IES/GATE

Electronics & Telecommunication Engineering

VOLUME-VI

Signals System, Control System

Contents

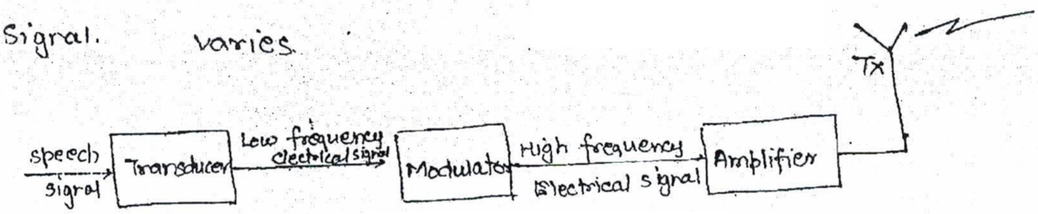
Toppersnotes
Unleash the topper in you

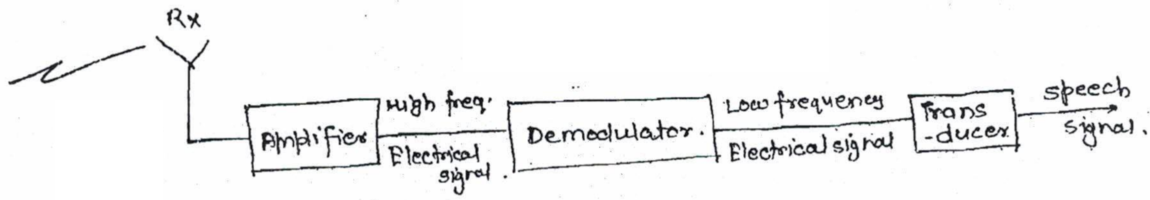
Speech Signal - 300 Hz - 3,400 Hz

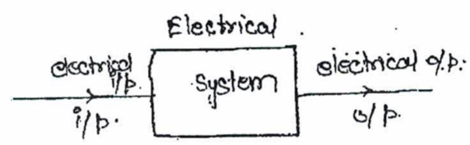
Audio " 20 Hz - 20, KHz.

Video Signal. 0 - 4.5 MHz (5 MHz).

Data Signal. varies.







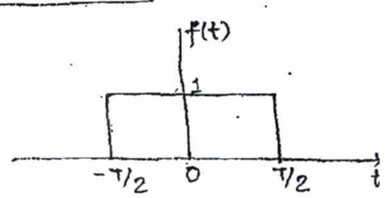
- Signal:- signal is any quantity having information associated with it.

$$\text{Signal} \begin{cases} \text{voltage } v(t) \\ \text{Current } i(t) \end{cases} f(t).$$

* RGB → colour coding
* still frames per sec → is standard
$\frac{1}{16}$ → frames per second is not visible for eye

* Signal need not be always single variable.
* Signal need not be always function of time.

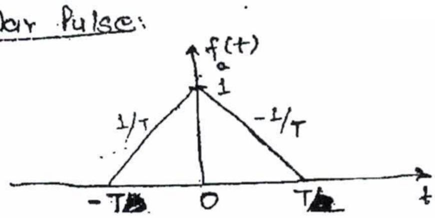
Rectangular Pulse:



$f(t) = 1, \quad -T/2 \le t \le T/2$

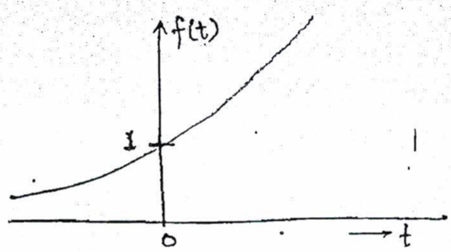
$0, \quad$ otherwise.

Triangular Pulse:



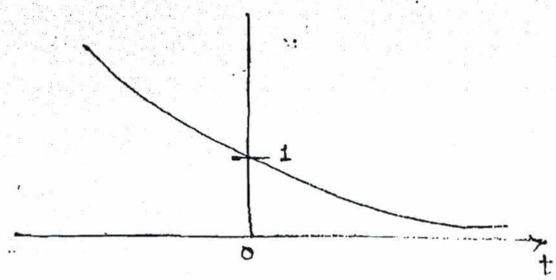
$$f(t) = \begin{cases} \frac{1}{T}t + 1, & -T \le t \le 0 \\ -\frac{1}{T}t + 1, & 0 \le t \le T \\ 0 & \text{otherwise.} \end{cases}$$

Exponentially Increasing

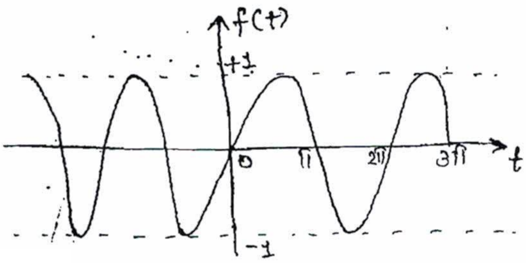


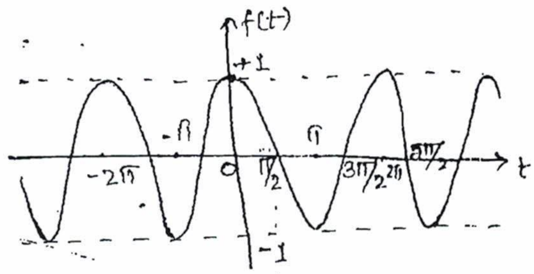
$\boxed{f(t) = e^{at}}$

Exponentially Decreasing



$\boxed{f(t) = e^{-at}}$





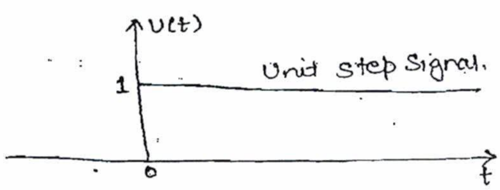
$\boxed{f(t) = \sin t}$ ⟶ Sinusoidal signals ⟵ $\boxed{f(t) = \cos t}$

Zero cross over $= K\pi$
where K = integer

Z.C.O. $= (2K+1)\pi/2$, K = integer

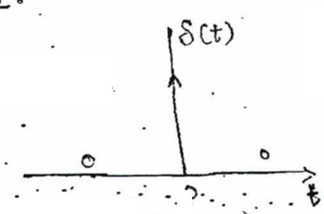
* The time instance at which the signal oscillating b/w positive & negative value's cross zero value are defined as zero cross over's of such oscillating signal's.

Unit step Signal: [u(t)]



$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0. \end{cases}$$

Impulse Signal :-



$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \neq 0, & t = 0 \end{cases}$$

or $\boxed{\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1}$ ⟶ Unit impulse

Toppersnotes
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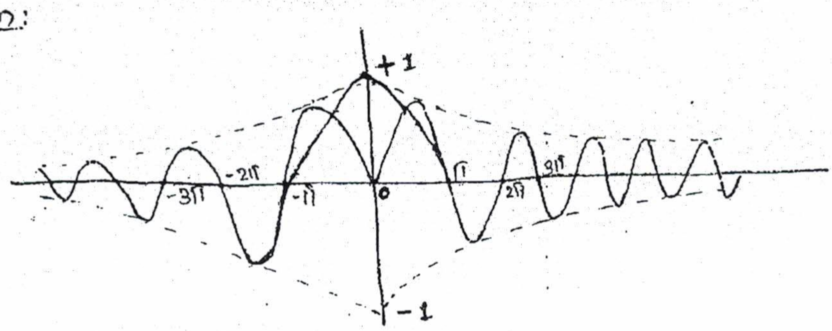
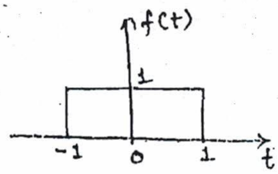
Q. Sketch the graph of signal defined as

$$Sa(t) = f(t) = \frac{\sin t}{t}$$, also called as sampling function.

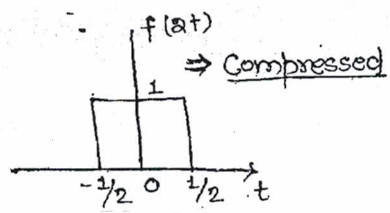
<u>Solution:</u>

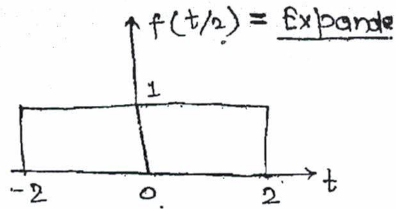


OPERATION OF SIGNAL ON TIME AXIS :-



⇒

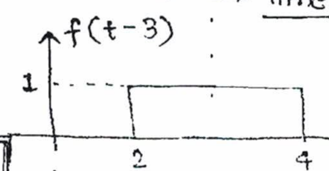




<u>Time Scaled</u>

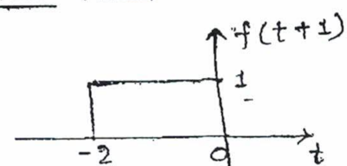
$$f(t) = \begin{cases} 1 \; ; & -1 \le t \le 1 \\ 0 \; ; & \text{otherwise} \end{cases}$$

⇒



⇓

Delayed version



⇓

Advanced Version

<u>Time Shifted</u>

<u>by putting $t \to 2t$</u> [Compression]

$$f(2t) = \begin{cases} 1 \; ; & -1 \le 2t \le 1 \\ 0 \; ; & \text{otherwise} \end{cases}$$

or

$$f(2t) = \begin{cases} 1 \; ; & -1/2 \le t \le 1/2 \\ 0 \; ; & \text{otherwise} \end{cases}$$

<u>By putting $t \to t/2$</u> [Time Expansion].

$$f(t/2) = \begin{cases} 1 \; ; & -1 \le t/2 \le 1 \\ 0 \; ; & \text{otherwise} \end{cases} \quad \text{or} \quad f(t/2) = \begin{cases} 1 \; ; & -2 \le t \le 2 \\ 0 \; ; & \text{otherwise.} \end{cases}$$

<u>Time Scaling</u>

$t \to at$
- $a > 1 \Rightarrow$ compression
- $a < 1 \Rightarrow$ Expansion.

* <u>Divide by 'a' at time scale directly.</u>

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TIME SHIFTING

Delayed or Right shifted [put $t \to (t-3)$]

$$f(t-3) = \begin{cases} 1 ; & -1 \le (t-3) \le 1 \\ 0 ; & \text{otherwise} \end{cases} \quad \text{or} \quad \begin{cases} 1 ; & 2 \le t \le 4 \\ 0 ; & \text{otherwise.} \end{cases}$$

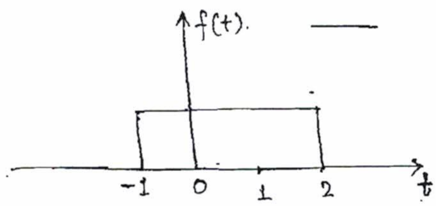
Advanced or Left Shifted [Put $t \to (t+1)$]:

$$f(t+1) = \begin{cases} 1 ; & -1 \le t+1 \le 1 \\ 0 ; & \text{otherwise} \end{cases} \quad \text{or} \quad \begin{cases} 1 ; & -2 \le t \le 0 \\ 0 ; & \text{otherwise.} \end{cases}$$

Time Shifting

t —
- $t \to t - t_0 \Rightarrow$ Delay (or) right shift transformation.
  $\Rightarrow$ Add 't_0' at time scale directly.
- $t \to t + t_0 \Rightarrow$ Advance (or) left shift transformation
  $\Rightarrow$ Subtract 't_0' on time scale directly.

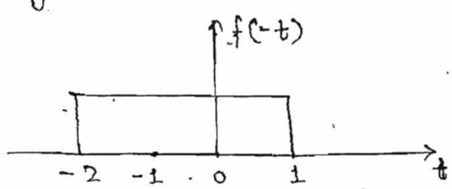
➡ **TRANSFORMATION OF SIGNALS BY GRAPHICAL METHOD.**



$$f(t) = \begin{cases} 1 & -1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Time Reversed [$f(-t)$]:

putting $t \to (-t)$



$$f(-t) = \begin{cases} 1 & -1 \le -t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

or

$$f(-t) = \begin{cases} 1 & -2 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Graphical Method :-

* Rotation of complete Graph by 180° around Y-axis
* Take mirror image around or across Y-axis.
* Add extra negative sign (-) to all values of t.

Toppersnotes
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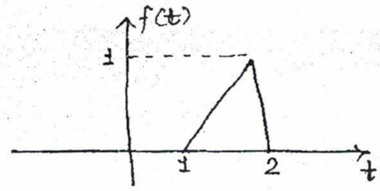
$$f(t-3) \xrightarrow{t \to 2t} f(2t-3)$$

$$f(t+5) \xrightarrow{t \to -t} f(-t+5).$$

2. Sketch the Graph of signal $f(-t+5)$ for the following signal $f(t)$ given in figure

imp

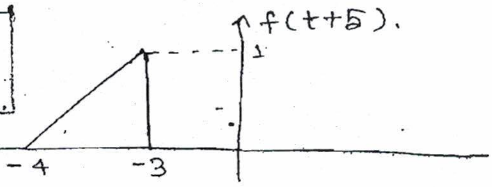


Solution :-

Step : 1 : Time Shifting

$$f(t) \xrightarrow{t \to t+5} f(t+5)$$

	lower limit	upper limit	
$f(t) \Rightarrow$	1	2	$\Leftarrow f(t)$
$f(t+5) \Rightarrow$	-4	-3	$\Leftarrow f(t+5)$
$f(-t+5) \Rightarrow$	4	3	$\Leftarrow f(-t+5)$

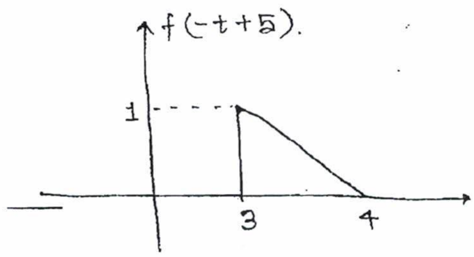


Step : 2 $\Rightarrow$ Time Reversal

$$f(t+5) \xrightarrow{t \to -t} f(-t+5).$$

$-4 \longrightarrow 4$

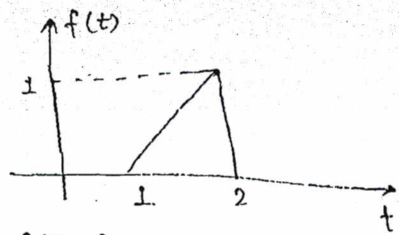
$-3 \longrightarrow 3$



Note :- when it is required to perform combination of time shifting & time reversal transformation's on a given signal $f(t)$, to get an expression of the form $f(-t+b)$

Note → the natural order is time shifting first followed by time reversal. However shifting can also be performed after reversal with the precaution of changing the visible shifting transformation to its opposite nature i.e. if it is visibly advance, change it to delay. & vice-versa.

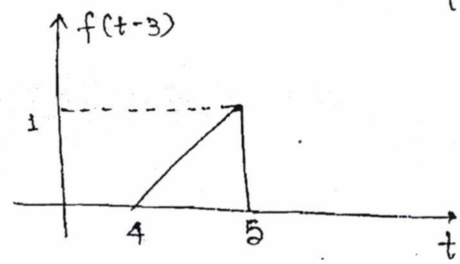
Q. For the above signal sketch the graph of $f(2t-3)$.



Solution :

* <u>Step: 1 ⇒ Time shifting</u>

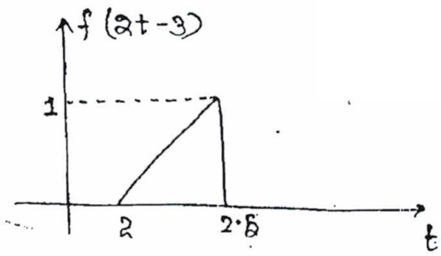
$$f(t) \xrightarrow{t \to t-3} f(t-3)$$



* <u>Step: 2 ⇒</u> <u>Time scaling</u>

$$f(t-3) \xrightarrow{t \to 2t} f(2t-3)$$

	lower limit	upper limit
$f(t) \Rightarrow$	1	$\boxed{2}$
$f(t-3) \Rightarrow$	4	5
$f(2t-3) \Rightarrow$	2	$\boxed{5/2}$



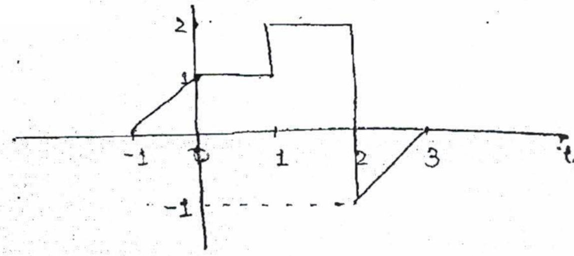
1) <u>Note</u>: * when it is required to perform combination of time shifting & time scaling to get an expression of the form $f(at-b)$ for the given signal $f(t)$, ~~the natural~~ order is <u>shifting first followed by scaling</u>. However shifting can also be performed after scaling with a precaution of dividing the visible shift amount by the scaling factor.

2) * when we have <u>combination of time scaling & time reversal</u> they can be <u>performed in any order without any precautions</u>.

Q. For the following signal $f(t)$ sketch the graph of $f(-2t-5)$.

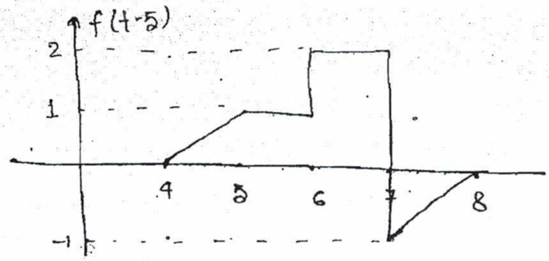
Step: 1 Time shifting

$$f(t) \xrightarrow{t \to t-5} f(t-5).$$



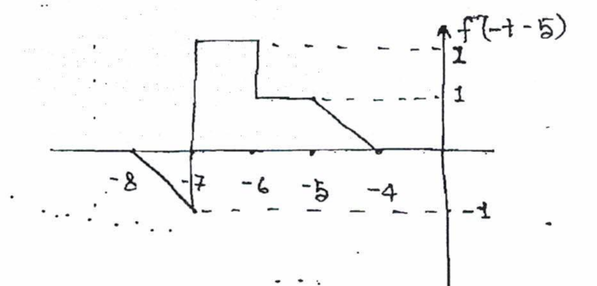
Step: 2 Time reversal

$$f(t-5) \xrightarrow{t \to -t} f(-t-5)$$

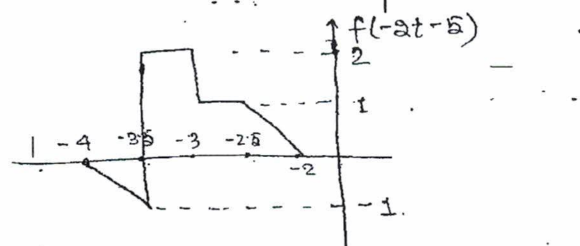


Step: 3 Time Scaling

$$f(-t-5) \xrightarrow{t \to 2t} f(-2t-5)$$



$f(t)$	-1	0	1	2	3
$f(t-5)$	4	5	6	7	8
$f(2t-5)$	2	$5/2$	3	$7/2$	4
$f(-2t-5)$	-2	$-5/2$	-3	$-7/2$	-4



Q: If in the above problem $f(-2t-5)$ is to be realized by performing the operations in the order of time reversal time scaling & then time shifting what are the transformations required.

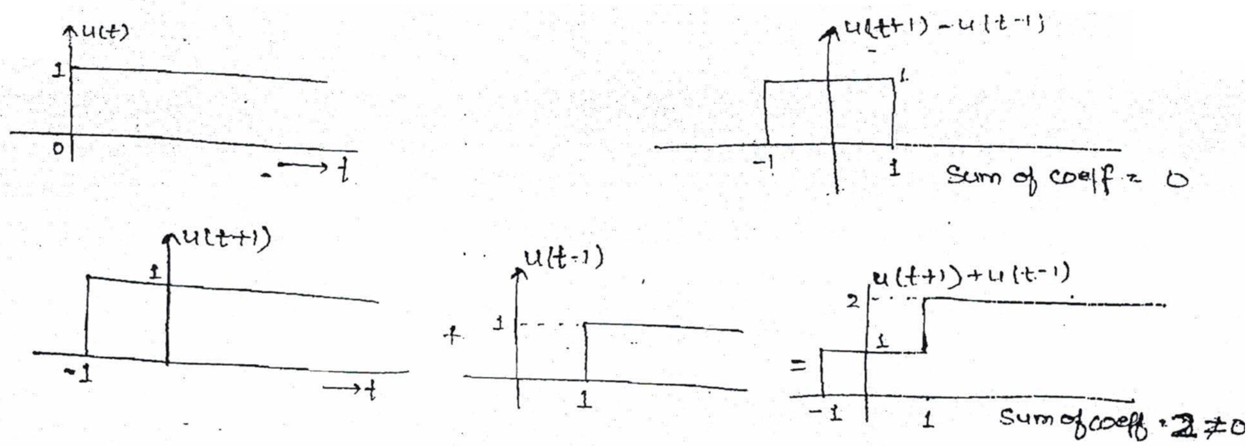
Solution:-

$$f(t) \longrightarrow f(-2t-5)$$

Time Reversal $\xrightarrow{t \to -t}$ $f(-t)$

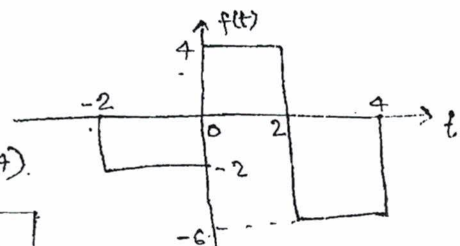
Time scaling $\xrightarrow{t \to 2t}$ $f(-2t)$

Time shifting $\xrightarrow[t \to \cancel{t-5}]{t \to t+5/2}$ $f[-2(t+5/2)] = f(-2t-5)$



* Any signal f(t) having step change's alone can always be defined as sum of number of shifted step signal's with suitable co-efficient's where the shift of u(t) represent's the timing instant at which a step change is required in the given signal & the co-efficient indicates the amount of step change required at the shift given to u(t).

Q. Represent the following signal f(t) using shifted step signals

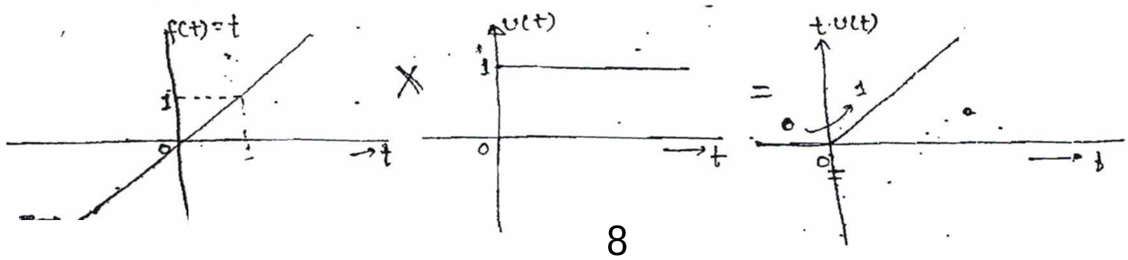


$f(t) = -2\,u(t+2) + 6\,u(t)$
$\qquad -10\,u(t-2) + 6\,u(t-4)$

$\boxed{\text{Sum of Co-efficient} = 12 - 12 = 0.}$

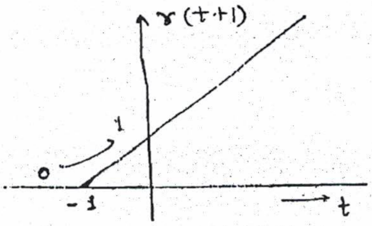
* In the above representation the sum of the co-efficients will be zero only if signal has a finite duration.

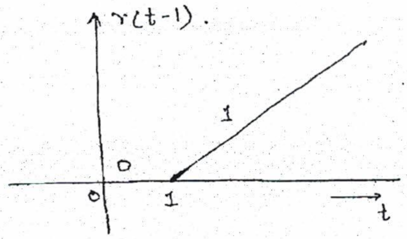
UNIT RAMP SIGNAL



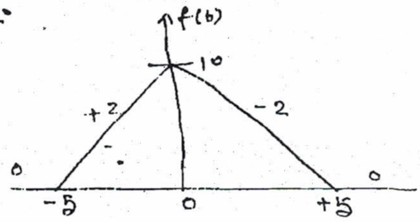
Toppersnotes
Unleash the topper in you

$r(t) = t\,u(t)$ $\qquad r(t) = \begin{cases} 0, & t<0. \end{cases}$





Ex.

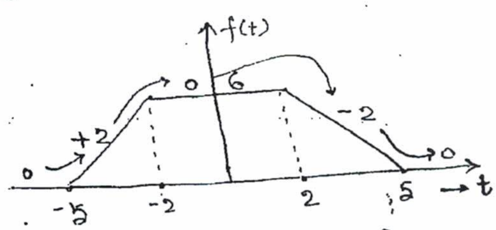


$f(t) = 2r(t+5) - 4r(t) + 2r(t-5)$

Note: Any signal f(t) which is a combination of straight lines can always be defined as sum of number of shifted ramp signal's with suitable co-efficient's where the shift of the ramp signal's indicates the time instant at which a change in slope is required in given signal & the co-efficient represent's the amount of change in slope required at the shift given to r(t).

Question Represent the following signal using ramp signal.

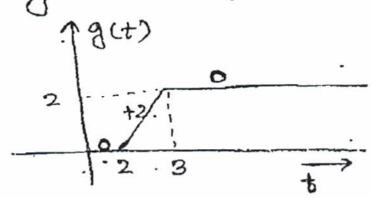
$f(t) = 2r(t+5) - 2r(t+2)$
$\qquad - 2r(t-2) + 2r(t-5).$



Trapezoidal Pulse.

Question: Represent the following signal g(t) using ramp signal's

$g(t) = 2r(t-2) - 2r(t-3).$



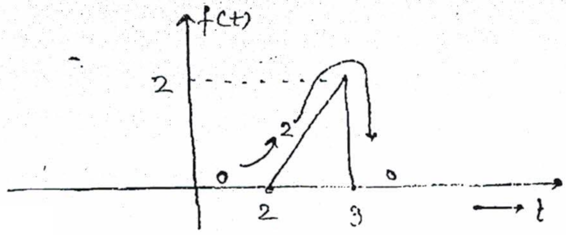
Toppersnotes
Unleash the topper in you

Q. Represent the following signal f(t) using shifted ramp and shifted step signals.

Solution:

$f(t) = 2r(t-2) - 2r(t-3)$

$- 2u(t-3)$.



Sawtooth Pulse.

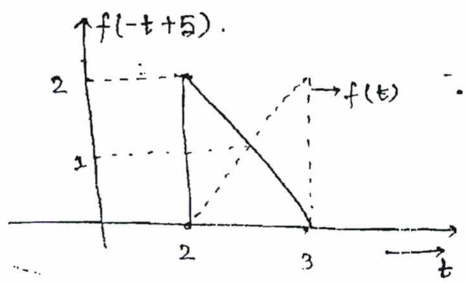
Q: For the above sawtooth pulse f(t), f(t) + f(-t+5), will be

(a) $u(t-2) - u(t-3)$

(b) $r(t-2) - r(t-3)$

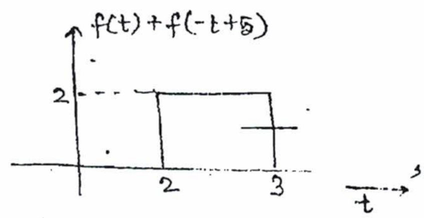
(c) $2[u(t-2) - u(t-3)]$

(d) $2[r(t-2) - r(t-3)]$.



$f(t) + f(-t+5) = 2u(t-2) - 2u(t-3)$

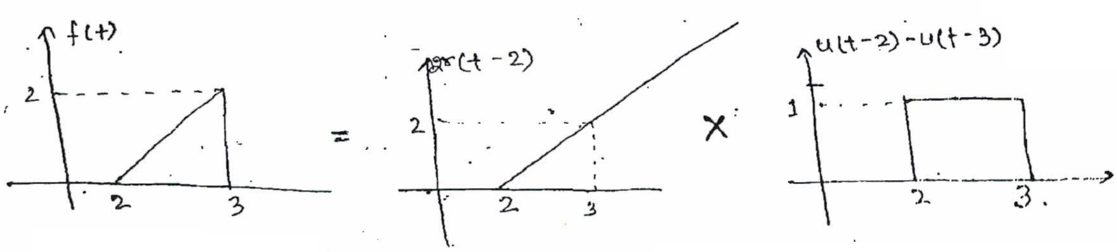
$= 2[u(t-2) - u(t-3)]$



Note: * Addition of two sawtooth pulses occuring between the same two time instance having the same slope but of opposite sign will always result in a rectangular pulse.

imp * whenever it is required to sustain only a portion of a given signal multiply the signal by a suitable pulse having a value one in the required portion and a value 0 in the unrequired portion.

imp



or $2t-4$

$$f(t) = 2r(t-2)\,[u(t-2) - u(t-3)]$$

$$= (2t-4)[u(t-2) - u(t-3)]$$

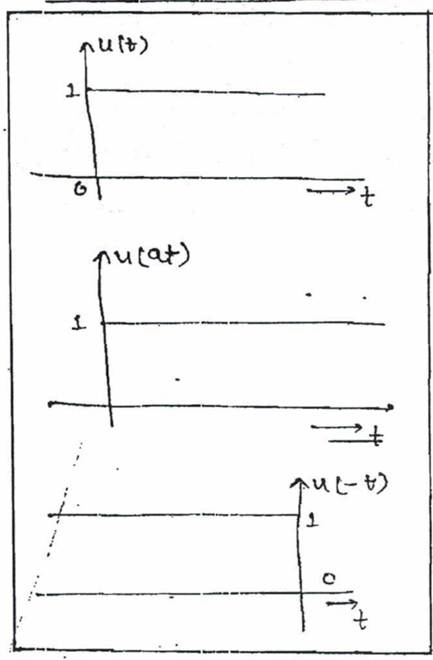
$$= (2t-4)u(t-2) - (2t-4)u(t-3)$$

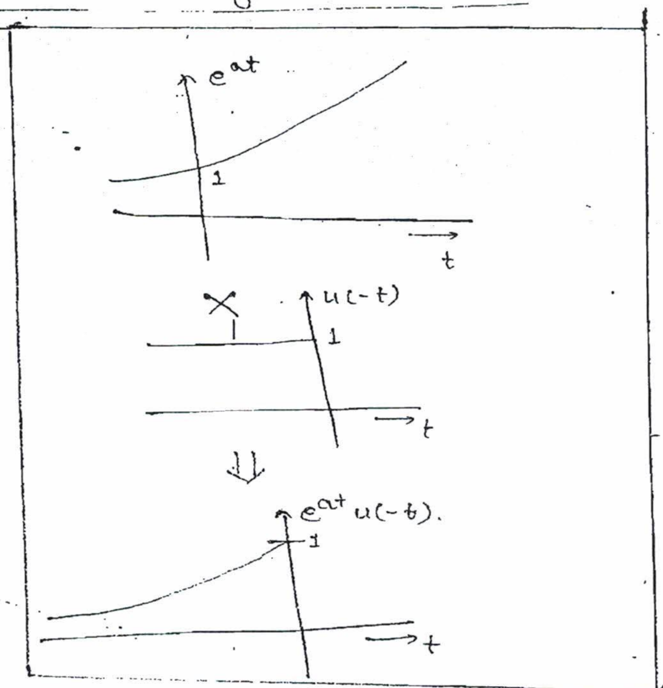
$$= (2t-4)u(t-2) - (2t-6+2)u(t-3)$$

$$= 2(t-2)u(t-2) - 2(t-3)u(t-3) - 2u(t-3)$$

$$\therefore \boxed{f(t) = 2r(t-2) - 2r(t-3) - 2u(t-3)}$$

Time Scaling + Time reversal change on unit step.





// . Sketch the Graph of the following signals.

(a) $e^{at}u(t)$

(b) $e^{-at}u(-t)$

(c) $e^{at}u(t)$

(d) $\sin t\,[u(-t)]$

(e) $\cos t\, u(t)$

(f) $u(t) + u(-t)$.

(g) $u(t) - u(-t)$.

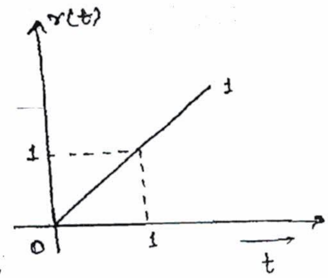
Time scaling effect for Ramp Signal

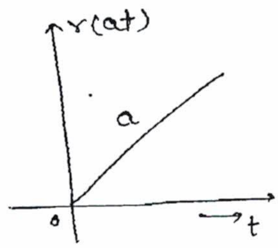
For a ramp signal there is absolutely no difference between time scaling + magnitude scaling

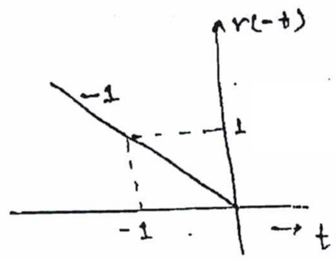
$$r(t) = \begin{cases} t & t \geq 0 \\ 0, & t \leq 0 \end{cases}$$

$$r(at) = \begin{cases} at, & at \geq 0 \\ 0, & t < 0 \end{cases} \quad \text{or} \quad \begin{cases} at, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$a\,r(t) = \begin{cases} at, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad \text{so} \quad \boxed{r(at) = a\,r(t)}$$







$$r(-t) = \begin{cases} 0, & t > 0 \\ -t & t \leq 0 \end{cases}$$

$r(3t) = 3\,r(t)$

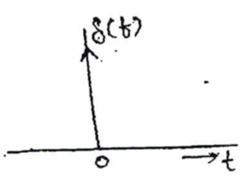
$r(3t) \neq -3\,r(t)$.

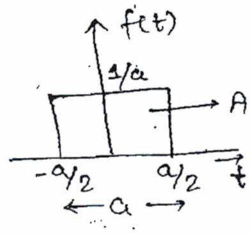
$r(at) = a\,r(t)$ → when a is (+)ve only.

Q. Sketch the Graph of following Signals.

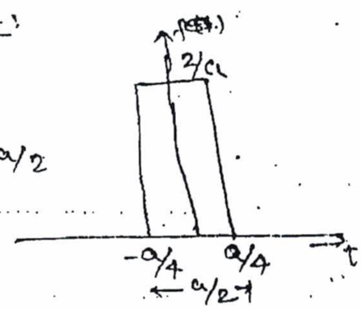
(1) $r(4t-6)$ (2) $r(-t+5)$

Effect of Scaling on Unit Impulse function:

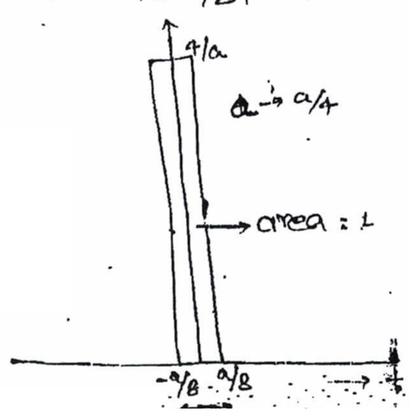


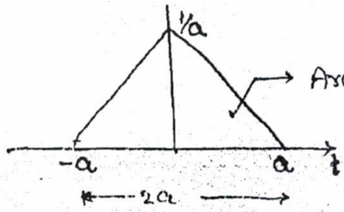


→ Area = 1 ⇒ $a \to a/2$



$$\boxed{\begin{gathered} \delta(t) = \begin{cases} 0; & t \neq 0 \\ \neq 0, & t = 0 \end{cases} \\ \int_{-\infty}^{\infty} \delta(t) \cdot dt = 1 \end{gathered}}$$

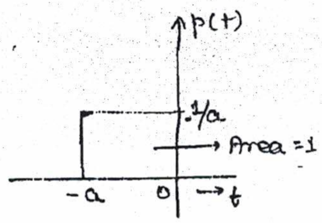




$\lim_{t \to 0} g(t) = \infty$ (undefined).

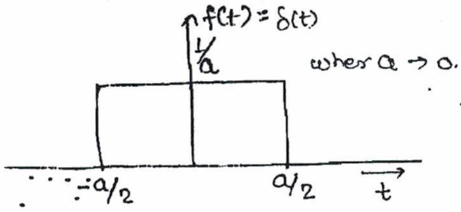
Still Area = 1

So $g(t) \approx \delta(t)$.

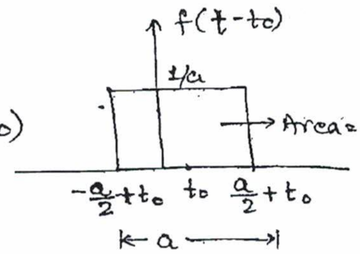


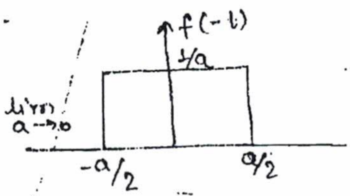
Note:- $\delta(t)$ signal does not have unique defenitio like other signals, it's defenition is only genera-lised by using a wide variety of signals for which all signal's the width's & height are inversaly proportional and area is equal to 1.

Hence $\delta(t)$ is also called as a generalised function

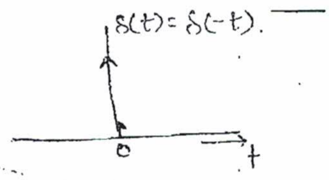


$\xrightarrow[\text{Time shifting}]{\delta(t-t_0)}$ $\lim_{a \to 0} f(t-t_0)$

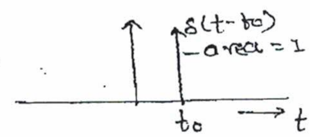




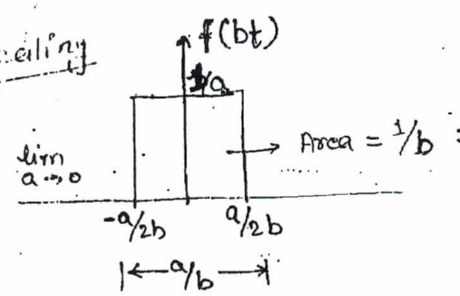
$\Rightarrow$



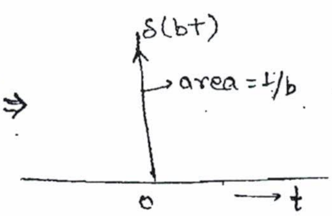
or



scaling



$\Rightarrow$



$$A\cdot\delta(t) \Rightarrow \int_{-\infty}^{\infty} A\cdot\delta(t)\cdot dt = A\int_{-\infty}^{\infty}\delta(t)dt = A$$

$$\boxed{\begin{array}{l} r(at) = a\,r(t) \\ \delta(bt) = \frac{1}{b}\delta(t) \end{array}}$$

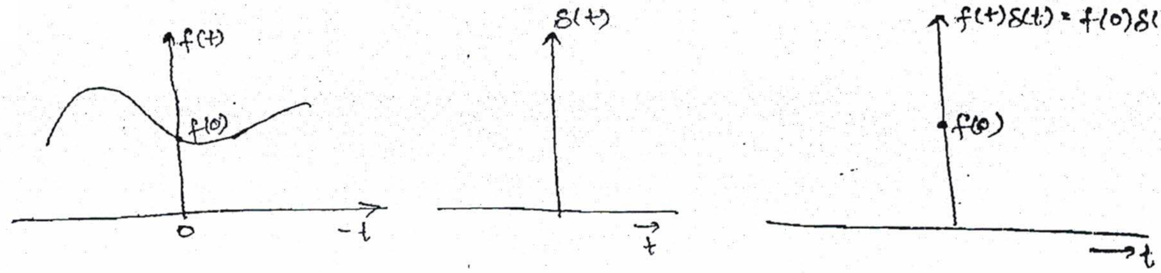
e.g. $\delta(3t) = \frac{1}{3}\delta(t)$

$\delta(-3t) = \frac{1}{3}\delta(t)$

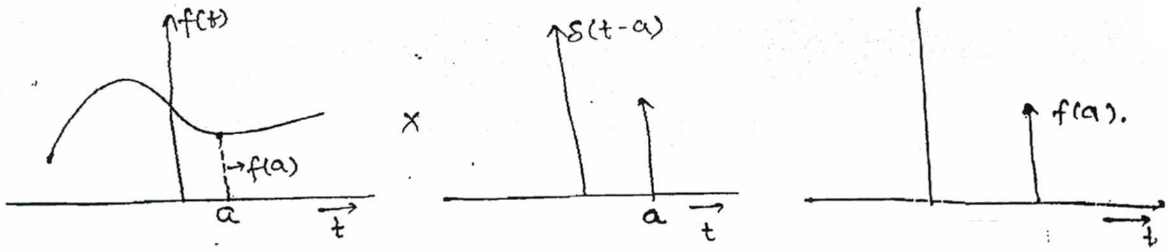
$$\boxed{\delta(bt) = \frac{1}{|b|}\delta(t)}$$

b can be (−)ve or (+)ve

Toppersnotes
Unleash the topper in you



$$\boxed{f(t)\cdot\delta(t) = f(0)\,\delta(t)}$$



$$\boxed{f(t)\cdot\delta(t-a) = f(a)\cdot\delta(t-a)}$$

This is known as sampling property of delta function.

1. $$\boxed{\int_{-\infty}^{\infty} f(t)\cdot\delta(t)\cdot dt = f(0)}$$ or $$\int_{-\infty}^{\infty} f(0)\cdot\delta(t)\cdot dt = f(0)\int_{-\infty}^{\infty}\delta(t)\,dt = f(0)\times 1 = f(0).$$

2. $$\boxed{\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\cdot dt = f(a)}$$ ⇒ This property is known as shifting property

$$\int_{-\infty}^{\infty} f(a)\cdot\delta(t-a)\cdot dt = f(a)\int_{-\infty}^{\infty}\delta(t-a)\,dt = f(a).$$

Q⇒ Evaluate the value of following integrals.

1) $\int_{-\infty}^{\infty}\cos t\,\delta(t)\cdot dt = \cos(0) = 1$ Ans

2) $\int_{-\infty}^{\infty}\sin(t-\pi/2)\,\delta(t-\pi)\cdot dt = \sin(\pi-\pi/2)$
$= \sin\pi/2 = 1$ Ans

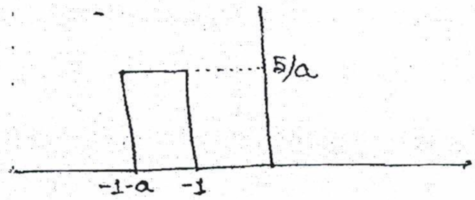
3) $\int_{-\infty}^{\infty}\sin(t-\pi/2)\,\delta(3t-\pi)\,dt = \frac{1}{|3|}\cancel{\sin(\pi-\pi/2)}$
$\delta(3t-\pi) = \delta(3(t-\pi/3)) = \frac{1}{3}\delta(t-\pi/3)$
$= \frac{1}{3}\cancel{\sin\pi/2} = 1/3$; $\frac{1}{3}\sin(\pi/3-\pi/2)$
$= \frac{1}{3}(-1/2) = -1/6$
Ans

4) $\int_{-\pi/6}^{\pi/6}\sin(t-\pi/2)\,\delta(3t-\pi)\,dt$
$= 0$ Ans

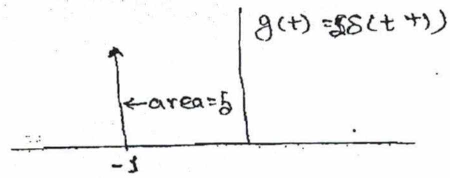
Evaluate the value of the following integral

V.Imp

$$\int_{-\infty}^{\infty} \frac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+1} \cdot g(t)\cdot dt$$ where $g(t)$ is the following impulse as $a \to 0$.



$$\int_{-1-a}^{-1} \frac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+1} \cdot \underbrace{\frac{5}{a}\cdot dt}$$



or

$$5\int_{-\infty}^{\infty} \frac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+1} \cdot \underbrace{\delta(t+1)\cdot dt}$$

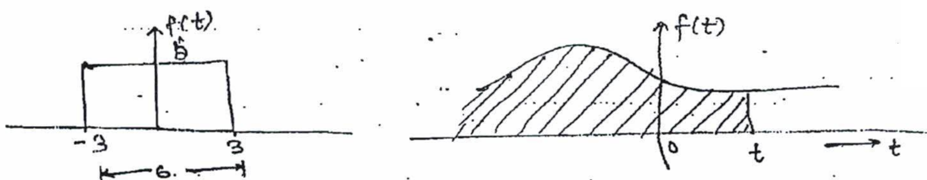
$x(t) = A\,\delta(t)$
↳ weight of impulse
or
→ Area under weighted impulse

$$5\,\frac{\sin\left[\frac{\pi}{2}(-1-2)\right]}{(-1)^2+1} = \frac{5}{2}\sin\left[-\frac{3\pi}{2}\right]$$

$$= -\frac{5}{2}\sin\frac{3\pi}{2} = -\frac{5}{2}\times -\sin\frac{\pi}{2} = \boxed{\frac{5}{2}.\ \text{Ans}}$$

Differentiation & Integration of Signals :- 13/08/2013

- Integral of any signal is physically the measure of the time varying area under the signal in the limits from $-\infty$ to t. where t can range from $-\infty$ to $+\infty$.



$$\int_{-\infty}^{\infty} f(t)\cdot dt = \begin{cases} 30 & \text{finite} \\ \infty & \text{infinite.} \end{cases}$$

$$\boxed{\int_{-\infty}^{t} f(t)\cdot dt = g(t) \to \text{Integral of } f(t)}$$

$t \to -\infty$ to $+\infty$