



**IES/GATE**

**CIVIL ENGINEERING**

**VOLUME – XI**

**S.O.M.**



# Index

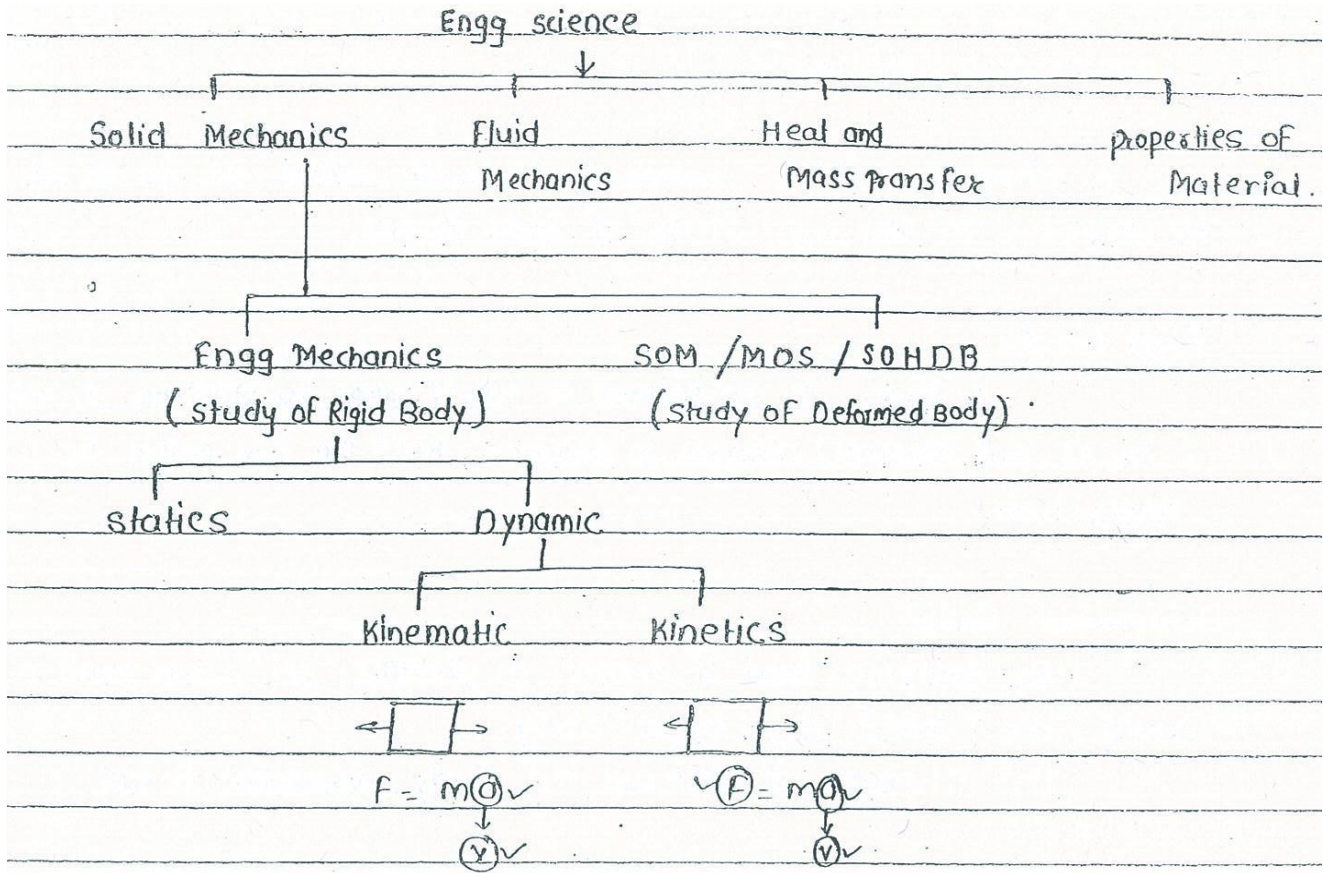
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SOM

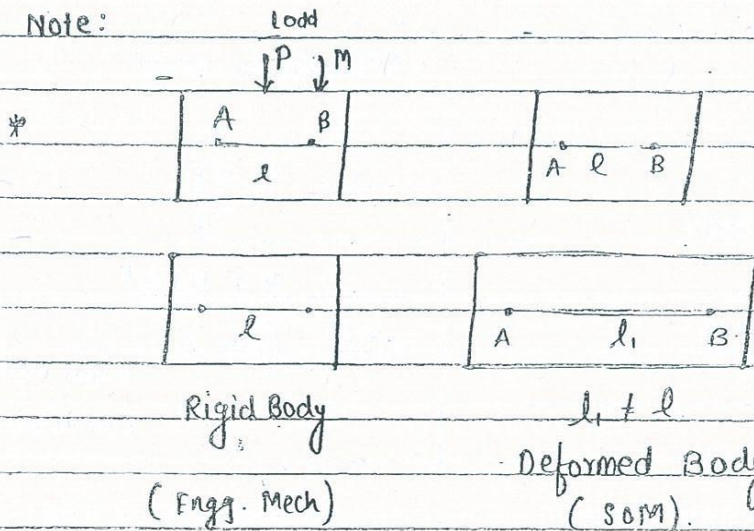
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**SOM**

# Chapter 1 - Introduction



Note:





## Engineering Mechanics

A physical science which deals with effect of forces of the object. In this we study of Rigid Bodies.

### Rigid Body

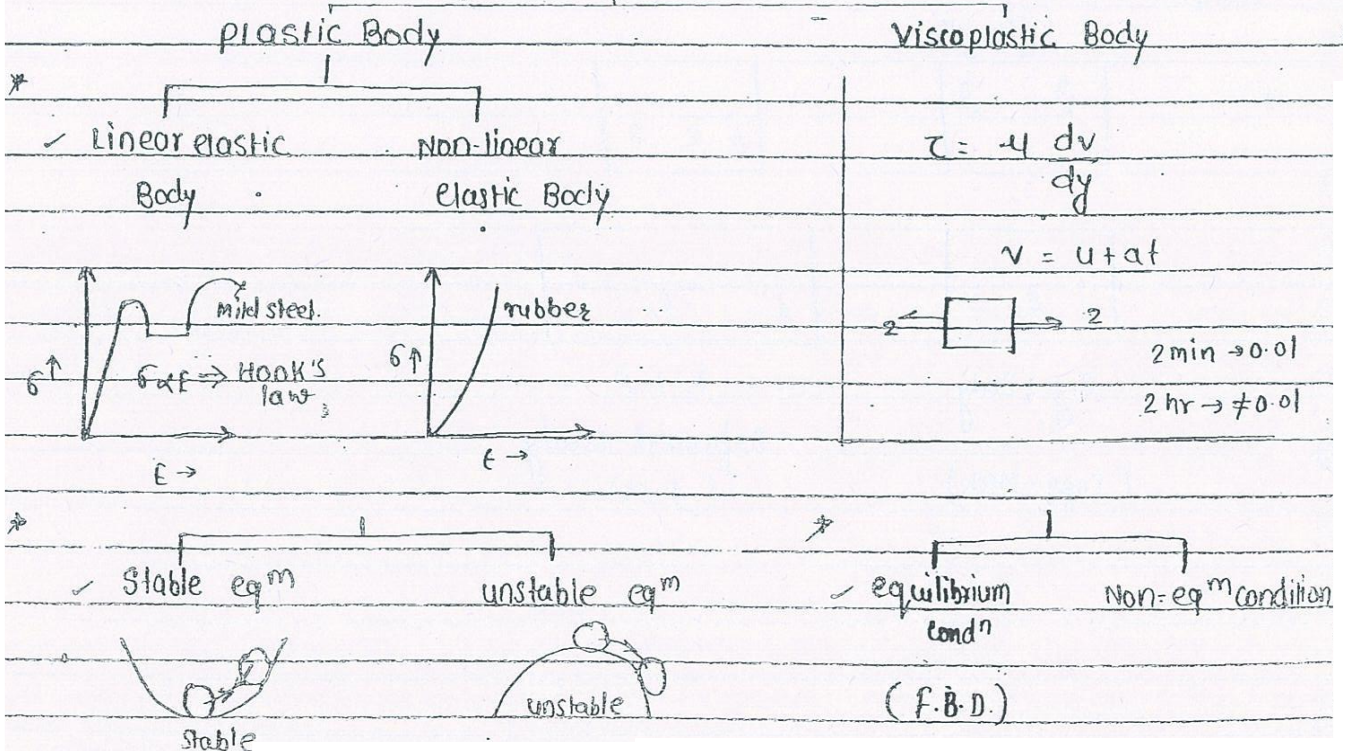
A Body is considered a rigid when by Applying load (forces and Moment) changes in distance bet<sup>n</sup> the two points is Negligible.

### Deformable Body

By Applying load if changes in distance bet<sup>n</sup> two points is considerable called deformable body.

In Strength of Material we study deformable bodies

### SOM

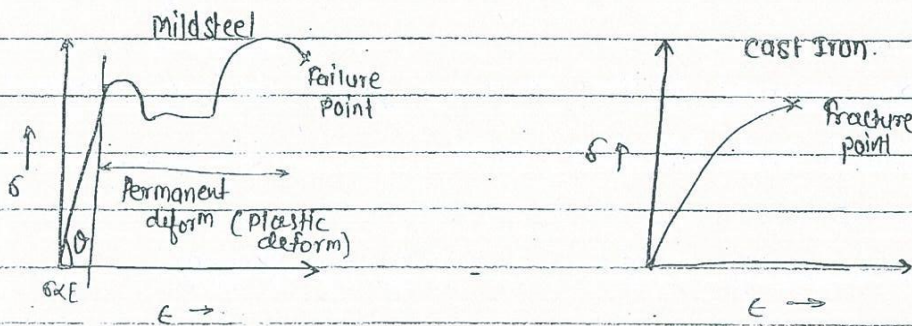
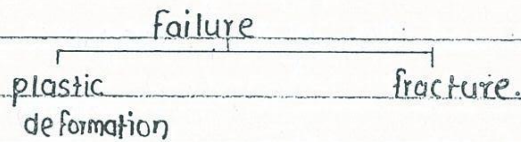




\* Solid Mechanics are developed a Relationship bet<sup>n</sup> the load Applied two Non Rigid Body, Internal forces and The deformation Induced in the Body.

\* Viscoplastic solid explain The Time dependant inelastic Behaviour of The solid (Viscoplastic and Viscoelastic solid) have high plastic deformation zone.

### \* Failure



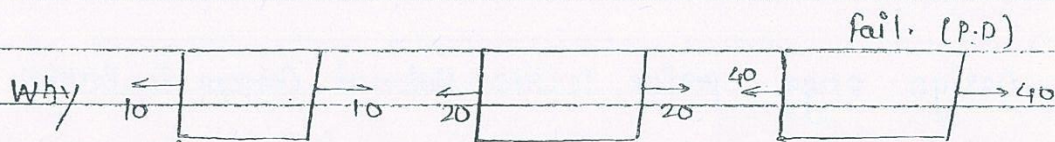
- Design engg. prefer Ductile Material Compare to Brittle Material
- In Ductile Material when plastic deformation starts (perm. deform or yielding) failure occurs.
- In Brittle Material There is no plastic deformation and failure occurs at the fractured point
- In Ductile Material Before fracture some indication to the designer in the form of plastic deformation that Mean Component will fail after some time But in Brittle Material There is no indication in the form of yielding before fracture.



That's why designer preferred ductile Material for designing purpose.

## Strength

- It is defined as max<sup>m</sup> value of stress that a Material can with stand without any failure.
- Strength is the Mechanical property of Material which does not depend on shape and size of the Component.
- It has a constant value (It is a true characteristic of Material).
- Stress can be carried in the Material which does not depend on the load Applied and c/s Area used.
- Mild steel is frequently used for the designing purpose.



$$\sigma = \frac{P_i}{A} = \frac{10}{2}$$

$$= 5 \text{ mpa}$$

$$\sigma = \frac{20}{2}$$

$$= 10 \text{ mpa}$$

$$\sigma = \frac{40}{2}$$

$$= 20 \text{ Mpa}$$

$\sigma = 20 \rightarrow$  strength of Material

\* stress only due to Internal force.



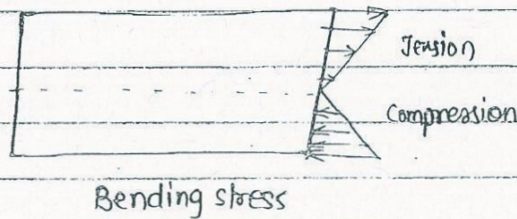
\* Why Mild steel is preferred for the Design purpose?

- It is ductile Material so it provide Indication to the designer in the form of yielding before fracture. (Brittle Material does not show yielding Behaviour).
- Coefficient of Thermal expansion of Mild steel is Almost equal to the coefficient of Thermal expansion of concrete.
- Young's Modulus of elasticity is same if Both Tension and Compression

$$E_{\text{Tension}} = E_{\text{Compression}}$$

$$E_p = \frac{\text{Stress}}{\text{Strain}}$$

$$E_c = \frac{\text{comp. stress}}{\text{Strain}}$$

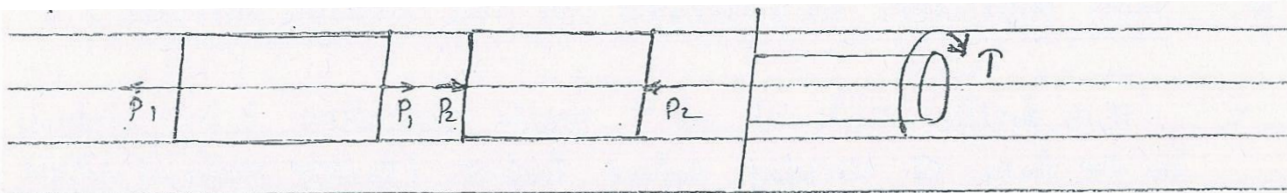


- Steel can be easily welded and joined
- It is easily Available and have less cost Compare to other Material.
- It is easily Recyclable
- Scrap value is less good.

\* Ductility

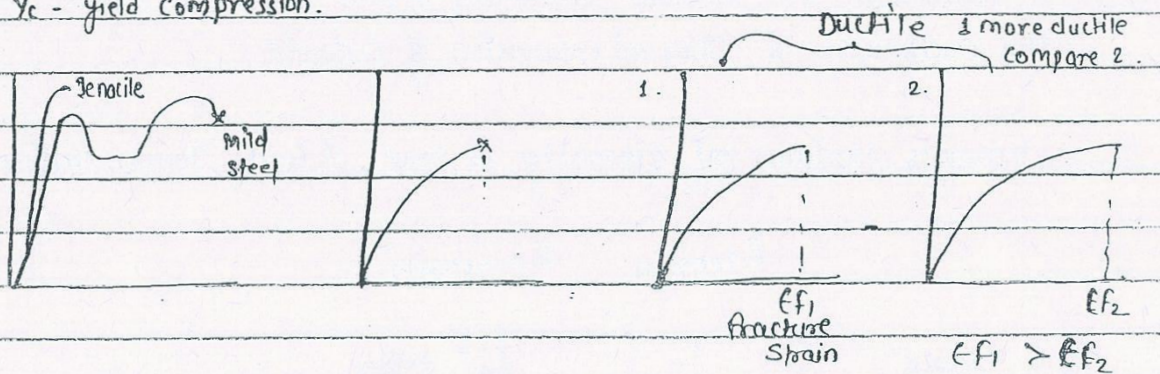
- Ductility Mean Can be drawn easily into thin wires.
- Ductile Material is strong in Tension, Moderate in Compression and weak in shear.  
(Any Material which have high tensile strength called as Tenacite Material)
- when the fracture strain is large it is called ductile material.





$$\sigma_{yt} > \sigma_{yc} > \tau_y$$

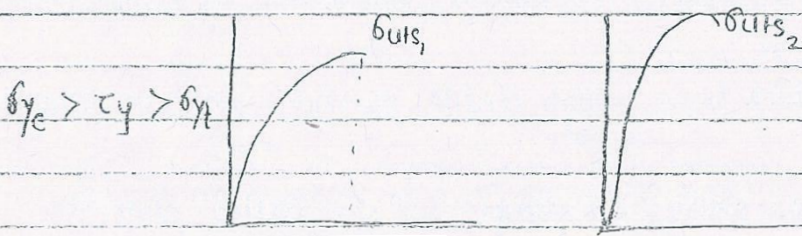
$\sigma_{yt}$  - yield tension  
 $\sigma_{yc}$  - yield compression.



Mild steel : Aluminium, Copper, Gold, silver, lead.

### \* Brittleness

- Lack of ductility is called Brittleness
- Brittle Material Strong in Compression, Moderate in shear and weak in Tension.
- Tungsten Brittle at Room Temperature and <sup>ductile</sup> elevated at elevated Temp
- Brittle Material have less fracture strain value compare to ductile Material



$\sigma_{UTS2} > \sigma_{UTS1}$   
 ← more brittle

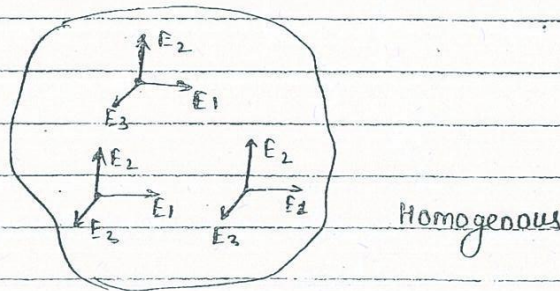


\* Assumption used in SOM

- Material is continuous and deformable that means No voids and cracks in the Material.
- All the Materials are Homogenous and Isotropic

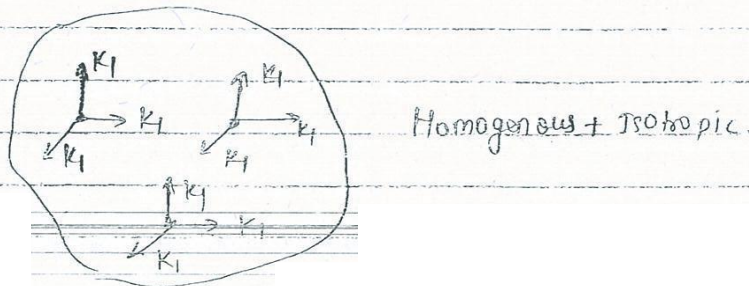
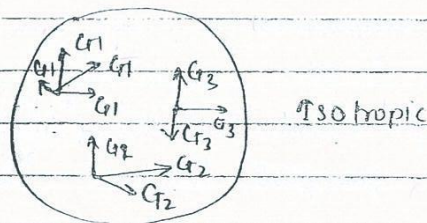
Homogenous :-

A Material is called homogenous when it have same elastic property  $E, G, K, \nu$  at any point in given direction.  
 ex. Wood, Iron, Aluminium, steel.



Isotropic :-

A Material is called Isotropic when it has same elastic property in Any Direction at a given point.  
 ex. glass, Brass, Aluminium, Copper, steel, Iron

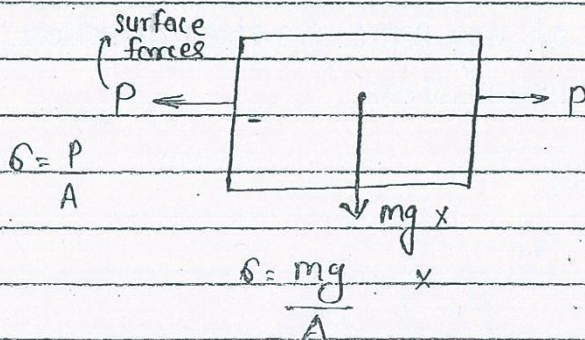




Note :- If Test is conducted on Average value Metal shows homogenous and isotropic property

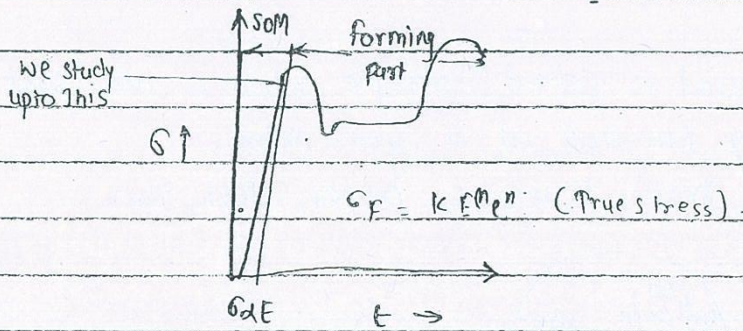
But if Test is conducted on Molecular level (Atomic level) Material are Non Homogenous and not show isotropic properties.

- All Member are weightless that mean effect of body forces are Neglected and Analysis is carried out by the surface forces only.



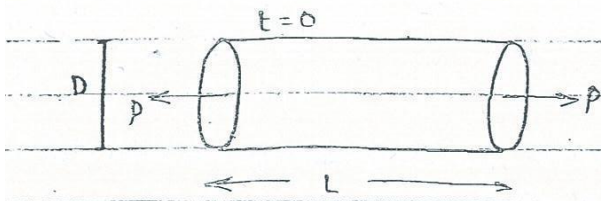
- Material follow hook's law that Means (Stress  $\propto$  Strain)

$$\epsilon = \frac{\text{Stress}}{\text{Strain}}$$

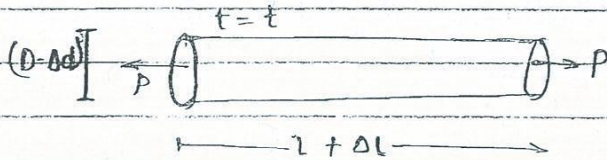


- Superposition principal is valid (Algebraic sum) is used for the Calculation purpose Because hook's law is Valid.



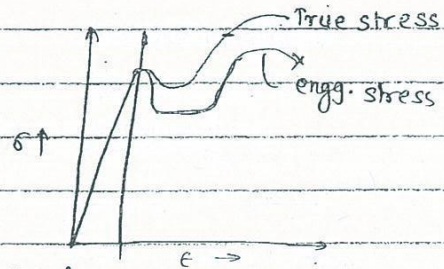
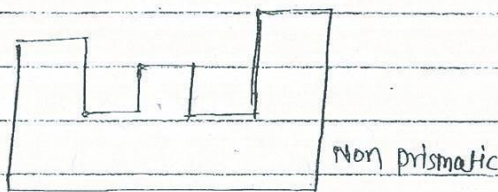


$$\text{engg stress} = \frac{P}{A_{\text{initial}}} = \frac{P}{\frac{\pi}{4} D^2}$$



$$\text{True stress} = \frac{P}{A_{\text{instantaniow}}} = \frac{P}{\frac{\pi}{4} (D - \Delta d)^2}$$

- Effect of True stress and True strain is Neglected.



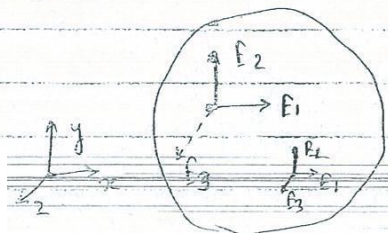
- All The Material should be prismatic in nature  
(prismatic - uniform cross section throughout the length)

- Effect of residual stresses (Initial stresses) are Neglected that Mean Material used in The Analysis are stressed free.

- Material follow Saint Venant's principal

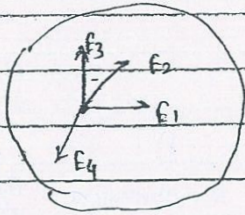
### Orthotropic

A Material is said to be orthotropic if it has different elastic properties at any point in a given 1<sup>er</sup> direction. (Orthogonal dir<sup>n</sup>)  
ex wood, graphite (All Material with layers).





\* Anisotropic



It has different elastic property in different direction at a given point

ex. Material with cracks, voids, will behave as Anisotropic Material.

\* Factor of Safety

Ductile Material

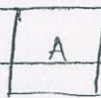
$$FOS = \frac{\text{Yield stress}}{\text{Design stress / working stress}}$$

Brittle Material

$$FOS = \frac{\text{ultimate Tensile strength}}{\text{Design stress / working stress}}$$

$$FOS > 1$$

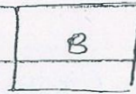
$$FOS = 1.6 \text{ to } 4$$



$$FOS = 2$$

$$\sigma_{Des} = \frac{100}{2}$$

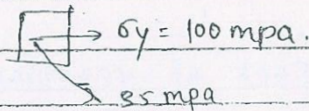
$$= 50 \text{ mpa}$$



$$FOS = 4$$

$$\sigma_{Des} = \frac{100}{4}$$

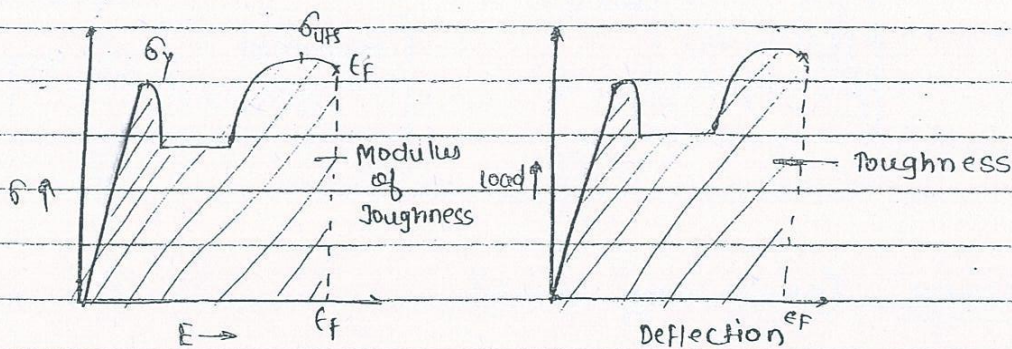
$$= 25 \text{ mpa}$$



- Since Assumption is used in SOM not possible practically so Factor of Safety is used for the design Calculation
- Higher The value of FOS lower will be the design efficiency that mean total cost for designing is increased
- If used High FOS then for full filling the Req<sup>m</sup> either High strength Material is used or %s Area of the same Material is increased. So both the Material <sup>condition</sup> cost will increase (for increasing the %s Area some extra material deposit which increase the cost)



## \* Toughness



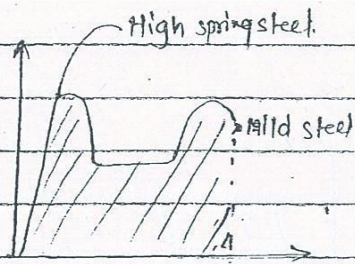
$$\text{Toughness} = \frac{\text{M.O.T}}{\text{Vol. of component}}$$

$$\text{M.O.T} = \frac{\sigma_u + \sigma_y}{2} \cdot \epsilon_f$$

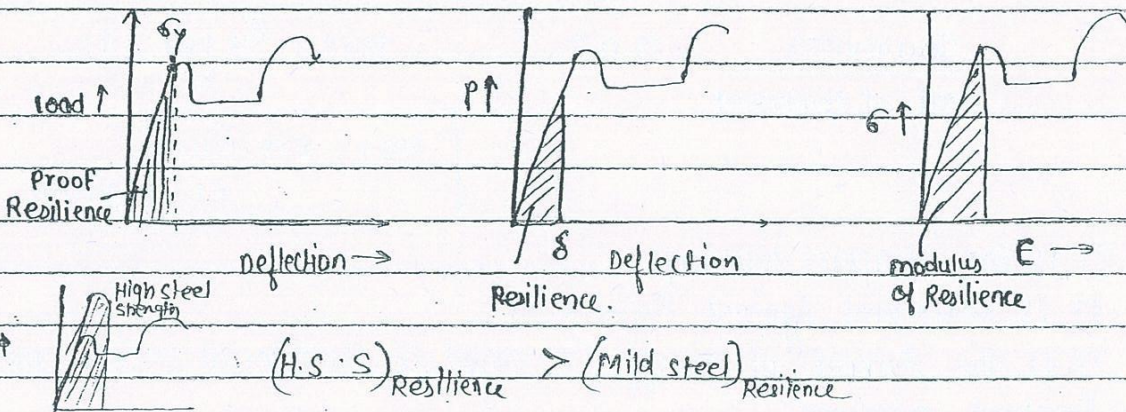
$$\text{M.O.T} = 0.75 \frac{\sigma_{UTS}}{\epsilon_f} \leftarrow \text{Brittle}$$

- It is for Impact loading
- It is Resistant Against Fracture.
- It is The Amount of energy i.e stored in The Material Before The Fracture Occures.
- To Determine The Toughness Tensile test is Conducted on The specimen and stress-strain dig. is obtained. (By load-deflection curve)
- The Total Area under stress-strain curve will Represent the energy Req. per unit volume to fracture The Material.
- Iron is much Tougher Than glass because when we applied Impact load glass will break immediately while iron piece absorbed substantial energy before fracture.
- For Designing of gears, clutches, Crane, chains, <sup>very</sup> High strength Material used that time designer used toughness property that so designed beyond the yield strain.
- since Ductile Material fracture strength is Higher than Brittle Material so generally toughness is more in ductile material compare to Brittle material.





\* Resilience (strain energy Density)



$$\begin{aligned}
 \text{Resilience} &= \frac{1}{2} \times P \times \delta & - P < \frac{P}{A} & \frac{P}{A} < \frac{P}{A} \\
 &= \frac{1}{2} \times (\sigma \cdot A) (E \cdot L) & (\sigma = \frac{P}{A}) & L + \Delta L \\
 &= \frac{1}{2} \times \sigma \cdot E (A \cdot L) & E = \frac{\sigma}{\epsilon} & \text{Strain} = \frac{\Delta L}{L} \\
 &= \frac{1}{2} \times \sigma \cdot \frac{\sigma}{E} \cdot V & \epsilon = \frac{\sigma}{E} & \Delta L = \delta
 \end{aligned}$$

\*  $\text{Resilience} = \frac{\sigma^2}{2E} \times \text{Volume}$

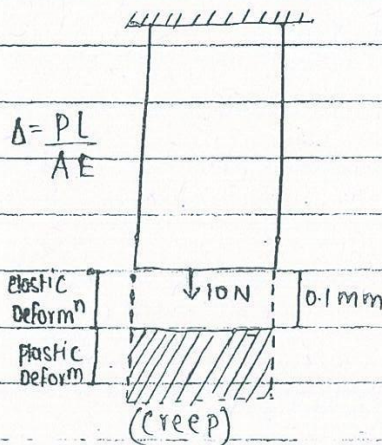
\*  $\text{proof Resilience} = \frac{\sigma_y^2}{2E} \times \text{Volume} \quad (\sigma_y = \sigma_{\max})$

M.O.R =  $\frac{\sigma_y^2}{2E}$

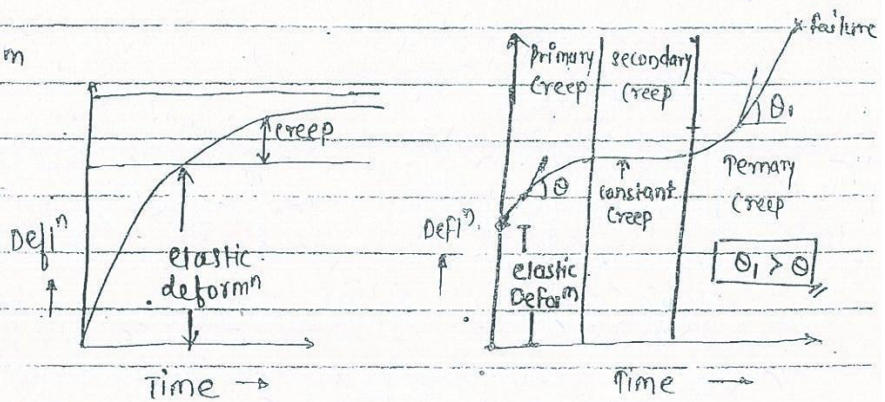


- The ability of Material to absorb energy when deformed elastically and to Return it when unloaded is called Resilience.
- This property is used for shock and vibration load.  
eg. used in spring.
- Since Modulus of Resilience does not depend on the dimension so it is True characteristic of the Material (It has constant value)

### \* Creep



- 1] Temperature      Fe  $\rightarrow$  1539°C
- 2] Time                 $\left(\frac{MP}{2}\right) = \frac{1539}{2}$
- 3] Magnitude of Load       $\downarrow$
- 4] Type of load.              Homologous Temp<sup>r</sup>



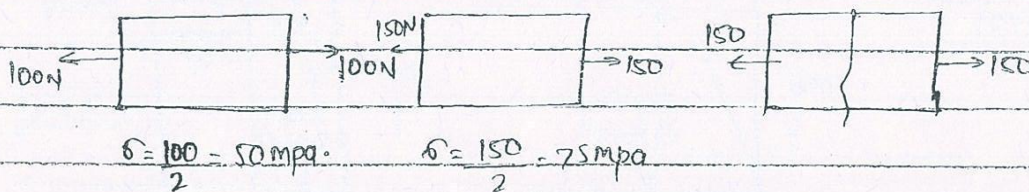
- It is a Material property due to this Material gets deformed progressively at a slow rate with time at constant stress (due to dead load)
- Most Metal show Creep at elevated Temp, when Temp. Reaches half of the Melting point Temp of Metal creep becomes unacceptable. Such Temp. is called Homologous Temp.

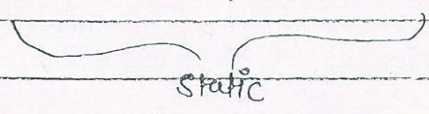


- Aircraft, pressure vessel at High Temp, Steam Turbine, gas turbine are subjected to constant stress for a long period under such condition material experience slow deformation which is called creep.
- In primary creep Rate of creep deformation decreases with time
- In secondary creep Rate of creep deformation is constant and called steady state.
- In tertiary creep Rate of creep deformation increases with time until fracture occurs (called progressive stage).

### \* Fatigue

yield strength =  $\sigma_y = 100 \text{ mpa}$



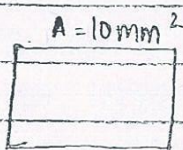

  
 static

force — vector quantity — }
  
Direction
  
Magnitude

$\sigma_y = 250 \text{ mpa}$  - mild steel

$\sigma_e = 181 \text{ mpa}$  - endurance limit (fatigue limit)

eg.



Static load

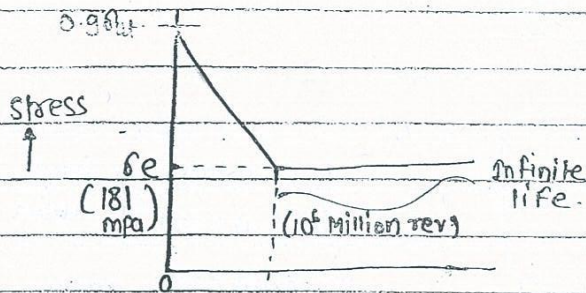
$f = 250 \times 10 = 2500 \text{ N}$

variable load

$f = 181 \times 10 = 1810 \text{ N}$

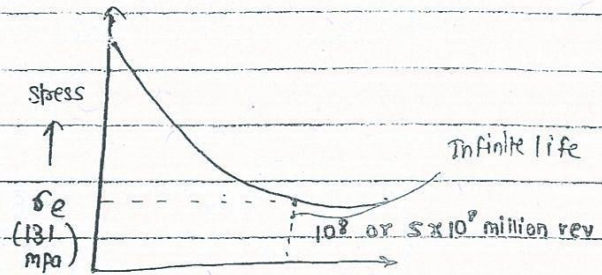


## S-N Diagram (fatigue).



No of Revolution Before  
failure

(Mild steel)



No of Revolution Before  
failure

(Aluminium)

- when The loading is cyclic (variable) and No of Repetitions are large a Metal Specimen may fail at a stress level below The yield stress such a failure is called fatigue failure.
- fatigue failure is to be Avoided when stress developed in The Component does not cross a particular limit called endurance limit or fatigue limit.
- fatigue failure always show brittle fracture in The ductile Material. (If variable load is Applied in The ductile Material it will not give any indication in the form of yielding (plastic deformation) before fracture.  
eg Aircraft wings, Bridges, Multistorey Building, steam and gas turbine Bear this type of variable load.
- endurance limit is quite below The yield strength of Material