IES/GATE

Electrical Engineering

VOLUME-VI

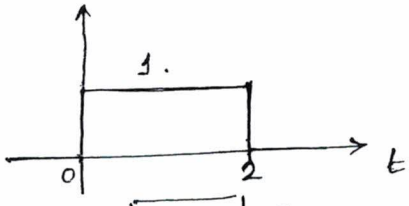
Signals & Systems

CONTENT

Different Operations on Signal

i) Time Shifting :

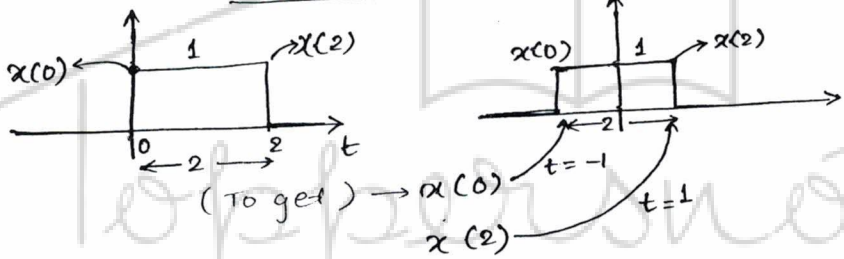
$$x(t) \longrightarrow y(t) = x(t+k)$$



Case (a) : when $k > 0$ → +ve (Left shifting)

ex : $k = 1$

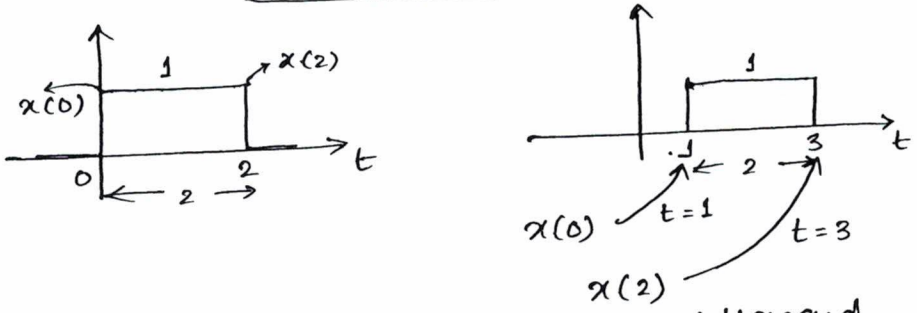
$$x(t) \longrightarrow y(t) = x(t+1)$$



Case (b) : When $k < 0$ → −ve (Right shifting)

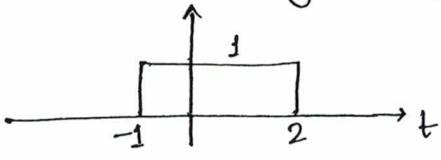
ex $k = -1$

$$x(t) \longrightarrow y(t) = x(t-1)$$



ii) Amplitude - Shifting : → Upward, Downward.

$$x(t) \longrightarrow y(t) = k + x(t)$$



Case (a) : When $k > 0$

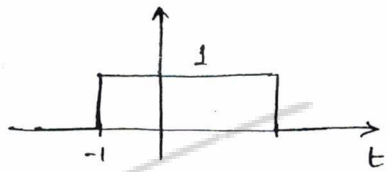
ex → $k = 1$.

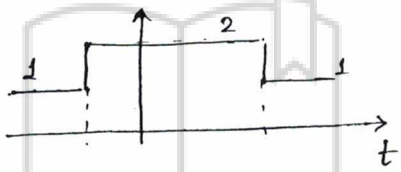
$$\rightarrow x(t) = \begin{cases} 0 ; & t < -1 \\ 1 , & -1 \le t \le 2 \\ 0 ; & t > 2 \end{cases}$$

$$\rightarrow y(t) = 1 + x(t)$$

$$= \begin{cases} 1+0 \Rightarrow 1 ; & t < -1 \\ 1+1 \Rightarrow 2 ; & -1 \le t \le 2 \\ 1+0 \Rightarrow 1 ; & t > 2 \end{cases}$$

$x(t) \rightarrow y(t) = 1 + x(t)$





Case (b) : when $k < 0$

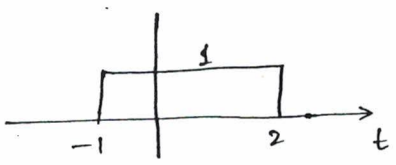
ex : $k = -1$.

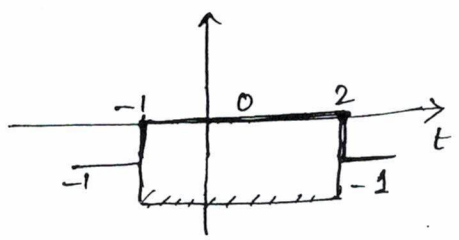
$$\rightarrow x(t) = \begin{cases} 0 , & t < -1 \\ 1 , & -1 \le t \le 2 \\ 0 , & t > 2 \end{cases}$$

$$\rightarrow y(t) = -1 + x(t)$$

$$= \begin{cases} -1+0 \Rightarrow -1 ; & t < -1 \\ -1+1 \Rightarrow 0 ; & -1 \le t \le 2 \\ -1+0 \Rightarrow -1 ; & t > 2 \end{cases}$$

$x(t) \rightarrow y(t) = -1 + x(t)$

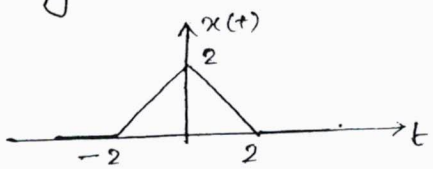




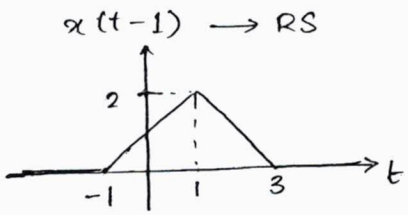
Q. Draw the waveform of

$$y(t) = -4 + x(t-1)$$

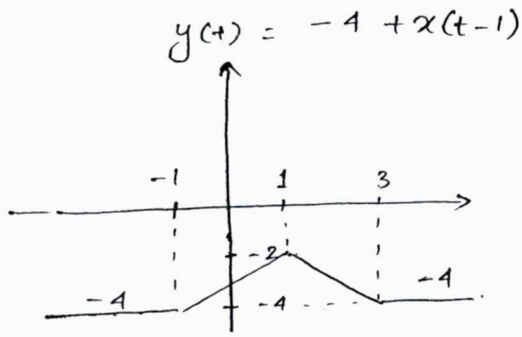
where



$\Rightarrow$ <i>

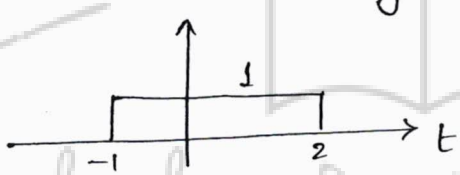


$\Rightarrow$



iii) Time Scaling :

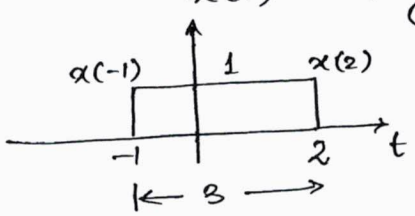
$$x(t) \longrightarrow y(t) = x(at) \; ; \; a \neq 0$$

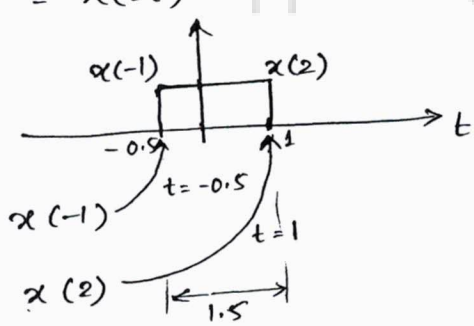


Case a) : when $\boxed{a > 1}$ → Compression.

Ex : $a = 2$

$$x(t) \longrightarrow y(t) = x(2t)$$

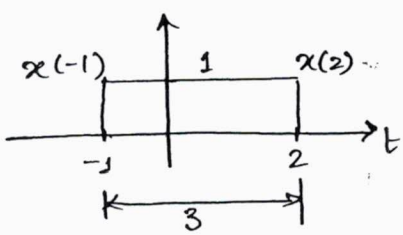


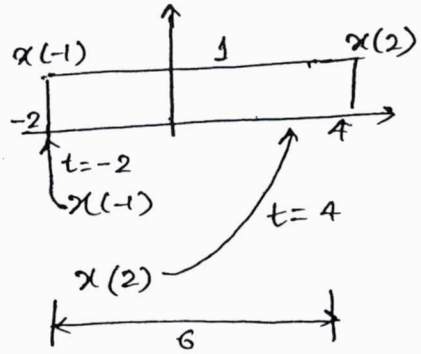


Case b) : when $\boxed{0 < a < 1}$ → Expansion

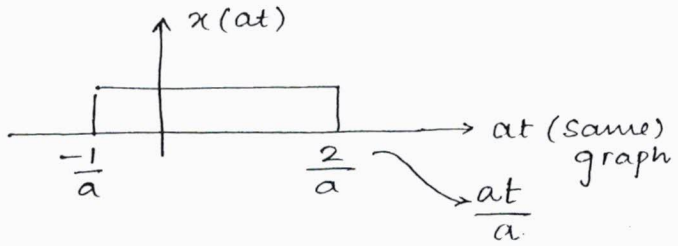
ex : $a = 0.5$

$$x(t) \longrightarrow y(t) = x(0.5t)$$

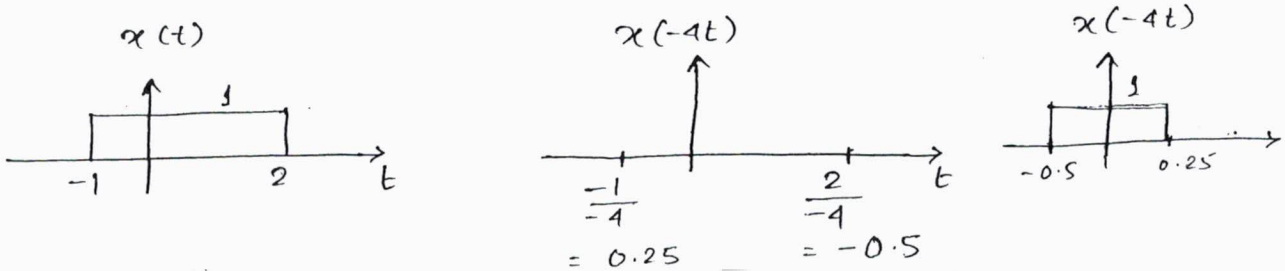




General Rule :

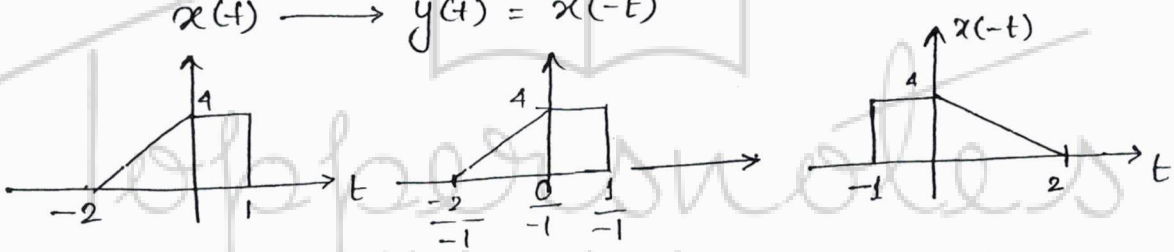


Ex : $x(-4t)$



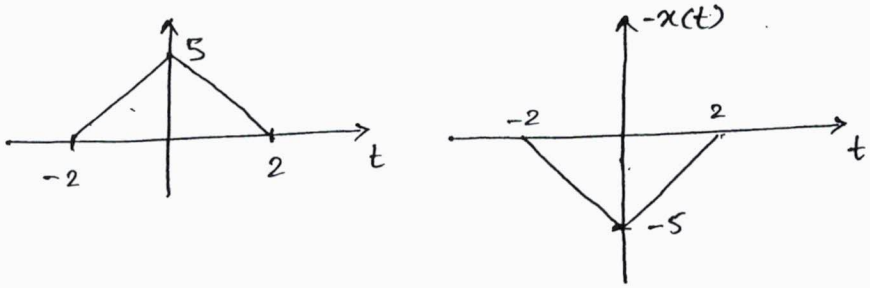
iv) Time Reversal : → folding about Y axis

$x(t) \longrightarrow y(t) = x(-t)$ → Time scaling $a = -1$

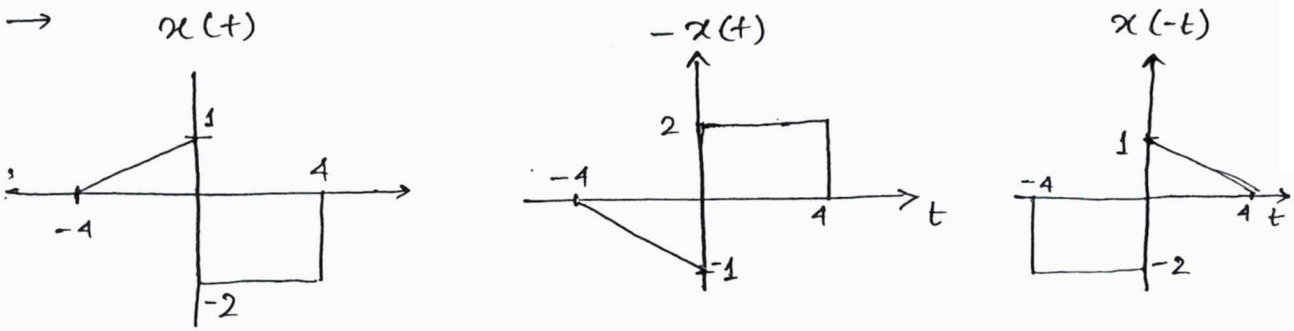


v) Amplitude Reversal :- → folding about X axis

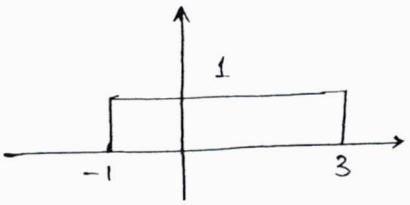
$x(t) \longrightarrow y(t) = -x(t)$



Ex → $x(t)$



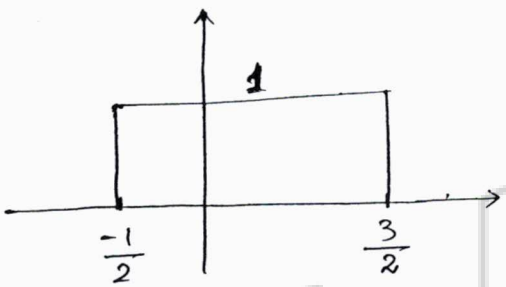
Q. $x(t)$



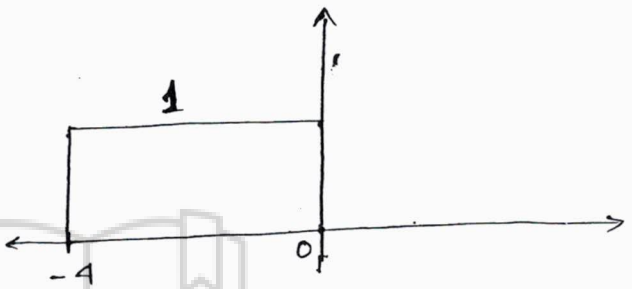
Draw the waveform of $y(t)$

$y(t) = x(2t+3)$

$x(2t)$



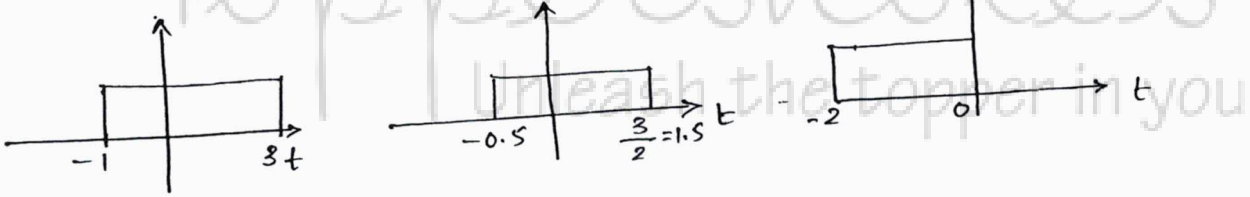
$x(2t+3)$



1st Method :

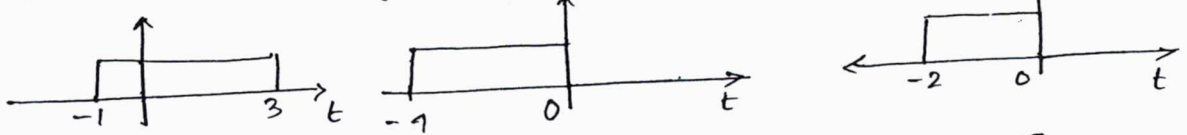
$y(t) = x(2t+3) = x[2(t+1.5)]$

$x(t) \xrightarrow{\text{Scaling}} x(2t) \xrightarrow{\text{Shifting}} y(t)$ ← LS by 1.5

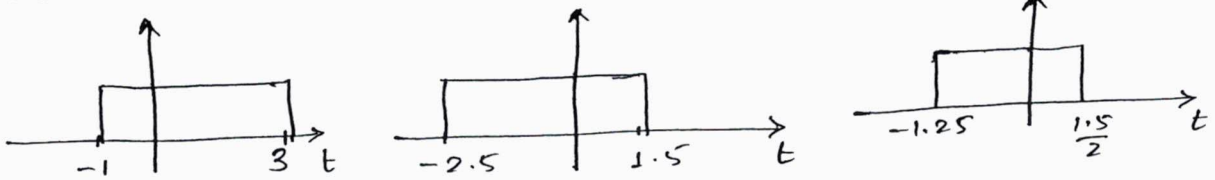


2nd Method : $y(t) = x(2t+3)$

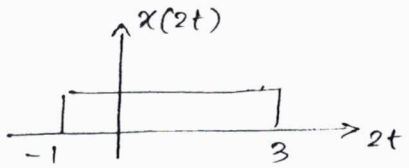
1. $x(t) \xrightarrow{\text{Shifting}} x(t+3) \xrightarrow{\text{Scaling}} x((2)t+3)$

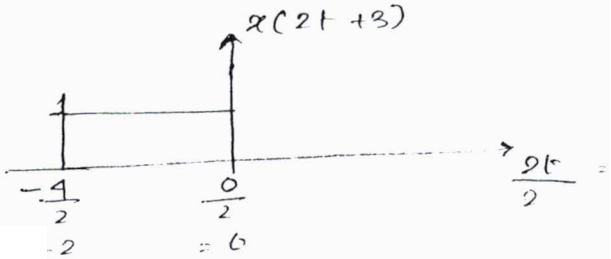


2. $x(t) \xrightarrow{\text{Shifting}} x(t+1.5) \longrightarrow x[2(t+1.5)]$

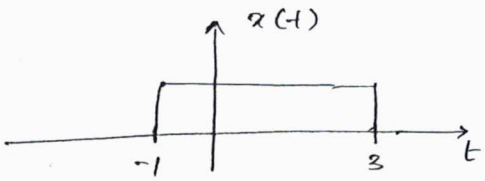


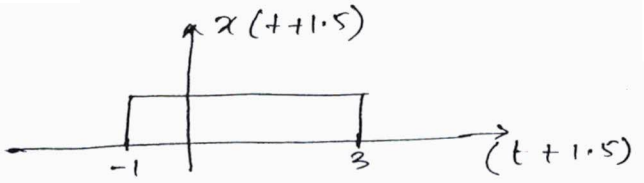
3rd Method :

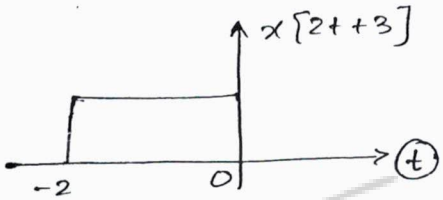


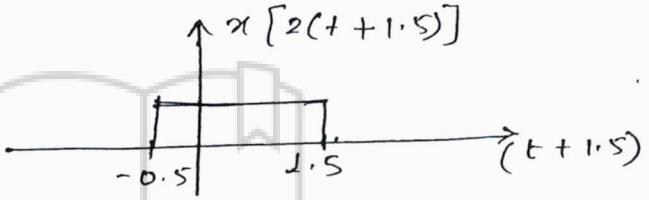


4th Method :

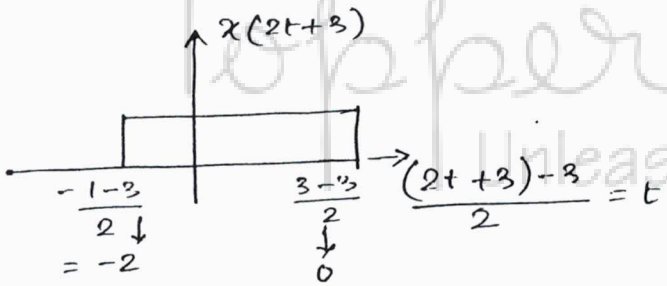


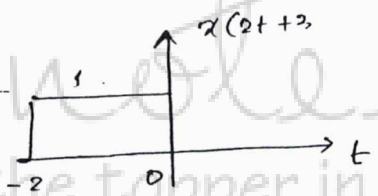




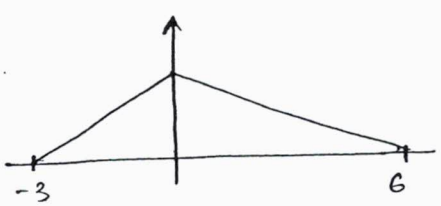


Shortcut :- (for waveform)





Q. x(t)

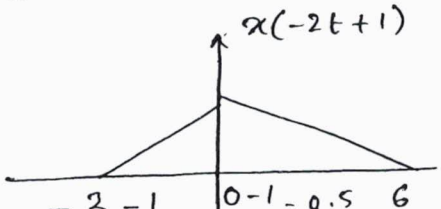


Draw the waveform of

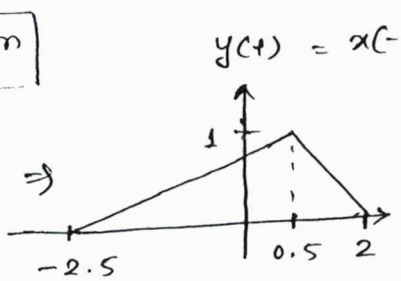
$y(t)$

$y(t) = x(-2t+1)$

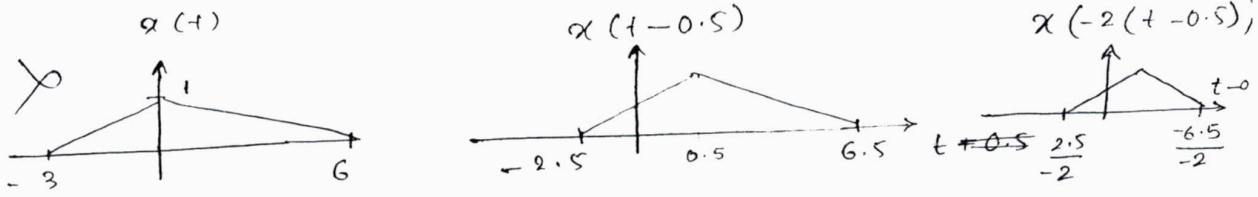
Shortcut :→ Slope change → operation



$\frac{(-2t+1)-1}{-2} = t$



4th Method: $y(t) = x(-2(t-0.5))$

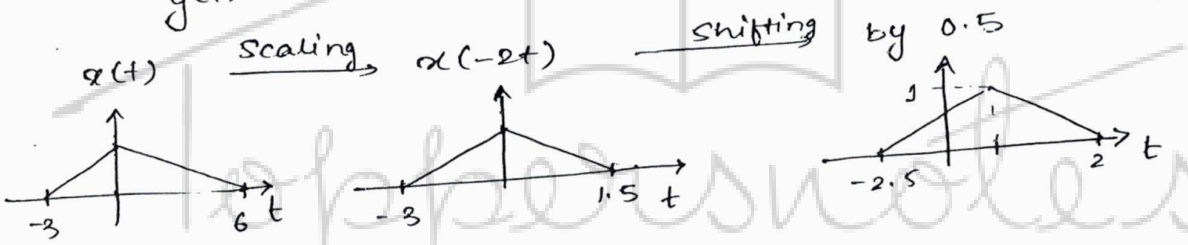


$x(-2t+1)$

1st Method :

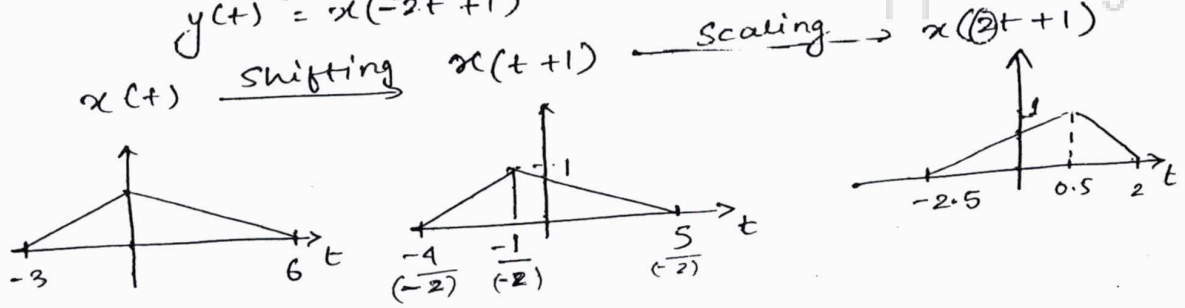
RS by $0.5 \rightarrow t$

$$y(t) = x(-2t+1) = x[-2(t-0.5)]$$



2nd Method :

$$y(t) = x(-2t+1)$$



1. SIGNAL DEFINITION & ITS CLASSIFICATION:

Signal:
A signal is a fn. which contains some information.

System:
A system is a medium which processes a signal. It is interconnection of devices or components which converts signal from one form to another form.

Classification of Signal:

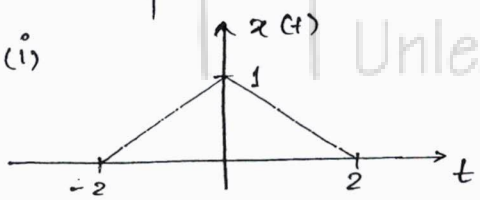
1. Even / Odd Signal:

a) <u>Even Signal</u>: Even signals are symmetrical or mirror image about Y axis.

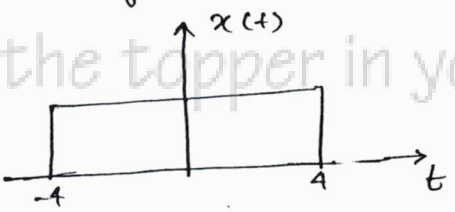
i.e $\boxed{x(t) = x(-t)}$ ⟶ Time Reversal.

* Even signals are independent of time reversal.

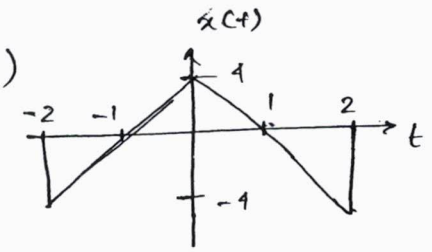
Ex — (i)



ii)



iii)



iv) $x(t) = \boxed{\cos\omega_0 t}$ ⟶ even signal.

$t = -t$

$x(-t) = \cos(-\omega_0 t)$

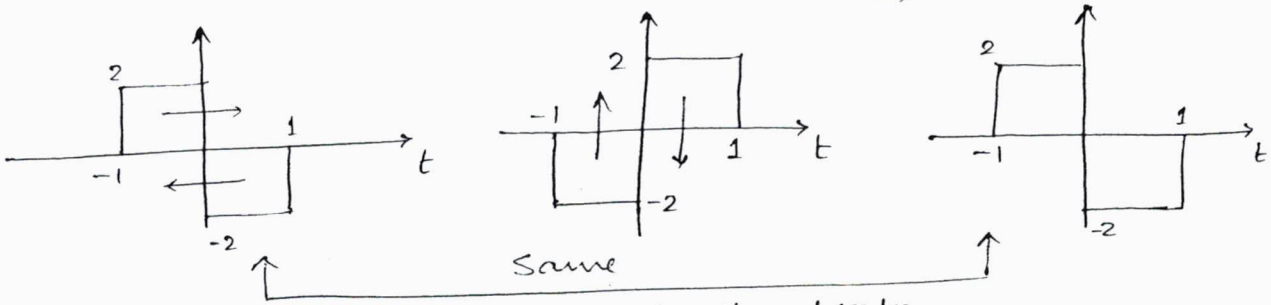
$= \cos\omega_0 t$

$\therefore x(t) = x(-t)$

b) <u>Odd Signal</u>: Odd signals are having <u>anti-symmetric LHS and RHS.</u>

i.e $\boxed{\begin{array}{c} x(t) = -x(-t) \\ \text{or} \\ x(-t) = -x(t) \end{array}}$ → Time Reversal, Amplitude Reversal.

Ex-1. $x(t) \longrightarrow x(-t) \longrightarrow -x(-t)$

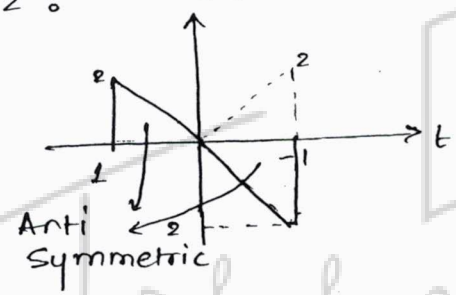


Same

So odd signal or fn.

Antisymmetric $\longrightarrow$ (Mirror of Mirror Image)
image

Ex-2 : $x(t) \longrightarrow$ odd.



Ex-3 : $x(t) = \sin\omega_0 t \longrightarrow$ odd signal.

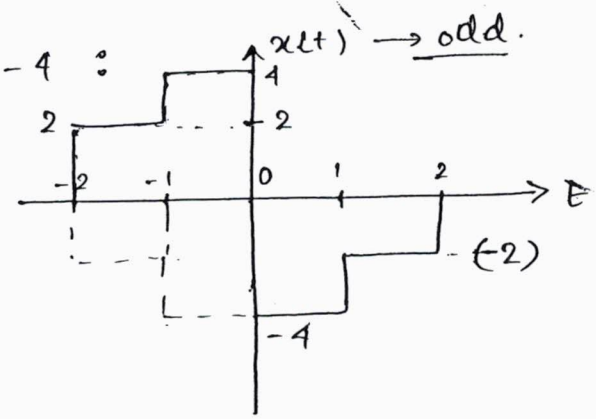
$$x(-t) = \sin(\omega_0(-t))$$

$$= -\sin\omega_0 t$$

$$= -x(t)$$

$$\therefore \boxed{x(-t) = -x(t)} \longrightarrow \text{Odd signal.}$$

Ex-4 : $x(t) \longrightarrow$ odd.



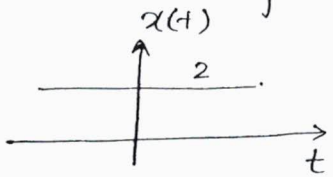
NOTE:

For any odd signal the avg value will be equal to 0.

Ex.5 → $x(t) = 2$ = DC signal → Even Signal

$t = -t$

$x(-t) = 2 = x(t)$



Ex. 6 → $x(k) = \sin[k^2]$

$k = -k$

$x(-k) = \sin[(-k)^2]$

$= \sin k^2$ = even signal.

$x(k) = (x(-k))$

Ex-7 → $x(z) = \sin\left(\frac{\pi}{2}\right) = 1$ = DC signal → even signal.

NOTE:

Any signal can be divided into 2 parts in which 1 part will be even and the other part will be odd.

i.e $x(t) = x_e(t) + x_o(t)$

where $x_e(t)$ = even part of $x(t)$

$$= \frac{x(t) + x(-t)}{2}$$

$x_o(t)$ = odd part of $x(t)$

$$= \frac{x(t) - x(-t)}{2}$$

15 Some important points :

1. $E \overset{\cdot}{\times} E = E$

$(t^2 \times t^4) = t^6$

2. $O \overset{\cdot}{\times} E = O$

$(t^3 \times t^2) = t^5$

3. $O \overset{\cdot}{\times} O = E$

$t^3 \times t = t^4$

4. $E \pm E = E$

ex : $x(t) = \cos t + t^2$

$\downarrow t = -t$

$x(-t) = \cos(-t) + (-t)^2$

$= \cos t + t^2$

$\therefore x(t) = x(-t) = \text{Even}$

5. $O \pm O = O$

ex. $x(t) = \sin t + t^3$

at $t = -t$

$x(-t) = (-t)^3 + \sin(-t)$

$= -t^3 + \sin(-t)$

$= -[t^3 + \sin t]$

$= -x(t) = \text{odd.}$

6. $E \pm O =$ Neither even nor odd.

ex - $x(t) = \cos t - \sin t$

at $t = -t$

$x(-t) = \cos t + \sin t \neq x(t) \neq \text{even}$

$= -(x(t)) \neq \text{odd.}$

Q. find $x_e(t)$ and $x_o(t)$ for signal

$$x(t) = \underset{\downarrow}{2} + \underset{\downarrow}{t^2}\underset{\downarrow}{\sin t} - \frac{t^3}{\cos t} + \frac{\cos^2 t}{t^2} - \frac{\sin^3 t}{t^5}$$

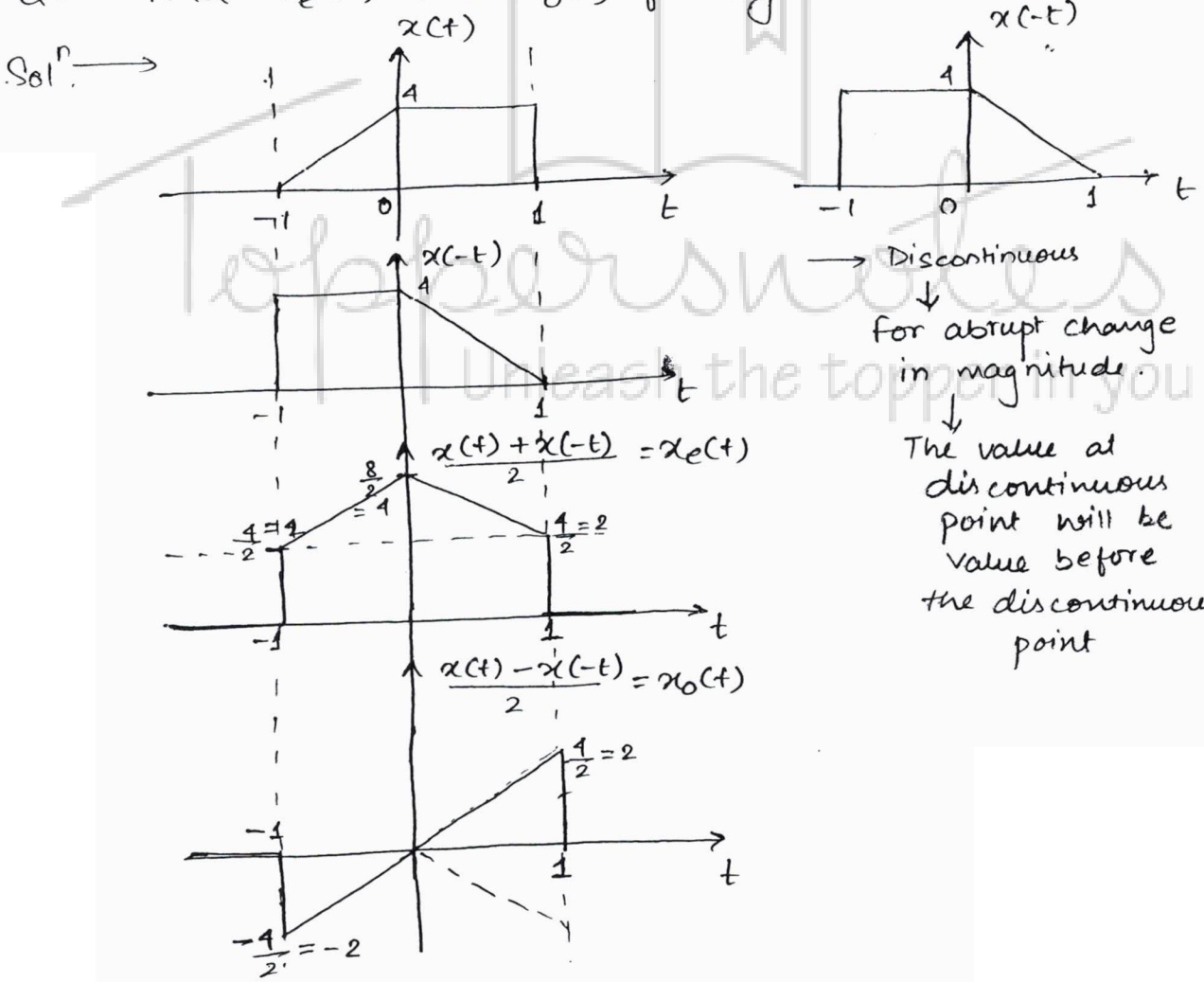
$2 \to t$; $t^2 \sin t \to (t \times 0) \to 0$; $\frac{t^3}{\cos t} \to \frac{0}{1} \to 0$; $\frac{\cos^2 t}{t^2} \to \frac{1}{t} \to t$; $\frac{\sin^3 t}{t^5} \to \frac{0}{0} \to t$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$x_o(t) = t^2 \sin t - \frac{t^3}{\cos t} \qquad x_e(t) = 2 + \frac{\cos^2 t}{t^2} - \frac{\sin^3 t}{t^5}$$

Q. find $x_e(t)$ and $x_o(t)$ for signal $x(t)$

Soln →



→ Discontinuous

↓

for abrupt change in magnitude.

↓

The value at discontinuous point will be value before the discontinuous point

2. Conjugate Symmetric (CS) and Conjugate - Antisymmetric (CAS) Signal :

a) CS Signal : $\boxed{x(t) = x^*(-t)}$

Let $x(t) = a(t) + jb(t)$ — (I)

$\downarrow$

$t = -t$

$\therefore x(-t) = a(-t) + jb(-t)$

$\downarrow *$

$x^*(-t) = a(-t) - jb(-t)$ — (II)

for CS : $\longrightarrow \boxed{x(t) = x^*(-t)}$ $\downarrow$ for conjugate Symmetry

from (I) & (II)

$a(t) = a(-t) \longrightarrow$ even

and $b(t) = -b(-t) \longrightarrow$ odd

Ex : $\underbrace{x(t)}_{CS.} = \underbrace{\cos t}_{E} + j\underbrace{t^3}_{O}$

b) CAS Signal : $\boxed{x(t) = -x^*(-t)}$

Let $x(t) = a(t) + jb(t)$

$\downarrow t = -t$

$x(-t) = a(-t) + jb(-t)$

$\downarrow *$

$x^*(-t) = a(-t) - jb(-t)$

for CAS : $x(t) = -x^*(-t)$

CAS $\longrightarrow \begin{cases} a(t) = -a(-t) \longrightarrow \text{odd} \\ b(t) = b(-t) \longrightarrow \text{Even} \end{cases}$

Ex : $\underbrace{x(t)}_{CAS} = \underbrace{t^3}_{O} + \underbrace{j\cos t}_{E}$

Q. Check CS/CAS signals

i) $x(t) = t^2$ = Real → E (CS) = R+E

ii) $x(t) = t^3$ = Real → O (CAS) = R+O

iii) $x(t) = j\cos t$ = Imaginary → E (CAS) = I+E

iv) $x(t) = j\sin t$ = Imaginary → O (CS) = I+O

NOTE:

→ Any signal can be divided into 2 parts in which 1st part will be conjugate symmetric and the other part will be conjugate antisymmetric.

i.e $x(t) = x_{CS}(t) + x_{CAS}(t)$

where $x_{CS}(t) = \frac{x(t) + x^*(-t)}{2}$ = CS part of $x(t)$

$x_{CAS}(t) = \frac{x(t) - x^*(-t)}{2}$ = CAS part of $x(t)$

→ for conjugate symmetic signal Real part should be Even and Imaginary part should be odd.

→ for CAS signal → Real part should be odd and imaginary part should be even.

5. Periodic And Non Periodic Signal:

a) Periodic Signal:

A signal is said to be periodic if it repeats itself after ~~sometime~~ Time period.

i.e $\boxed{x(t) = x(t \pm n T_0)}$

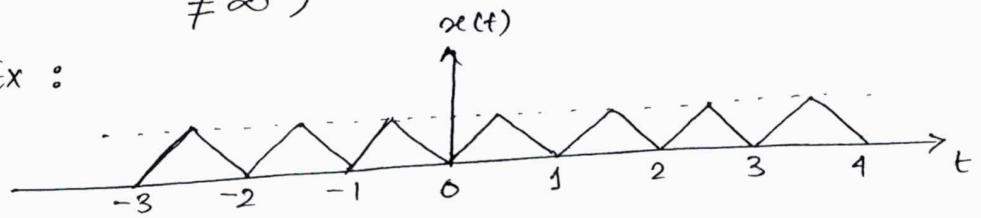
where n = an integer
= 1, 2, 3 - - -

T_0 = FTP (fundamental Time Period)

= It is the smallest, +ve and fixed value of time for which signal is periodic

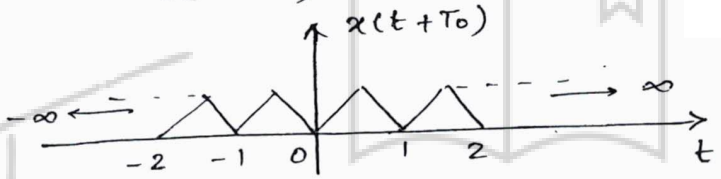
$\neq 0$
$\neq \infty$)

Ex :



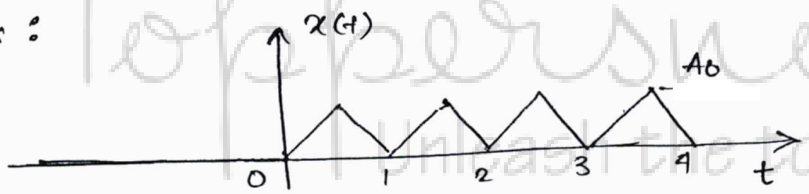
⟶ FTP = $T_0 = 1$.

$x(t + T_0) = x(t+1)$



⟶ Periodic.

Ex :



for a system to be periodic the signal should be from $-\infty$ to ∞

⟶ P ⟶ Non periodic

⟶ $T_0 = 1$ $x(t + T_0) = x(t+1)$