ToppersNotes

IES/GATE

MECHANICAL ENGINEERING

HEAT & MASS TRANSFER, MACHINE, MECHANICAL DESIGN & FLUID MACHINERY & INTRODUCTION & BFA INVENTORY

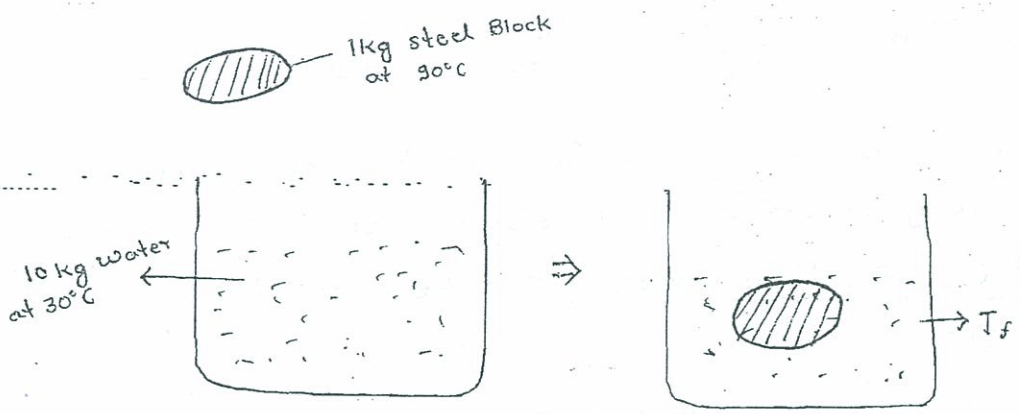
VOLUME-II

Sierra Innovations Pvt. Ltd.

Contents

Heat & Mass Transfer

Heat → Thermodynamics (Q measured in Joules)
Heat → HMT (Q measured in Joules/sec = Watt)



Heat lost by steel = Heat gained by water

$$m_s \times C_{ps}(90 - T_f) = m_w \times C_{pw}(T_f - 30) \rightarrow \text{Energy Balance Eq}^n$$

* The main difference b/w TD analysis & Heat transfer analysis of a problem is that in TD we deal with systems in equilibrium i.e to bring a system from one equilibrium state to another how much heat is required, is the main criteria in TD analysis.

  But in heat transfer analysis we evaluate at what rate change of state occurs by calculating the rate of heat transfer in Joule/sec or Watt.

* Modes of Heat Transfer :-

1) Conduction
2) Convection
3) Radiation

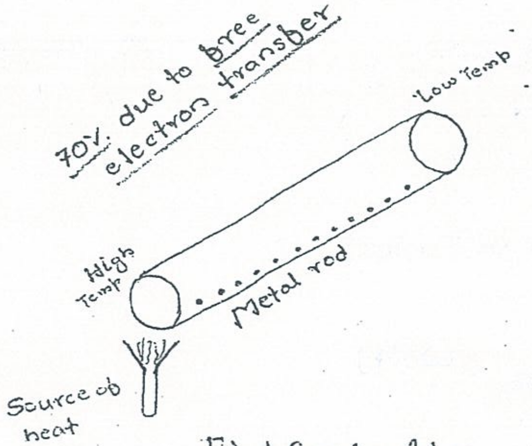


Fig: Conduction

30% of conduction occurs due to molecular lattice Vibrational Energy.

Silver , K = 405 W/mK
Copper . K = 385 W/mK
Gold . K = 319 W/mK
Aluminium. K = 200 W/mK
Steel . K = 17 to 45 W/mK

1) Conduction :- Conduction is a mode of heat transfer which generally occurs in solids due to temperature difference by molecular lattice vibrational energy transfer and also by free electron transfer.

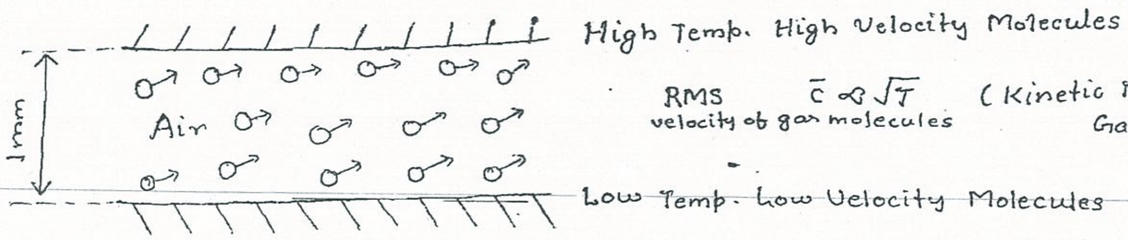
The reason behind all electrically good conductors of electricity are also good conductors of heat is that because of the presence of abundant free electrons.

Exception – Diamond is a good conductor of heat with highest value of K = 2300 W/mK which is a non metal. The highest conductivity of diamond is due to its perfect crystalline molecular lattice arrangement.

Non-Metals
- Crystalline (Diamond, quartz, graphite)
- Amorphous (Glass, K_{glass} = 1.5 W/mK)

Insulators have very low thermal conductivity thereby prevent the heat transfer rate. Eg.- Asbestos (K = 0.2 W/mK), Glasswool (K = 0.075 W/mK) Refractory brick (K = 0.9 W/mK) (Furnaces), Polyurethene foam (PUF, K = milli 'K') (used in refrigerator walls)

Conduction also occurs in gases as shown below:

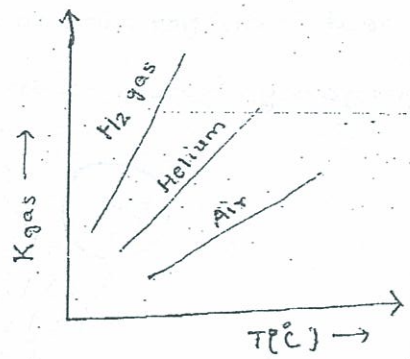


RMS velocity of gas molecules $\bar{c} \propto \sqrt{T}$ (Kinetic Theory of Gases)

Conduction occurs in gases by 'molecular momentum transfer' when high velocity · high temp. molecules collide with low temp. low velocity molecules. But in general gases are 'very bad' conductors of heat. Eg. – Air with $K = 0.026\ W/mK$)

* Thermal conductivity of gases increase with increase in their temperature. The reason being at higher temp. greater molecular activities results in more no. of collisions/unit time & hence higher momentum transfer.

As T_{gas} ↑
ν (Kinematic Viscosity) ↑
C_p (J/KgK) ↑
K (W/mK) ↑
ρ (Density, Kg/m^3) ↓



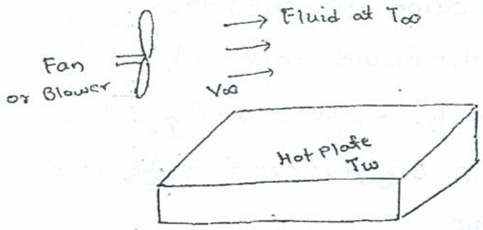
Liquids also conduct heat but they are better conductors than gases. Eg. – $K_{water} = 0.61\ W/mK$,

* Hg (Mercury) (Liquid Metal) has highest conductivity among liquids ($K_{Hg} = 8.3\ W/mK$)

Liquid sodium is also a good conductor of heat among liquids.

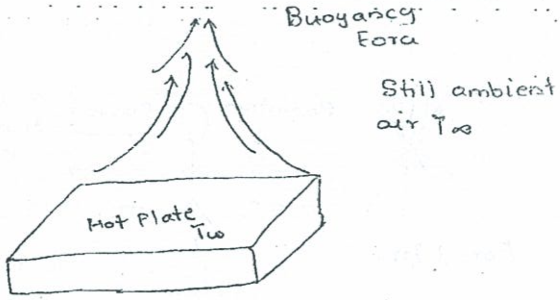
* Thermal Conductivity, K :- It is a thermophysical property of the material which tells about the ability of the material to allow heat energy to get conducted throug the material.

2) Convection :-



$T_w > T_\infty$

Forced convection

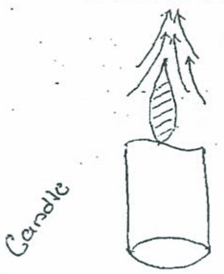


$T_w > T_\infty$

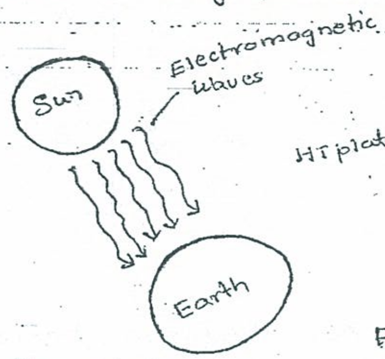
Free or Natural Convection

Defn :-

Convection is the mode of heat transfer which generally occurs b/w a solid surface and the surrounding fluid due to temp. difference associated with macroscopic bulk displacement of the fluid transporting thermal energy. In case of forced convection heat transfer, this motion of fluid is provided by an external energy like ~~a fan~~ or a blower or a pump, whereas in free convection heat transfer, the motion of the fluid occurs naturally due to the buoyancy forces arising out of density changes of fluid.



Eg: Free Convection



Eg - Radiation

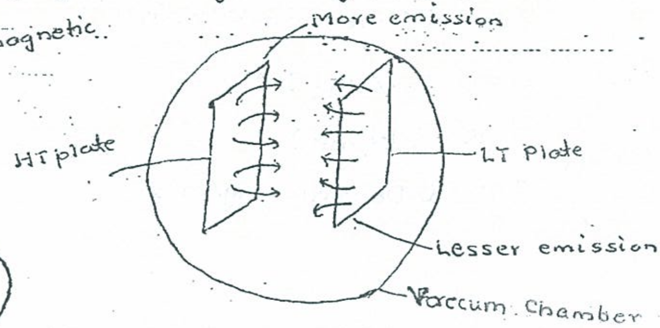


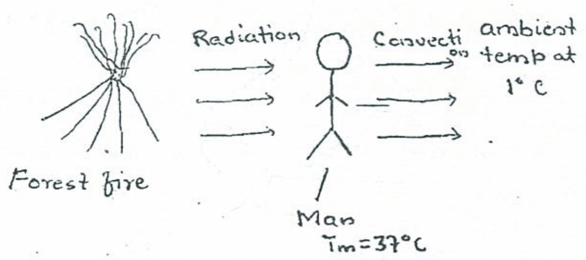
Fig : Radiation

3) Radiation :- All bodies at all temperatures emit thermal radiation except the body at 0 K (−273.15°C)

The rate of emission from body being a strong function of absolute temperature of body.

Defn :

Radiation is the mode of heat transfer which do not require any material medium for its propagation & hence occurs by electro-magnetic wave propagation travelling with a speed of light.



Radiation becomes the predominant mode of heat transfer particularly when temp. difference is sufficiently large.

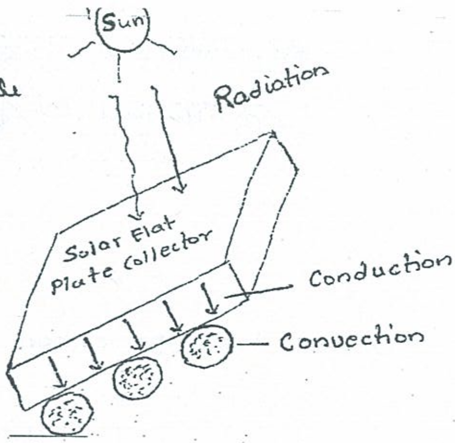
Eg.- The mode of heat transfer b/w hot fluids & gases & refractory brick walls in a large pulverised fuel fired Power boiler is predominantly by thermal radiation (Reason - Large ΔT)

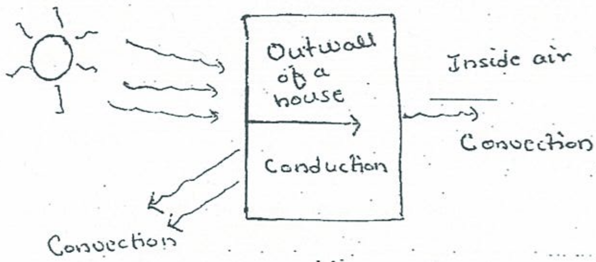
$q_{radiation} \propto (T_1^4 - T_2^4)\ \sigma$

in Kelvin Scale

$q_{conduction} \propto (T_1 - T_2)$

$q_{convection} \propto (T_1 - T_2)$



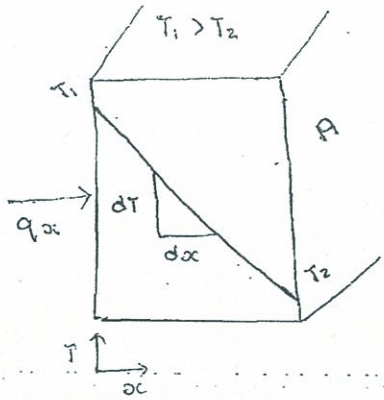


All three modes of heat transfer are present.

* Governing Laws of Heat Transfer :-

1) Fourier's Law of Conduction -

The law states that the rate of heat transfer by conduction along a given dir^n is directly '$\propto$' to temp. gradient along that dir^n and is also directly proportional to the area of heat transfer lying ⊥ar to the dir^n of heat surface.



$$\left[q_x \propto -\left(\frac{dT}{dx}\right) \right]$$

'–' sign shows that the temp. of slab decreases in the increasing dir^n of x. i.e to satisfy the Claussius law of II^nd law of TD.

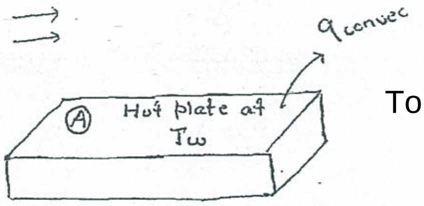
$$[q_x \propto A]$$

$$\boxed{q_x = -KA\left(\frac{dT}{dx}\right) \text{ watt}}$$

where K = thermal conductivity of material.

2) Newton's law of Cooling (for convection HT) :

The law states that the rate of heat transfer by convection b/w a solid body & surrounding fluid is directly '$\propto$' to the temp. difference b/w them and is also directly proportional to the area of exposure or area of contact b/w them.



$$q_{conv} \propto (T_w - T_\infty)$$
$$\propto A$$
$$\Rightarrow q_{conv} = hA\,\Delta T$$
$$= hA\,(T_w - T_\infty)$$

where h = Convection HT constant or Film HT coefficient in W/m^2K

Unlike thermal conductivity 'K', h is not a property of material but it depends on some of the thermophysical properties of the fluid like ρ, μ, C_p, K.

Properties of Fluid
- ρ - density kg/m^3,
- μ - dynamic viscosity (Pa-sec)
- C_p - Specific heat (J/Kg K)
- K - thermal conductivity (W/mK)

* In any forced convection

$$h = f(\vec{V}, D, \rho, \mu, C_p, K)$$

$\vec{V}$ = velocity of fluid (m/s)

D = characteristic dimension of body

* In free or natural convection heat transfer

$$h = f(g, \beta, \Delta T, L, \mu, \rho, C_p, K)$$

where g = acc^n due to gravity

β = Isobaric volume expansion coeff of fluid $= \frac{1}{V}\left(\frac{\delta V}{\delta T}\right)_P$

$\Delta T = T_w - T_\infty$

L = characteristic dimension of body.

* Range of Convection Coefficient (h) :-

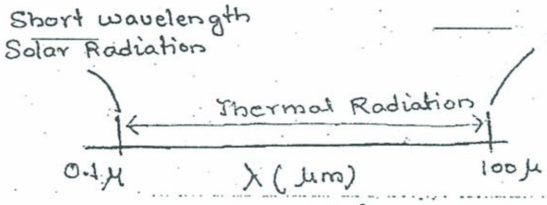
1) Free convection in gases & vapours $\Rightarrow$ h = 3 to 25 W/m^2K
2) Forced convection in gases $\Rightarrow$ h = 25 to 450 W/m^2K
3) Free convection in liquids $\Rightarrow$ h = 300 to 650 W/m^2K
4) Forced convection in liquids $\Rightarrow$ h = 600 to 4000 "
5) Condensation heat transfer $\Rightarrow$ h = 3000 to 25000 W/m^2K
   (vapour to liquid)
6) Boiling HT $\Rightarrow$ h = 5000 to 50,000 W/m^2K
   (Liquid to vapour)

Water can convect away heat 25 times more rapidly as compared to air

$C_{pw} = 4.18\, C_{pa}$
$\rho_w = 1000\, \rho_a$
$\mu_w > \mu_{air}$
$K_w > K_{air}$

3) Stefan-Boltzman Law of Radiation :-

The law states that the radiation energy emitted from a black body per unit time and per unit area is directly proportional to the fourth power of the absolute temperature of the black body.



λ & T have a relation

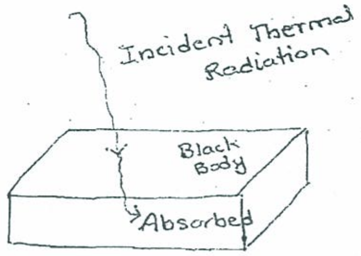
$\boxed{T \uparrow \Rightarrow \lambda \downarrow}$

Electromagnetic Spectrum

$1\ \mu m = 10^{-6}\ m$

Optical pyrometer is an instrument used to measure high temperatures.

*



Black Body :-

Black body is the body which absorbs all the thermal radiation incident or falling upon the body.

Eg. - Ice & Snow.

A thermally black body absorbing all the incident thermal radiation may not appear black to the human eye.

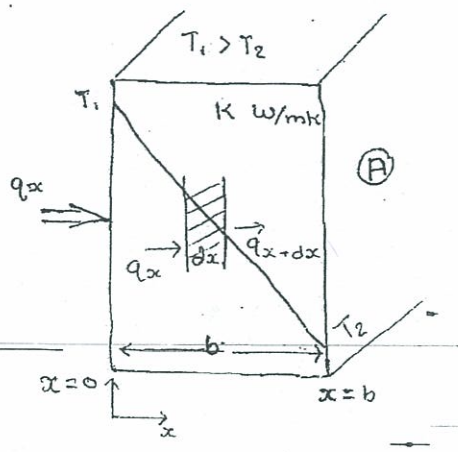
$$E_b \propto T^4$$

$$\boxed{E_b = \sigma T^4\ J/sec\ m^2 = Watt/m^2}$$

σ = Stefan Boltzman constant = $5.67 \times 10^{-8}\ W/m^2K^4$

* Integration of Fourier's Law of Conduction

Case I) Conduction HT through a slab :-



$$q_x = -KA\frac{dT}{dx} \text{ watt}$$

$$\left.\begin{aligned} \text{At } x=0, \quad T=T_1 \\ x=b, \quad T=T_2 \end{aligned}\right\} \text{boundary cond}^{ns}$$

Assumptions :-

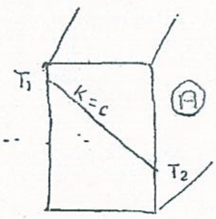
1) Steady state heat transfer [$q \neq f$(time)]

2) One dimensional heat conduction. $T = f(x)$ only

3) Uniform 'K' value

$$\int_0^b q_x\,dx = \int_{T_1}^{T_2} -KA\,dT = \int_{T_2}^{T_1} KA\,dT \quad [q_x \neq f(x) \text{ to satisfy steady state cond}^{ns}.$$
$$\text{i.e } q_x = q_{x+dx}]$$

$$\Rightarrow \boxed{q_x = \frac{KA(T_1 - T_2)}{b}} \quad \text{Watt}$$



$$q_x = -KA\frac{dT}{dx}$$

If $K = c \Rightarrow \frac{dT}{dx} = $ const

Temp. drops linearly with 'x'

Q) For a given heat flow & for the same thickness, the temperature drop across the material will be max^m for

a) Copper

b) Steel

~~c)~~ Glasswool

d) Refractory brick

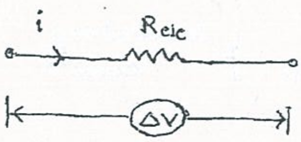
If K is min^m then ΔT is max^m

$K_{glasswool} = 0.075$ W/mK is min^m.

* Electrical Analogy of HT (Concept of Thermal Resistance)

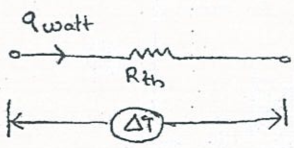
Electrical		Thermal
i Amps	$\longrightarrow$	q Watt
emf or ΔV	$\longrightarrow$	ΔT °C
R_{elec} in Ω	$\longrightarrow$	$R_{thermal}$

Electric Circuit



$$R_{ele} = \frac{\Delta V}{i}\ \Omega$$

Thermal Circuit



$$R_{th} = \left(\frac{\Delta T}{q}\right) \text{ K/watt}$$

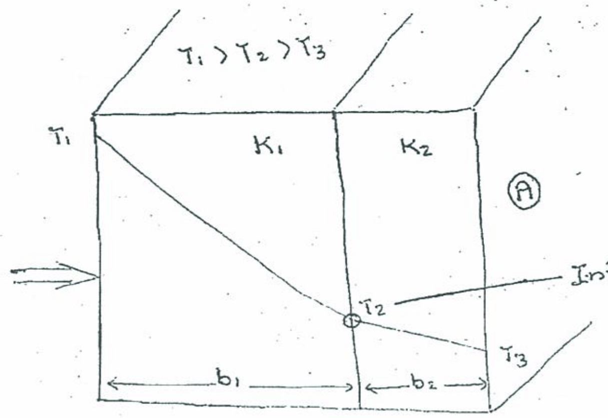
For a single slab,

$$q = KA\frac{\Delta T}{b} \quad \text{watt}$$

$$(R_{th})_{slab} = \frac{\Delta T}{q} = \frac{b}{KA} \quad K/\text{watt}$$

* More the thickness of the slab, lesser the value of its thermal conductivity, higher the thermal resistance offered by the slab. & heat transfer rate will be lesser.

* Conduction Heat Transfer through a Composite Slab :-



$$q = -KA\frac{dT}{dx}$$

If K is more $\Rightarrow \frac{dT}{dx}$ is lesser

$K_2 > K_1$

$\because$ q & A are same

Assumptions :

Steady state, one dimensional conduction heat transfer through composite slab.

$$T_1 \xrightarrow{q} \underset{\frac{b_1}{K_1A}}{\text{wwv}} T_2 \underset{\frac{b_2}{K_2A}}{\text{wwv}} T_3$$

$\therefore$ Rate of heat transfer through composite slab $= q = \dfrac{T_1 - T_3}{\frac{b_1}{K_1A} + \frac{b_2}{K_2A}}$

Heat rate / unit area $= \dfrac{q}{A}$ = Heat Flux

$$= \frac{T_1 - T_3}{\frac{b_1}{K_1} + \frac{b_2}{K_2}}$$

T_2 can be obtained from

$$q = \frac{T_1 - T_2}{\frac{b_1}{K_1A}} \Rightarrow T_2 = \text{----}\ ^\circ C \quad / \quad q = \frac{T_2 - T_3}{\frac{b_2}{K_2A}} \Rightarrow T_2 = \text{----}\ ^\circ C$$

Toppersnotes

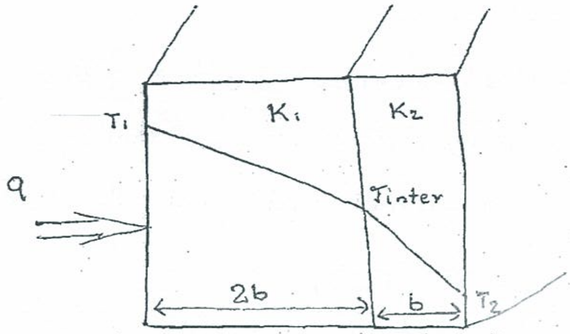
Gate 2006) In a composite slab, the temp. at the interface (P_{inter}) b/w two materials is equal to the avg of the temp of the two ends - Assuming steady one-dimensional heat conduction, which of the following statement is true about the respective thermal conduction.

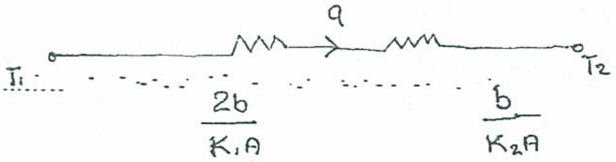
a) $2K_1 = K_2$

b) $K_1 = K_2$

c) $2K_1 = 3K_2$

d) $K_1 = 2K_2$





$$q = \frac{T_1 - T_{inter}}{2b/K_1A} = \frac{T_{inter} - T_2}{b/K_2A}$$

Put, $T_{inter} = \frac{T_1 + T_2}{2}$

$\therefore \; K_1 = 2K_2$

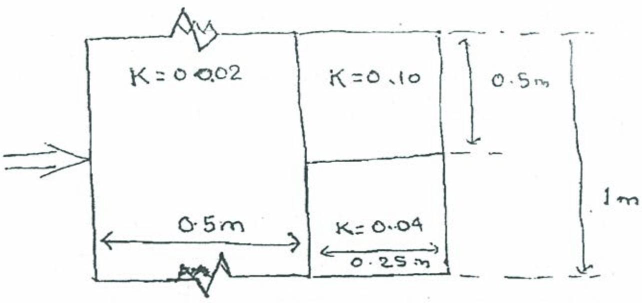
Gate 2005) Heat flows through a composite slab as shown below. The depth of the slab is 1m. The K values are in W/mK. The overall thermal resistance is in K/W

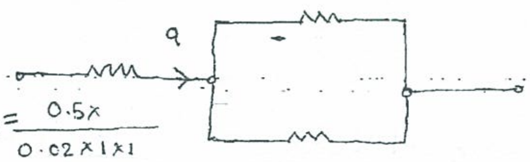
a) 17.2

b) 21.9

c) 28.6

d) 39.2





$$\frac{0.25}{0.10 \times 0.5 \times 1} = R_1$$

$$R_3 = \frac{0.5}{0.02 \times 1 \times 1}$$

$$\frac{0.25}{0.04 \times 0.5 \times 1} = R_2$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$R_1 = 5$

$R_2 = 12.5$ $\qquad R_{eq} = 3.57$

[Area of conduction is the area ⊥ar to the dir^n of Heat Transfer]

$R_3 = 25$

Overall thermal resistance $= R_{eq} + R_3$

$= 25 + 3.57$

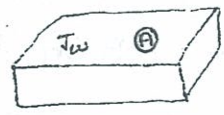
$= 28.57$

$\simeq 28.6$ K/W

$q_{conv} = hA(T_w - T_\infty)$

* Convection Thermal Resistance

$$(R_{th})_{convec} = \frac{\Delta T}{q} = \frac{T_w - T_\infty}{hA(T_w - T_\infty)}$$

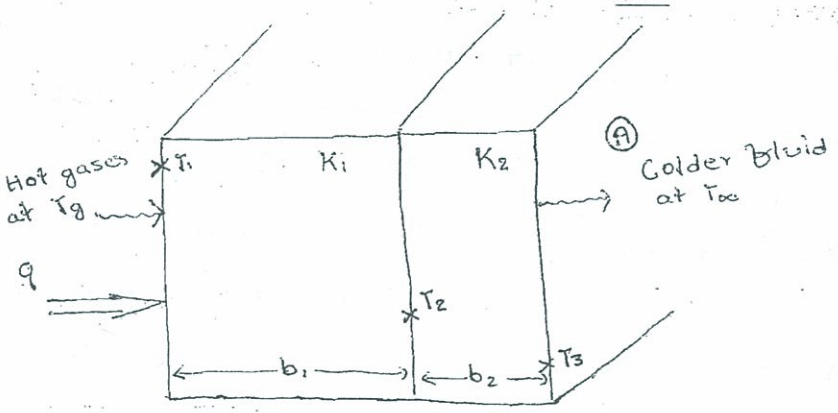


$$(R_{th})_{convection} = \frac{1}{hA} \quad \text{Kelvin/watt}$$

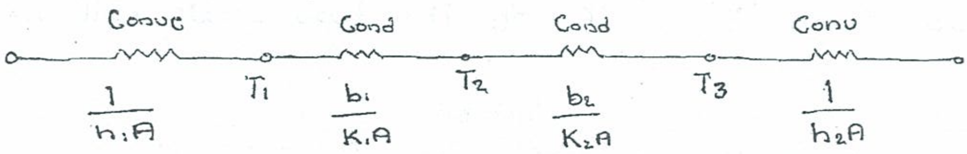
Higher the ~~coeff~~ convection heat transfer constant or more the convection heat transfer area or contact area ~~higher~~ lesser is the convection thermal resistance and higher will be the heat transfer rate.

Since 'h' values associated with water very high, it means the convection heat transfer ~~coeff~~ thermal resistance associated with water will be lesser.

* Conduction Convection heat transfer through a composite slab.



Assume one-dimensional steady state convection conduction HT b/w the hot gases and the ambient fluid through composite slab.



Rate of HT b/w hot gases and ambient colder fluid

$$q = \frac{T_G - T_\infty}{\frac{1}{h_1 A} + \frac{b_1}{K_1 A} + \frac{b_2}{K_2 A} + \frac{1}{h_2 A}} = \frac{T_G - T_\infty}{\frac{1}{A}\left[\frac{1}{h_1} + \frac{b_1}{K_1} + \frac{b_2}{K_2} + \frac{1}{h_2}\right]}$$

$\Rightarrow$ Heat Flux, $$\frac{q}{A} = \frac{T_G - T_\infty}{\frac{1}{h_1} + \frac{b_1}{K_1} + \frac{b_2}{K_2} + \frac{1}{h_2}} \text{ Watt/m}^2 \quad \text{---} \ (1)$$

* Defining overall heat transfer coefficient 'U' as the parameter which takes into account all the modes of HT into a single entity, i.e from the equation

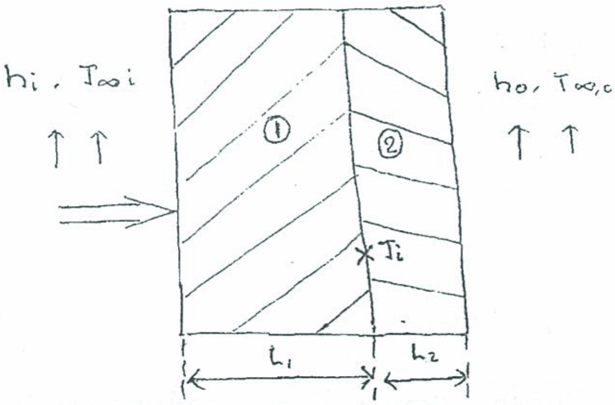
$$q = UA\Delta T = UA(T_G - T_\infty) \quad \text{Watt}/m^2K \quad \text{---- ②}$$

U & h have same units i.e W/m^2K

Comparing eqⁿ ① & ②

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{b_1}{K_1} + \frac{b_2}{K_2} + \frac{1}{h_2}}$$

Gate 2009) Consider the steady state heat conduction across the thickness in a plane composite wall as shown in fig exposed to convection, conduction on both sides.



Given data

$h_i = 20 W/m^2$; $h_o = 50 W/m^2$

$T_{\infty i} = 20°C$; $T_{\infty,o} = -2°C$

$K_1 = 20 W/mK$; $K_2 = 50 W/mK$

$L_1 = 0.3m$; $L_2 = 0.15m$

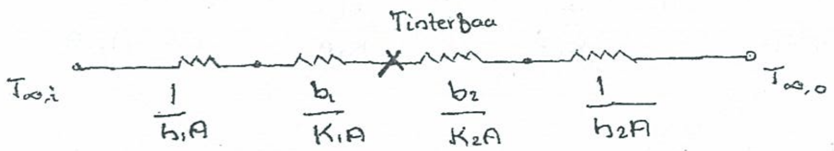
Assuming negligible contact resistance b/w the wall surfaces the interface Temp 'T' in °C of the two walls will be

a) − 0.50

b) 2.75

c) 3.75

d) 4.75



$$q = \frac{T_{\infty i} - T_{inter}}{\frac{1}{h_i A} + \frac{L_1}{K_1 A}} = \frac{T_{inter} - T_{\infty,o}}{\frac{b_2}{K_2 A} + \frac{1}{h_2 A}}$$

$$\Rightarrow \frac{20 - T_{int}}{\frac{1}{20} + \frac{0.3}{20}} = \frac{T_{inter} + 2}{\frac{0.15}{50} + \frac{1}{50}} \Rightarrow \frac{400 - 20T_{int}}{1.3} = \frac{50T_{int} + 100}{1.15}$$

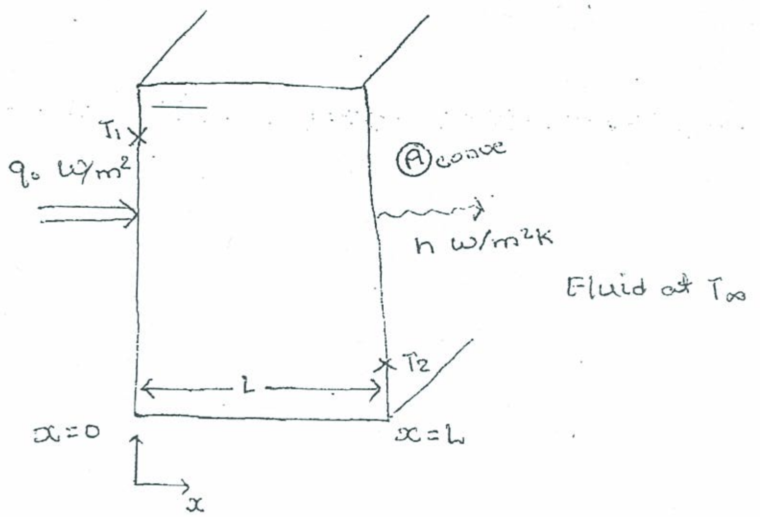
$$\Rightarrow 460 - 23T_{int} = 65T_{int} + 130$$

$\boxed{T_{int} = 3.75°C}$ Ans

ES,2003) An iron plate of thickness 'L' and thermal conductivity 'K' is subjected to a constant heat flux 'q_o W/m²' at the boundary surface at $x=0$. From the other boundary surface at $x=b$, the heat is dissipated by convection into a fluid at a temp T_∞ with a HT coeff 'h'. Develop the expression for the surface temp T_1 & T_2 at the surfaces $x=0$ and $x=L$ respectively for the following data. Calculate the surface temp. T_1 & T_2 if

$L = 2$ cm, $K = 20$ W/mK $q_o = 10^5$ W/m² $T_\infty = 50°C$ & $h = 500$ W/m²K

Sol^n :-



Answer should contain figure, thermal circuit, calculation & comments

Assume steady state. One dimensional convection-conduction HT

$$T_1 \xrightarrow[\frac{L}{K\times 1}]{q_o\ \text{W/m}^2} T_2 \xrightarrow[\frac{1}{h\times 1}]{} T_\infty$$

$$q_o = \frac{T_1 - T_2}{\frac{L}{K\times 1}} = \frac{T_2 - T_\infty}{\frac{1}{h\times 1}}$$

$$\Rightarrow 10^5 = \frac{T_2 - 50}{\frac{1}{500}} \Rightarrow 10^5 = 500T_2 - 25000$$

$$\Rightarrow \frac{125000}{500} = T_2$$

Put the value of T_2

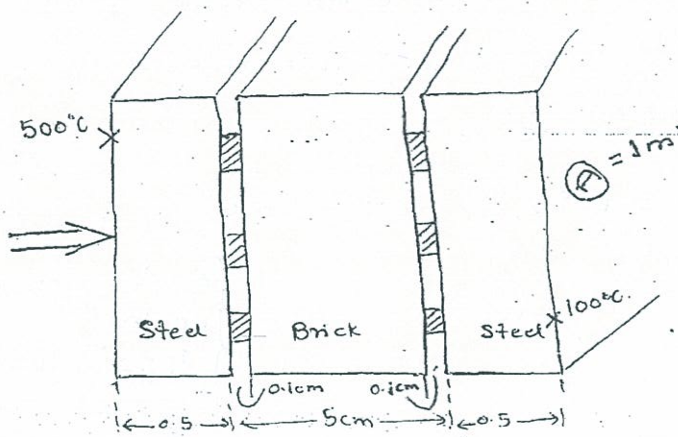
$$10^5 = \frac{T_1 - 250}{L/K} \qquad T_1 = 350°C \qquad T_2 = 250°C$$

$$\boxed{\begin{aligned} T_1 &= 350°C \\ T_2 &= 250°C \end{aligned}}$$

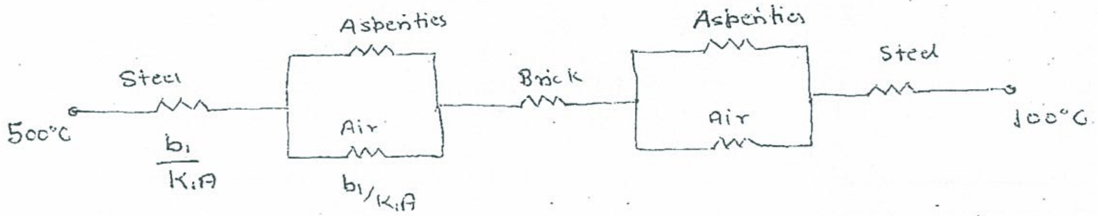
Since the convection heat transfer coeff. is very high, the fluid on the right hand side of ~~slab~~ slab may be a liquid.

*ES-2001) A layer of 5 cm thick insulating brick having conductivity of 1.5 W/mK is placed b/w two 0.5cm thick steel plates. The conductivity of MS is 50 W/mK. The faces of brick adjacent to the plates are rough having solid to solid contact of 30% of total area. The avg. height of asperities is 0.1cm. If the outer plate surface temps are 100°C & 500°C respectively, Calculate the rate of Heat transfer/area. The conductivity of air is 0.02 W/mK. —— [10.95 kW/m²]

Soln :-



Assume steady state, one dimensional HT



$$(R_{th})_{steel} = \frac{\cancel{0.5}^{\,0.01}}{\cancel{50}\times 100\times 1} = 0.0001$$

$$(R_{th})_{asp} = \frac{0.1}{100\times 1.5\times 0.3\times 1} = 0.0022$$

$$(R_{th})_{air} = \frac{0.1}{100\times 0.02\times 0.7\times 1} = 0.071$$

$$(R_{th})_{brick} = \frac{4.8}{100\times 1.5\times 1} = 0.032$$

$$\frac{1}{R_{eq}} = \frac{1}{(R_{th})_{asp}} + \frac{1}{(R_{th})_{air}} = \frac{1}{0.0022} + \frac{1}{0.071} = 454.54 + 14.08$$

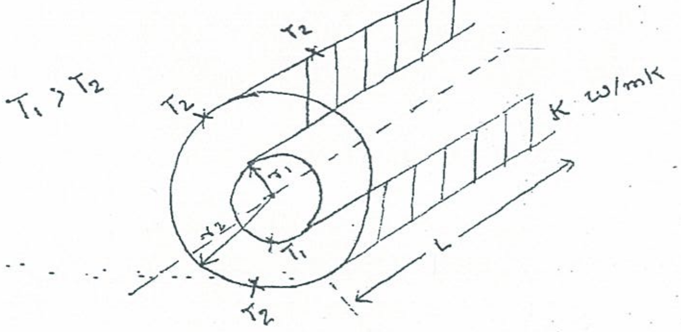
$$R_{eq} = 1/468.62 = 0.0021$$

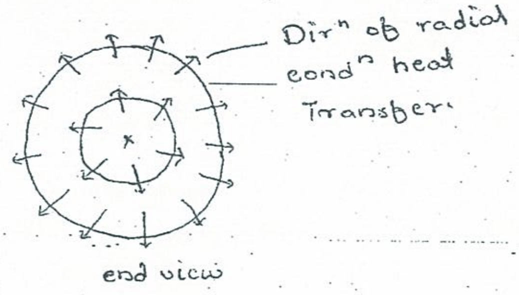
$$\text{Rate of HT/area} = \frac{T_1 - T_2}{\Sigma R_{th}} = \frac{500-100}{\Sigma R_{th}} = \frac{400}{0.0365} = 10950\ \text{W/m}^2$$

$$= 10.95\ \text{kW/m}^2$$

Since there is no macroscopic bulk motion possible for air (being trapped in a narrow gap), it is only conduction mode of heat transfer through the air.

* Radial Conduction HT through a hollow cylinder.





NOTE –

Conduction heat transfer occurs radially outward from inside cylindrical surface at R_1, which is at temp T_1 to a outside cylinder surface at 'r_2' which is at temp. T_2 $(T_1 > T_2)$

Unlike the case of slab, here the area of conduction HT changes in the dir^n of heat flow.

$T = f(r)$

At $r = r_1 \Rightarrow T = T_1$
At $r = r_2 \Rightarrow T = T_2$ } Boundary Cond^n

$T_1 > T_2$

Fourier's Law of Conduction:

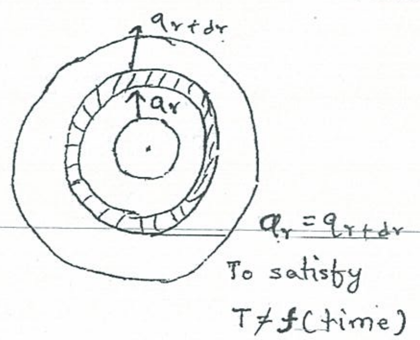
q = Rate of radial ~~conduction~~ HT $= -KA\left(\frac{dT}{dr}\right)$ watt

At any radius r, area of conduction HT $= A = 2\pi r L$

$\therefore q = -K 2\pi r L\left(\frac{dT}{dr}\right)$ watt

Assume steady state one-D, radial HT with constant thermal conductivity

$$\int_{r_1}^{r_2} q\,\frac{dr}{r} = \int_{T_1}^{T_2} -2\pi K L\, dT$$



To satisfy the steady state cond^n $q \neq f(r)$

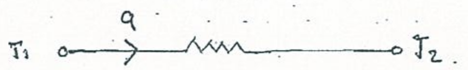
$$q \int_{r_1}^{r_2} \frac{dr}{r} = 2\pi K L \int_{T_1}^{T_2} dT$$

Toppersnotes

$$q \times \ln \frac{r_2}{r_1} = -2\pi K L (T_2 - T_1) = 2\pi K L (T_1 - T_2)$$

$$\boxed{q = \frac{2\pi K L (T_1 - T_2)}{\ln(r_2/r_1)}}$$

The equivalent thermal circuit for radial conduction through a hollow cylinder is



$$(R_{th})_{\text{hollow cylinder}} = \frac{\Delta T}{q}$$

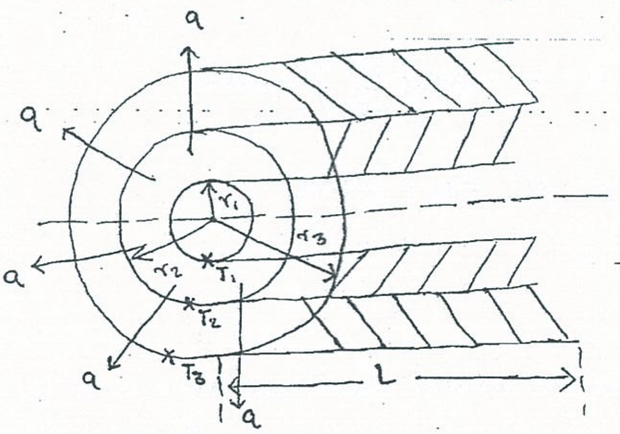
$$= \frac{T_1 - T_2}{q}$$

$$\boxed{(R_{th})_{HC} = \frac{\ln(r_2/r_1)}{2\pi K L}} \quad K/\text{watt}$$

NOTE :-

If the thickness of the hollow cylinder is very less and also if the thermal conductivity of the material is very high, then the thermal ~~conduction~~ resistance for this radial conduction almost becomes zero. Eg. - A thin copper pipe.

* Radial Conduction HT through a Composite Cylinder :-



Assume steady state One-D, radial HT from the innermost cylindrical surface at T_1 to the outermost cylindrical surface at T_3.