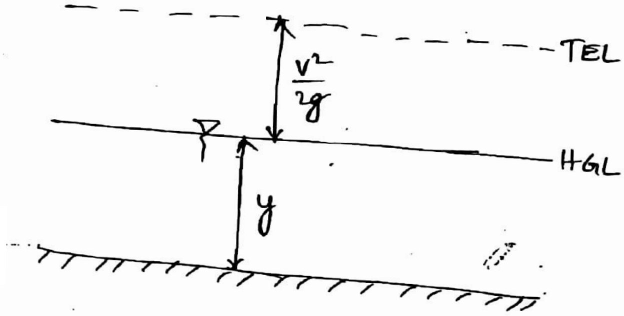
$P_A = \gamma_w (y\cos\theta)$

when bedslopes are small.

$\Rightarrow \theta \approx 0 \Rightarrow \cos\theta \approx 1$

$\therefore \boxed{P_A = \gamma_w y}$

→



HGL → Hydraulic gradient line

TEL → Total Energy line

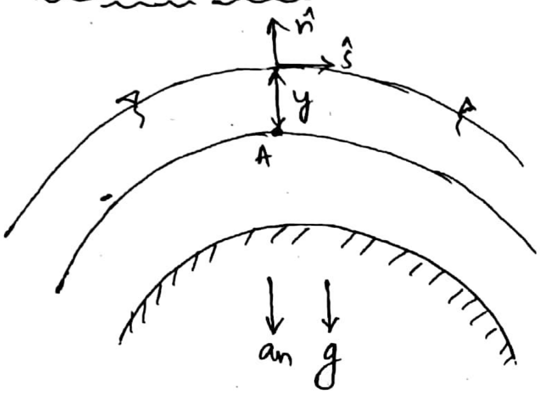
→ When $\left(\frac{P}{\gamma_w} + z\right)$ is plotted along the length we get HGL

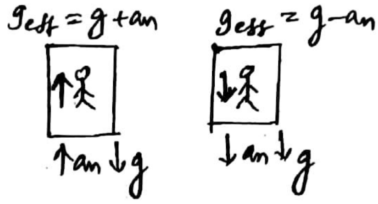
→ When $\left(\frac{P}{\gamma_w} + z + \frac{v^2}{2g}\right)$ is plotted along the length we get TEL.

→ For smaller slopes HGL will coincide with free surface and for large slopes HGL will be below the free surface. ($y\cos\theta < y$).

For smaller slopes HGL will coincide with the free surface and for the ~~Large slope~~ HGL will be below the free surface ($\because y\cos\theta < y$) [Repeated].

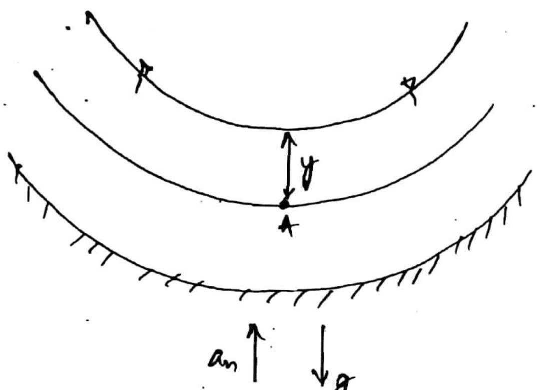
Curvilinear flow :-





$g_{eff} = g - a_n$

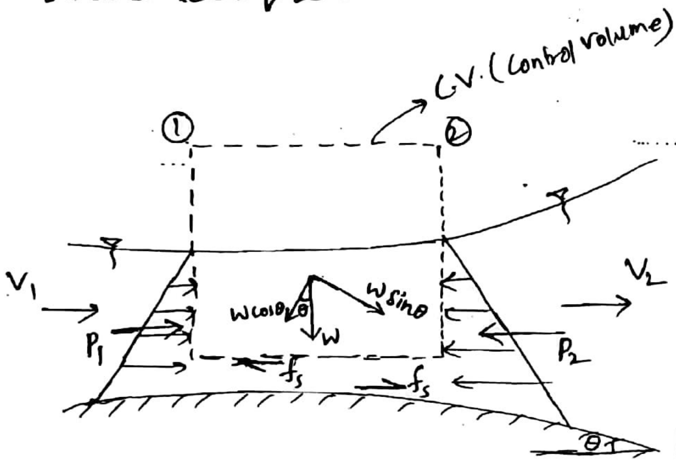
$P_A = \rho g_{eff} y$



$g_{eff} = g + a_n$

$P_A = \rho g_{eff} y$

⇒ Momentum equation :-



Net force on C.V $= P_1 - P_2 + W\sin\theta - f_s = M_2 - M_1$

$P_1, P_2 \rightarrow$ Pressure force

$M \rightarrow$ Momentum flux (kg·m/s²)

$M_1 = \rho Q V_1$ & $M_2 = \rho Q V_2$

↳ Rate of change of momentum $= M_2 - M_1$

$P_1 = \frac{1}{2}(\gamma_w y) \times y \times B$

$P_1 = \gamma_w \times y/2 \times y \times B$

→ $\boxed{P_1 = \gamma_w \bar{y}_1 A_1}$

Where, $\bar{y}_1$ = depth of C.G of area from free surface.

→ Thus pressure force on an area is found out by multiplying that pressure at the C.G. of the area with the complete area.

Note:-

→ In actual form for steady flow the momentum eqⁿ is written as,

"Net external force acting in x-direction on the control volume = Momentum flux going out of control volume in x-direction − Momentum flux coming inside the control volume in x-direction".

→ Specific force :-

$$\boxed{\text{Specific force} = \frac{P+M}{\gamma_w}}$$

where, P = pressure force
M = momentum flux ($\rho Q V_1$)

→ Specific force is constant for a horizontal frictionless channel

→ From Momentum eqⁿ,

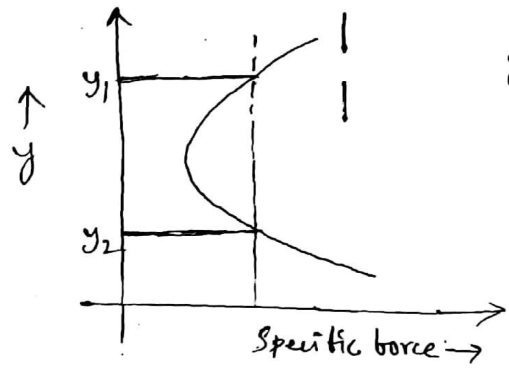
$$P_1 - P_2 = M_2 - M_1$$

$$\frac{P_1 + M_1}{\gamma_w} = \frac{P_2 + M_2}{\gamma_w} = \text{Specific force}$$

$\therefore$ Specific force = constant.

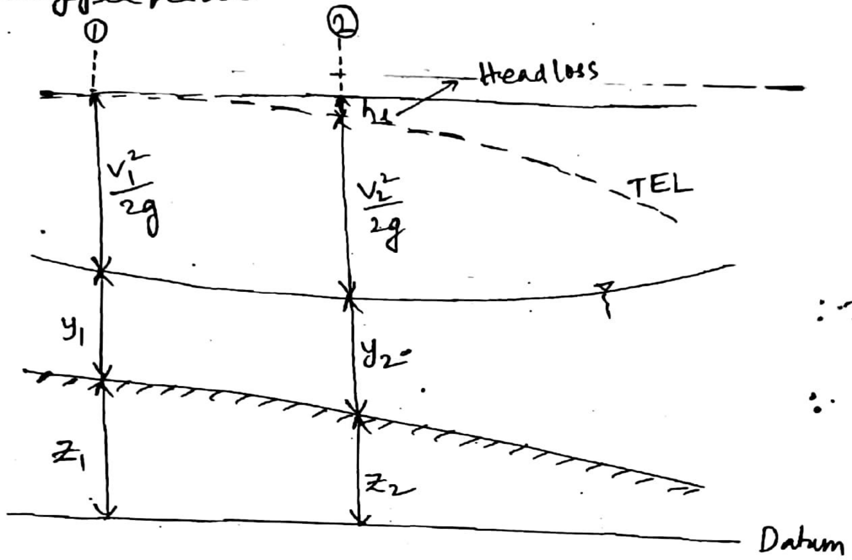
→ In the Analysis of RVF, friction does not play an important role, Hence for smaller slopes of bed RVF Analysis is done by making the specific force to be constant.

→ At a constant specific force, we have two possible depth of flow and they are called as "Conjugate depth or Sequent depth."



y_1 & y_2 → Conjugate depths or Sequent depth

Energy Equation :-



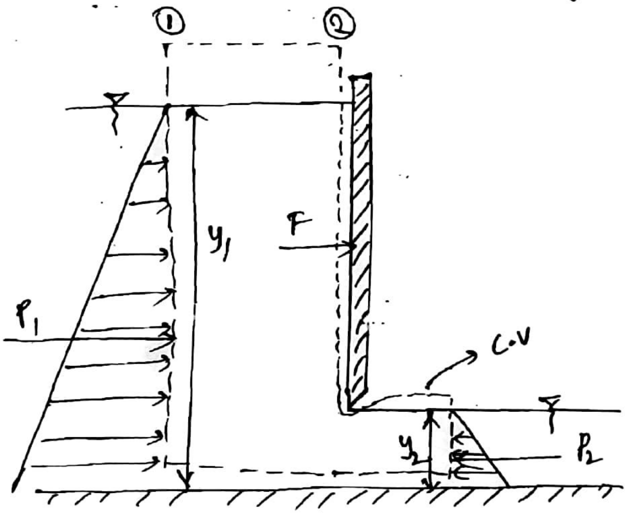
$\therefore$ Total Energy per unit wt. $= H$.

$\therefore H = y + z + \frac{V^2}{2g}$

$$\rightarrow \boxed{y_1 + z_1 + \frac{V_1^2}{2g} = y_2 + z_2 + \frac{V_2^2}{2g} + h_L}$$

→ Every term of the eq^n is representing "Energy Head", i.e, "Energy per unit wt."

Q: Find the force per unit width on the sluice gate due to water forces. neglect energy loss & consider channel to be rectangular.



Sol:- By applying momentum eq^n on C.V,

$$P_1 - P_2 - F = M_2 - M_1$$

$P \rightarrow$ Pressure force, $M \rightarrow$ Momentum

$$P_1 = \gamma_w \bar{y}_1 A_1$$

$P_1 = \gamma_w \cdot \frac{y_1}{2} \times (y_1 \times 1)$ ------- Considering unit width inside

$P_1 = \frac{\gamma_w y_1^2}{2}$, $P_2 = \frac{\gamma_w \cdot y_2^2}{2}$

$\rightarrow M_1 = \rho Q V_1 = \rho Q \times \frac{Q}{y_1 \times 1} = \frac{\rho Q^2}{y_1}$

$\rightarrow M_2 = \frac{\rho Q^2}{y_2}$

$\Rightarrow \frac{\gamma_w}{2}(y_1^2 - y_2^2) - F = \rho Q^2 \left(\frac{1}{y_2} - \frac{1}{y_1}\right)$

$\therefore F = \frac{\gamma_w}{2}(y_1^2 - y_2^2) - \rho Q^2 \left(\frac{1}{y_2} - \frac{1}{y_1}\right) \longrightarrow ①$

By applying energy equation,

$$y_1 + \cancel{z_1}^{\,0} + \frac{V_1^2}{2g} = y_2 + \cancel{z_2}^{\,0} + \frac{V_2^2}{2g} + \cancel{h_L}^{\,0}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

(By continuity equation, $V_1 (y_1 \times 1) = V_2 y_2 = Q$)

$$\Rightarrow y_1 + \frac{Q^2}{2g y_1^2} = y_2 + \frac{Q^2}{2g y_2^2}$$

$$\frac{Q^2}{2g}\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right) = y_1 - y_2$$

$$Q^2 = \frac{2g\, y_1^2 y_2^2}{y_1 + y_2}$$

By substituting in eq^n ①,

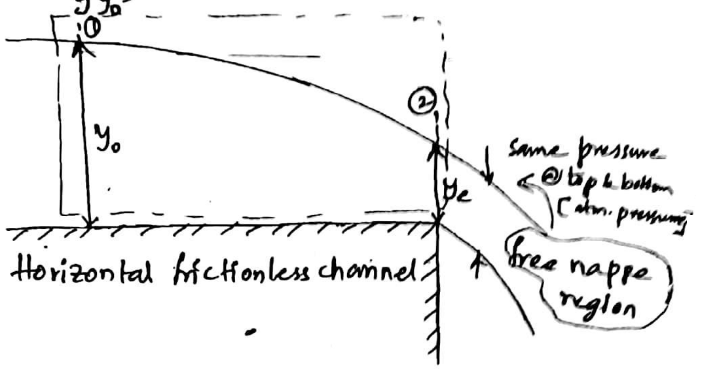
$$F = \frac{\gamma_w}{2}(y_1^2 - y_2^2) - 2\rho g \frac{y_1^2 y_2^2}{y_1 + y_2}\left(\frac{1}{y_2} - \frac{1}{y_1}\right)$$

$$F = \frac{\gamma_w (y_1 - y_2)}{2}\left(y_1 + y_2 - \frac{4 y_1 y_2}{y_1 + y_2}\right)$$

$$\therefore \boxed{F = \frac{\gamma_w (y_1 - y_2)^3}{2(y_1 + y_2)}}$$ (remember)

→ The above eq^n is valid for a Rectangular channel when energy loss & friction is neglected

Q:- Prove that, $\frac{y_e}{y_0} = \frac{2F_0^2}{2F_0^2+1}$ & $F_0^2 = \frac{q^2}{g y_0^3}$



Sol:- $F_r = \frac{V}{\sqrt{\frac{gA}{T}}} = \frac{Q/A}{\sqrt{\frac{gA}{T}}}$

$$\rightarrow F_r^2 = \frac{Q^2 T}{g A^3}$$

for rectangular channel, $\Rightarrow F_r^2 = \frac{Q^2 B}{g(By)^3} = \frac{(Q/B)^2}{g y^3} = \frac{q^2}{g y^3}$

[q → discharge per unit width]

→ In the free nappe, Pressure through out the depth is Atmospheric.

→ Applying Momentum eq^n between ① & ②:

$$P_1 - \cancel{P_2}^{\,0} = M_2 - M_1$$

$$\gamma_w \frac{y_0}{2} \times (y_0 \times B) = \rho Q V_e - \rho Q V_0$$

(By continuity eq^n, $B y_0 V_0 = B y_e V_e = Q$)

$$\frac{\gamma_w y_0^2 B}{2} = \frac{\rho Q^2}{B}\left(\frac{1}{y_e} - \frac{1}{y_0}\right)$$

$$\frac{g y_0^2}{2} = \frac{(Q/B)^2}{y_0}\left(\frac{y_0}{y_e} - 1\right) \quad \text{-----} \quad (q = Q/B) \; \& \; (\gamma_w = \rho g)$$

$$\frac{1}{\frac{2q^2}{g y_0^3}} = \left(\frac{y_0}{y_e} - 1\right) \quad \text{------} \quad \left(F_0^2 = \frac{q^2}{g y_0^3}\right)$$

$$\Rightarrow \frac{y_0}{y_e} - 1 = \frac{1}{2F_0^2}$$

$$\Rightarrow \boxed{\frac{y_e}{y_0} = \frac{2F_0^2}{1+2F_0^2}}$$

(Proved)

Note:- Generally eq^n used in the analysis of Open channel flow are,

i) Energy equation
ii) Momentum equation
iii) Continuity equation.

2. Uniform flow

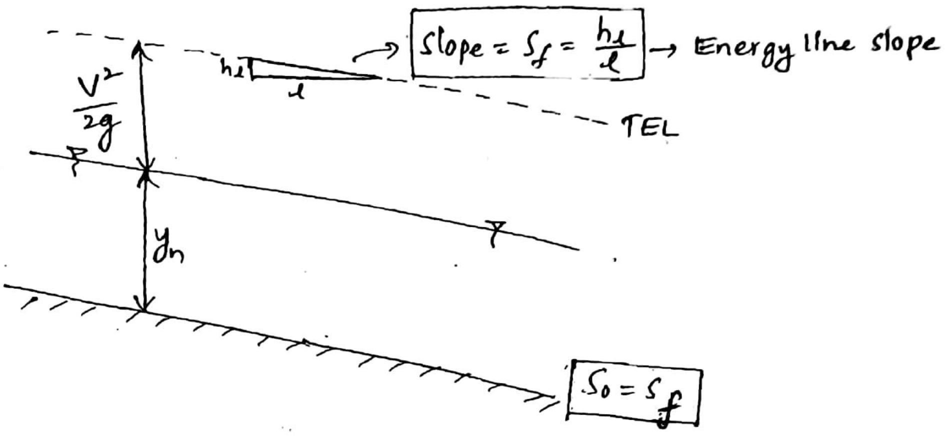
→ **Uniform flow ;**

→ Flow in an open channel is said to be uniform, if the depth of flow is same at every section of the channel, and this constant depth is called "normal depth of flow (y_n)".

→ for uniform flow, bed slope, Cross-section & surface roughness must remains constant thus the depth, Velocity & Discharge remains constant.

→ In the Analysis of uniform flow, flow is assumed to be steady.

→ As the depth & Velocity is same at every cross-section, the Bed slope, water-surface slope, Energy Line slope will all be same.



→ Formulae for determining Velocity and discharge are;

1. Chezy's equation ;

$$\boxed{V = C\sqrt{RS}}$$

V → Avg. velocity of flow

C → Chezy's constant

$C = \sqrt{\frac{8g}{f}}$ (f → friction factor)

$R = \frac{A}{P}$ (hydraulic radius)

S → Energy Line slope

- **Manning's Equation ;**

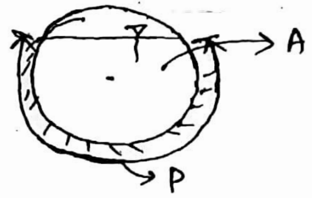
$$\boxed{V = \frac{1}{n} R^{2/3} S^{1/2}}$$

$$\boxed{h_L = \frac{f L V^2}{2gD}}$$ f ?

V = Avg. Velocity

n = manning's constant or roughness coefficient

$$\boxed{R = A/P}$$



$A \rightarrow$ Area of c/s

$P \rightarrow$ Wetted Perimeter

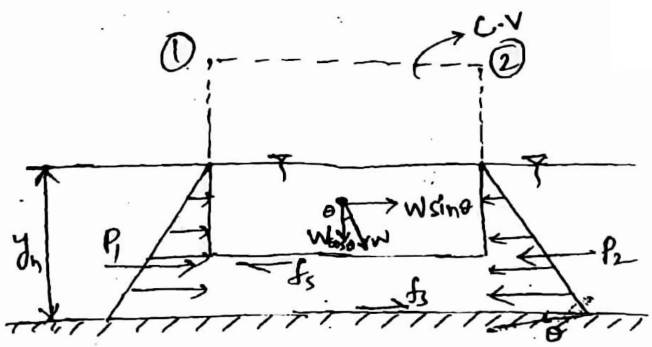
$S \rightarrow$ Energy line slope.

→ Energy line slope, However since in uniform flow, Bed slope, water surface slope, and energy line slope are equal so, $S = S_0$

Note :-

→ If "S" is used as "S_f" → We get actual depth of flow, Hence in varied flow actual depth of flow may not be equal to normal depth of flow.

Momentum equation of uniform flow ;



→ By momentum eqn on C.V. (control volume)

$$P_1 - P_2 + W\sin\theta - f_s = M_2 - M_1$$

for uniform flow, $P_1 = P_2 = \gamma_w \cdot \bar{y}_n \cdot A$

$$M_1 = M_2 = \rho Q V$$

$$\Rightarrow \boxed{f_s = W\sin\theta}$$

Energy derived from gravity = Energy loss due to friction.