

IES / GATE

**Electronics &
Telecommunication
Engineering**

VOLUME-VI

Signals System, Control System

Contents

Signal System

1-219

Control System

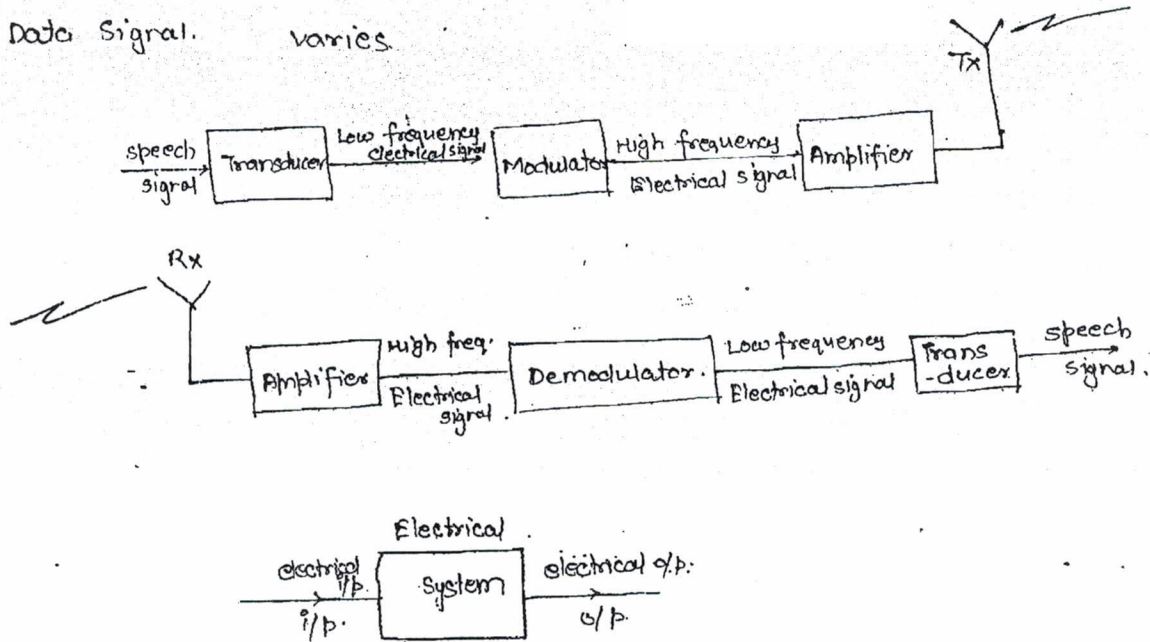
220-358

Speech Signal - 300 Hz - 3,400 Hz

Audio " 20 Hz - 20, KHz.

Video Signal. 0 - 4.5 MHz (5 MHz).

Data Signal. varies.



Signal:- signal is any quantity having information associated with it.

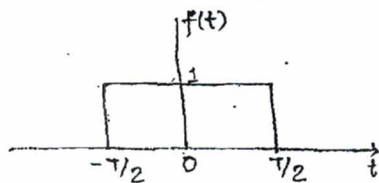
$$\left. \begin{array}{l} \text{Signal} \left\{ \begin{array}{l} \text{voltage } v(t) \\ \text{current } i(t) \end{array} \right\} f(t) \end{array} \right\}$$

* Signal need not be always single variable.

* Signal need not be always function of time.

* RGB → colour coding
 * still frames per sec. is standard
 $\frac{1}{16}$ → frames per
 ⇒ second is not visible for eye

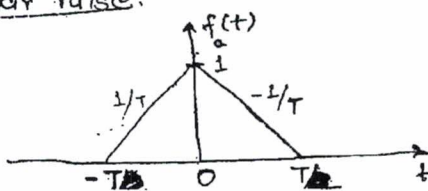
Rectangular Pulse:



$$f(t) = 1, \quad -T/2 \leq t \leq T/2$$

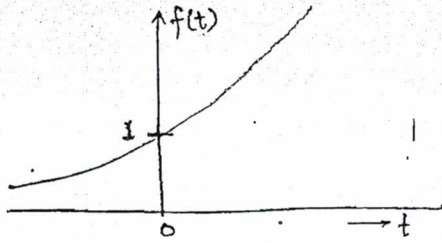
0, otherwise.

Triangular Pulse:



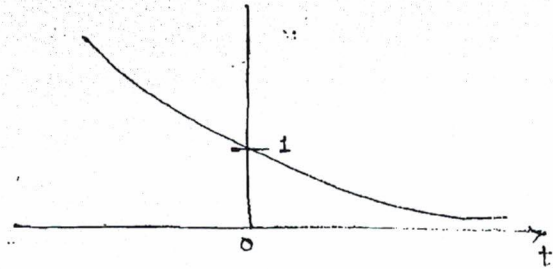
$$f(t) = \begin{cases} \frac{1}{T}t + 1, & -T \leq t \leq 0 \\ -\frac{1}{T}t + 1, & 0 \leq t \leq T \\ 0 & \text{otherwise.} \end{cases}$$

Exponentially Increasing

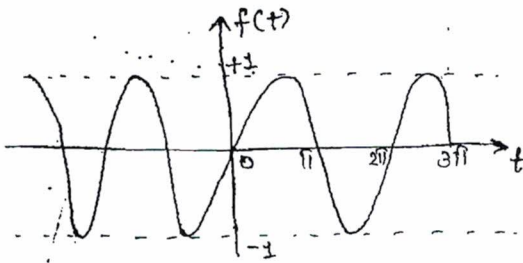


$$f(t) = e^{at}$$

Exponentially Decreasing

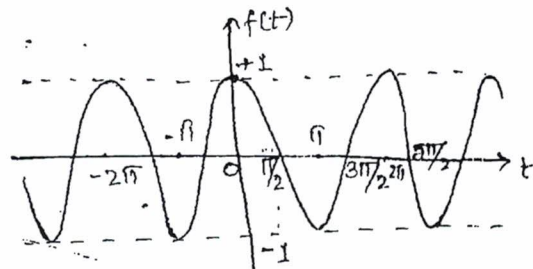


$$f(t) = e^{-at}$$



$$f(t) = \sin t$$

Sinusoidal Signals



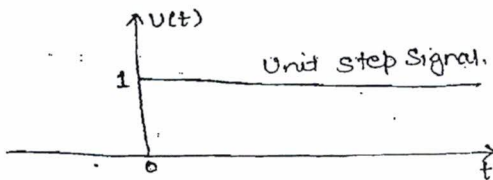
$$f(t) = \cos t$$

Zero cross over = $K\pi$
where $K = \text{integer}$

Z.C.O. = $(2K+1)\pi/2$, $K = \text{integer}$

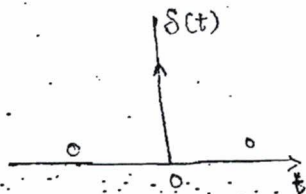
* The time instance at which the signal oscillating b/w positive & negative values cross zero value are defined as zero cross over's of such oscillating signals.

Unit step Signal: $u(t)$



$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0. \end{cases}$$

Impulse Signal:-



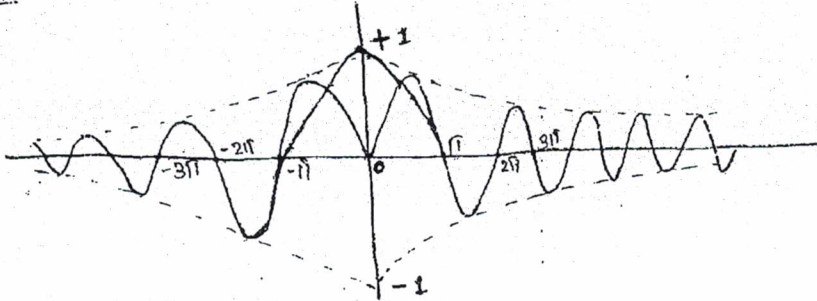
$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \neq 0 & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1 \Rightarrow \text{Unit Impulse}$$

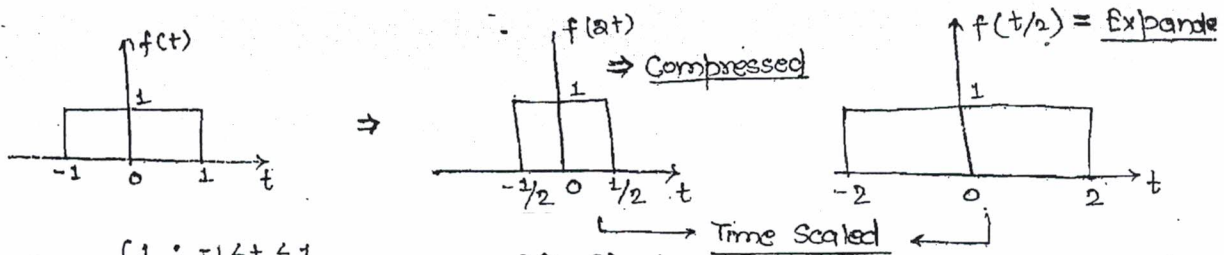
Q. Sketch the graph of signal defined as

$$s(t) = f(t) = \frac{\sin t}{t} \quad \text{also called as sampling function.}$$

Solution:



OPERATION OF SIGNAL ON TIME AXIS :-

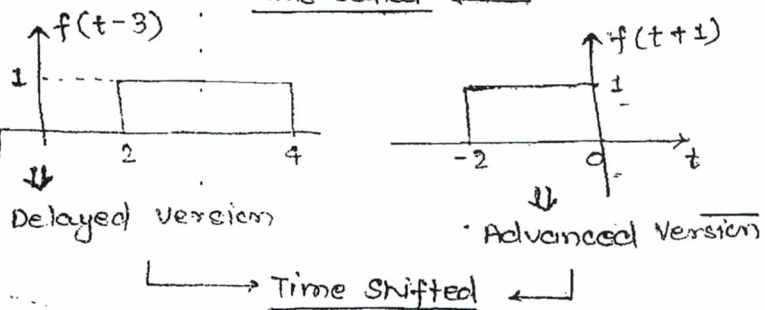


$$f(t) = \begin{cases} 1 & ; -1 \leq t \leq 1 \\ 0 & ; \text{otherwise} \end{cases}$$

By putting $t \rightarrow at$ [Compression]

$$f(at) = \begin{cases} 1 & ; -1 \leq at \leq 1 \\ 0 & ; \text{otherwise} \end{cases}$$

or

$$f(at) = \begin{cases} 1 & ; -1/2 \leq t \leq 1/2 \\ 0 & ; \text{otherwise} \end{cases}$$


By putting $t \rightarrow t/2$ [Time Expansion]

$$f(t/2) = \begin{cases} 1 & ; -1 \leq t/2 \leq 1 \\ 0 & ; \text{otherwise} \end{cases} \quad \text{or} \quad f(t/2) = \begin{cases} 1 & ; -2 \leq t \leq 2 \\ 0 & ; \text{otherwise} \end{cases}$$

Time Scaling

$$t \rightarrow at \begin{cases} a > 1 \Rightarrow \text{compression} \\ a < 1 \Rightarrow \text{expansion} \end{cases}$$

* Divide by 'a' at time scale directly.

TIME SHIFTING

Delayed or Right shifted [put $t \rightarrow (t-3)$]

$$f(t-3) = \begin{cases} 1 & ; -1 \leq (t-3) \leq 1 \\ 0 & ; \text{otherwise} \end{cases} \quad \text{or} \quad \begin{cases} 1 & ; 2 \leq t \leq 4 \\ 0 & ; \text{otherwise} \end{cases}$$

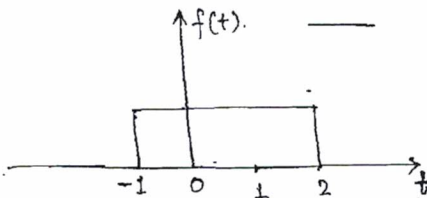
Advanced or Left shifted [Put $t \rightarrow (t+1)$]:

$$f(t+1) = \begin{cases} 1 & ; -1 \leq t+1 \leq 1 \\ 0 & ; \text{otherwise} \end{cases} \quad \text{or} \quad \begin{cases} 1 & ; -2 \leq t \leq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

Time Shifting

$t \rightarrow t - t_0 \Rightarrow$ Delay (or) right shift transformation.
 \Rightarrow Add ' t_0 ' at time scale directly.
 $t \rightarrow t + t_0 \Rightarrow$ Advance (or) left shift transformation.
 \Rightarrow Subtract ' t_0 ' on time scale directly.

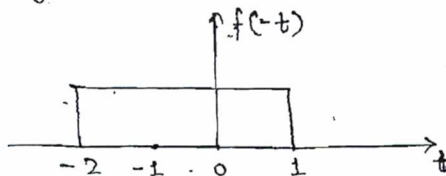
TRANSFORMATION OF SIGNALS BY GRAPHICAL METHOD.



$$f(t) = \begin{cases} 1 & -1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Time Reversed [$f(-t)$]:

putting $t \rightarrow (-t)$



$$f(-t) = \begin{cases} 1 & -1 \leq -t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

or

$$f(-t) = \begin{cases} 1 & -2 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Graphical Method :-

- * Rotation of complete Graph by 180° around Y-axis
- * Take mirror image around or across Y-axis.
- * Add extra negative sign (-) to all values of t.

$$f(t-3) \xrightarrow{t \rightarrow at} f(at-3)$$

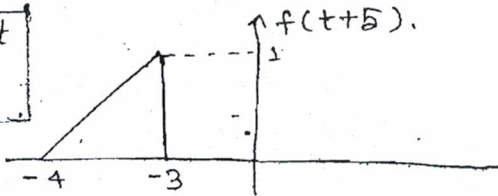
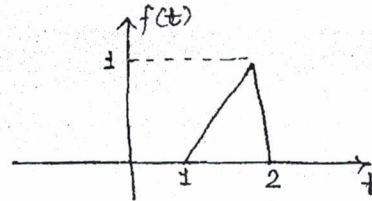
$$f(t+5) \xrightarrow{t \rightarrow -t} f(-t+5)$$

Q. Sketch the Graph of signal $f(-t+5)$ for the following signal $f(t)$ given in figure

Solution :-

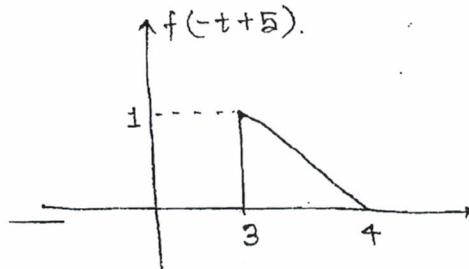
Step:1: Time Shifting

$f(t)$	$t \rightarrow t+5$	$f(t+5)$
lower limit		upper limit
$f(t) \Rightarrow 1$		$2 \leftarrow f(t)$
$f(t+5) \Rightarrow -4$		$-3 \leftarrow f(t+5)$
$f(-t+5) \Rightarrow 4$		$3 \leftarrow f(-t+5)$



Step:2: Time Reversal

$f(t+5)$	$t \rightarrow -t$	$f(-t+5)$
$-4 \rightarrow 4$		
$-3 \rightarrow 3$		



Note :- when it is required to perform combination of time shifting & time reversal transformation's on a given signal $f(t)$, to get an expression of the form $f(-t+b)$

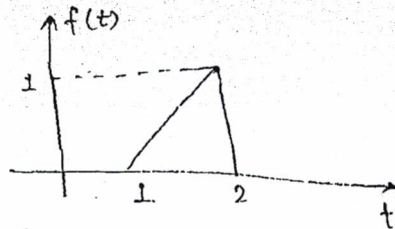
Note \rightarrow the natural order is time shifting first followed by time reversal. However shifting can also be performed after reversal with the precaution of changing the visible shifting transformation to its opposite nature i.e. if it is visibly advance, change it to delay. & vice-versa.

Q. For the above signal sketch the graph of $f(2t-3)$.

Solution:

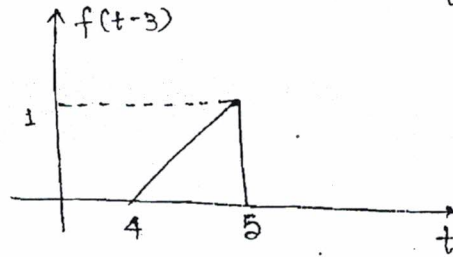
* Step: 1 \Rightarrow Time shifting

$$f(t) \xrightarrow{t \rightarrow t-3} f(t-3)$$

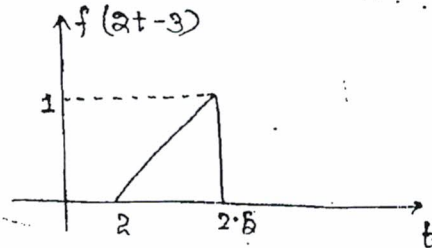


* Step: 2 \Rightarrow Time scaling

$$f(t-3) \xrightarrow{t \rightarrow 2t} f(2t-3)$$



lower limit	upper limit
$f(t) \Rightarrow$ ①	②
$f(t-3) \Rightarrow$ 4	5
$f(2t-3) \Rightarrow$ ②	⑤/2



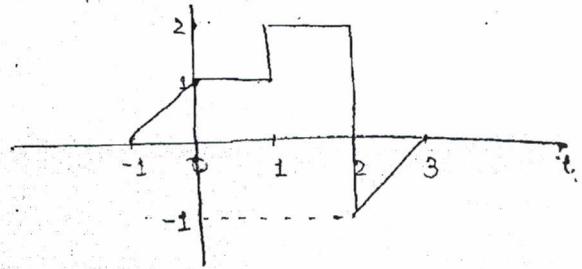
1) Note: * when it is required to perform combination of time shifting & time scaling to get an expression of the form $f(at-b)$ for the given signal $f(t)$, the natural order is shifting first followed by scaling. However shifting can also be performed after scaling with a precaution of dividing the visible shift amount by the scaling factor.

2) * when we have combination of time scaling & time reversal they can be performed in any order without any precautions.

Q. For the following signal $f(t)$ sketch the graph of $f(-2t-5)$.

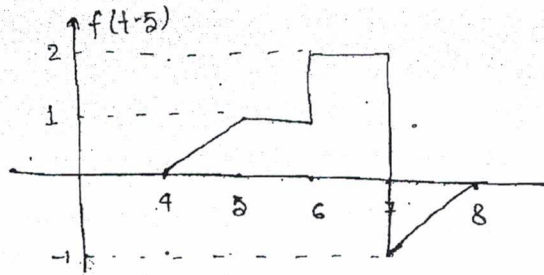
Step: 1 Time shifting

$$f(t) \xrightarrow{t \rightarrow t-5} f(t-5)$$



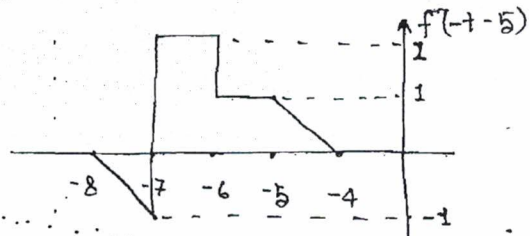
Step: 2 Time reversal

$$f(t-5) \xrightarrow{t \rightarrow -t} f(-t-5)$$

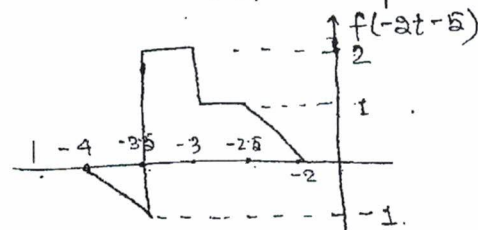


Step: 3 Time scaling

$$f(-t-5) \xrightarrow{t \rightarrow at} f(-at-5)$$



$f(t)$	(-1)	(0)	(1)	(2)	(3)
$f(t-5)$	4	5	6	7	8
$f(at-5)$	2	5/2	3	7/2	4
$f(-at-5)$	(-2)	(-5/2)	(-3)	(-7/2)	(-4)

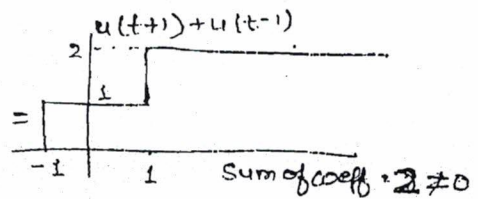
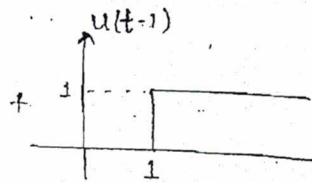
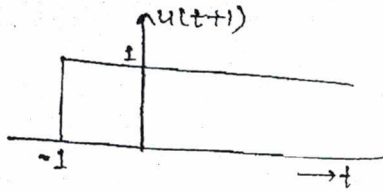
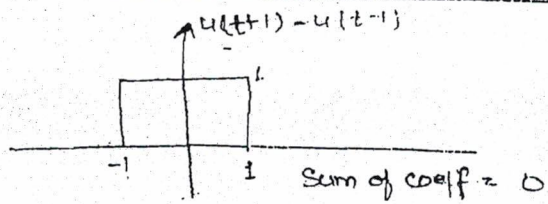
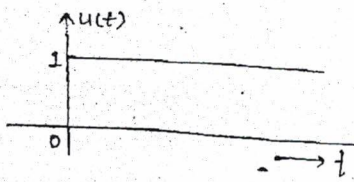


Q: If in the above problem $f(-at-5)$ is to be realized by performing the operations in the order of time reversal time scaling & then time shifting what are the transformations required.

Solution:

$$f(t) \longrightarrow f(-at-5)$$

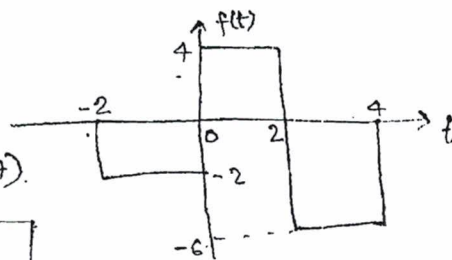
$$\left\{ \begin{array}{l} \text{Time Reversal} \quad t \rightarrow -t \quad f(-t) \\ \text{Time scaling} \quad t \rightarrow at \quad f(-at) \\ \text{Time shifting} \quad \begin{array}{l} t \rightarrow t+5/2 \\ t \rightarrow t-5 \end{array} \quad f[-a(t+5/2)] = f(-at-5) \end{array} \right.$$



* Any signal $f(t)$ having step changes alone can always be defined as sum of number of shifted step signals with suitable co-efficient's where the shift of $u(t)$ represent's the timing instant at which a step change is required in the given signal & the co-efficient indicates the amount of step change required at the shift given to $u(t)$.

Q: Represent the following signal $f(t)$ using shifted step signals.

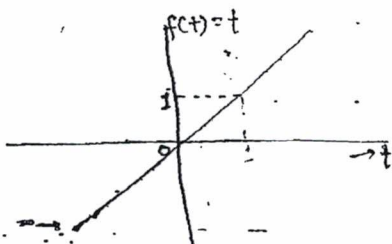
$$f(t) = -2u(t+2) + 6u(t) - 10u(t-2) + 6u(t-4)$$



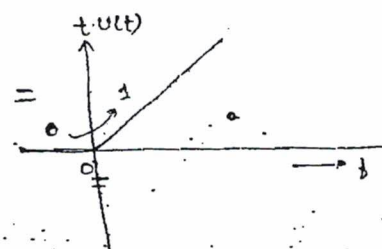
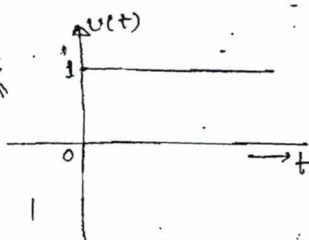
Sum of Co-efficient = $12 - 12 = 0$.

* In the above representation the sum of the co-efficient's will be zero only if signal has a finite duration.

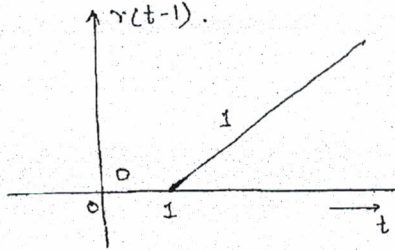
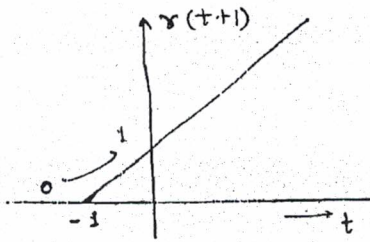
UNIT RAMP SIGNAL



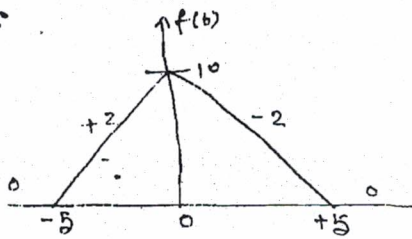
×



$$r(t) = t u(t) \quad r(t) = \begin{cases} 0, & t < 0. \end{cases}$$



Ex.

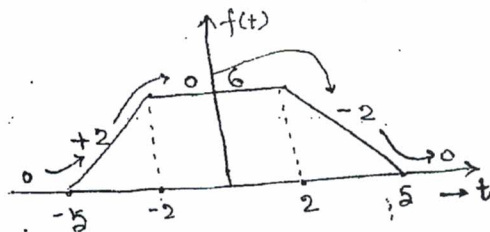


$$f(t) = 2r(t+5) - 4r(t) + 2r(t-5)$$

Note: Any signal $f(t)$ which is a combination of straight lines can always be defined as sum of number of shifted ramp signals with suitable co-efficients where the shift of the ramp signals indicates the time instant at which a change in slope is required in given signal & the co-efficient represents the amount of change in slope required at the shift given to $r(t)$.

Question Represent the following signal using ramp signal.

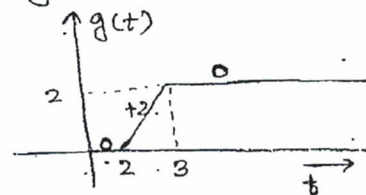
$$f(t) = 2r(t+5) - 2r(t+2) - 2r(t-2) + 2r(t-5)$$



Trapezoidal Pulse.

Question: Represent the following signal, $g(t)$ using ramp signals

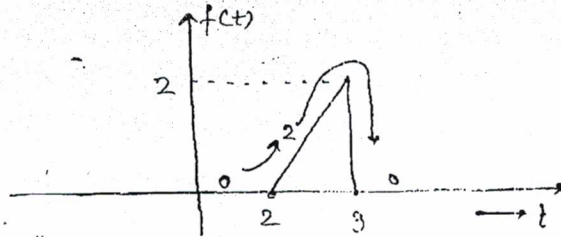
$$g(t) = 2r(t-2) - 2r(t-3)$$



Q. Represent the following signal $f(t)$ using shifted ramp and shifted step signals.

Solution:

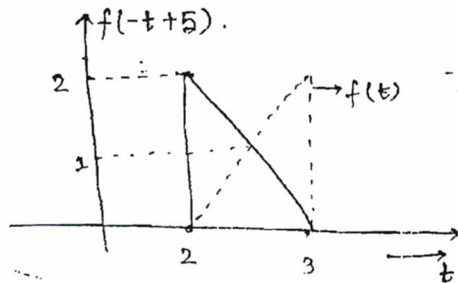
$$f(t) = 2r(t-2) - 2r(t-3) \\ + 2u(t-3)$$



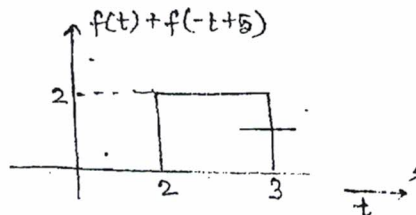
Sawtooth Pulse.

Q. For the above sawtooth pulse $f(t)$, $f(t) + f(t+5)$ will be

- (a) $u(t-2) - u(t-3)$
- (b) $r(t-2) - r(t-3)$
- (c) $2[u(t-2) - u(t-3)]$
- (d) $2[r(t-2) - r(t-3)]$



$$f(t) + f(t+5) = 2u(t-2) - 2u(t-3) \\ = 2[u(t-2) - u(t-3)]$$

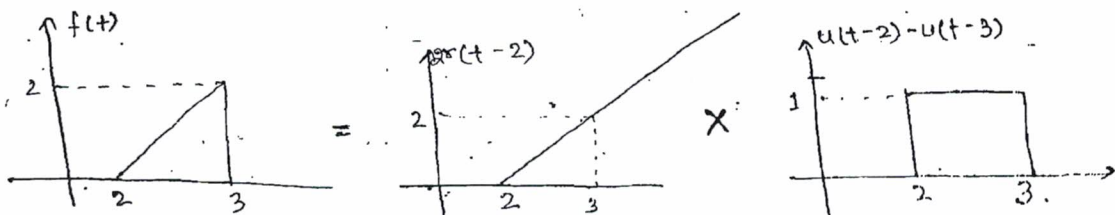


Note: * Addition of two sawtooth pulses occurring between the same two time instances having the same slope but of opposite sign will always result in a rectangular pulse.

imp

* whenever it is required to sustain only a portion of a given signal multiply the signal by a suitable pulse having a value one in the required portion and a value 0 in the unrequired portion.

imp

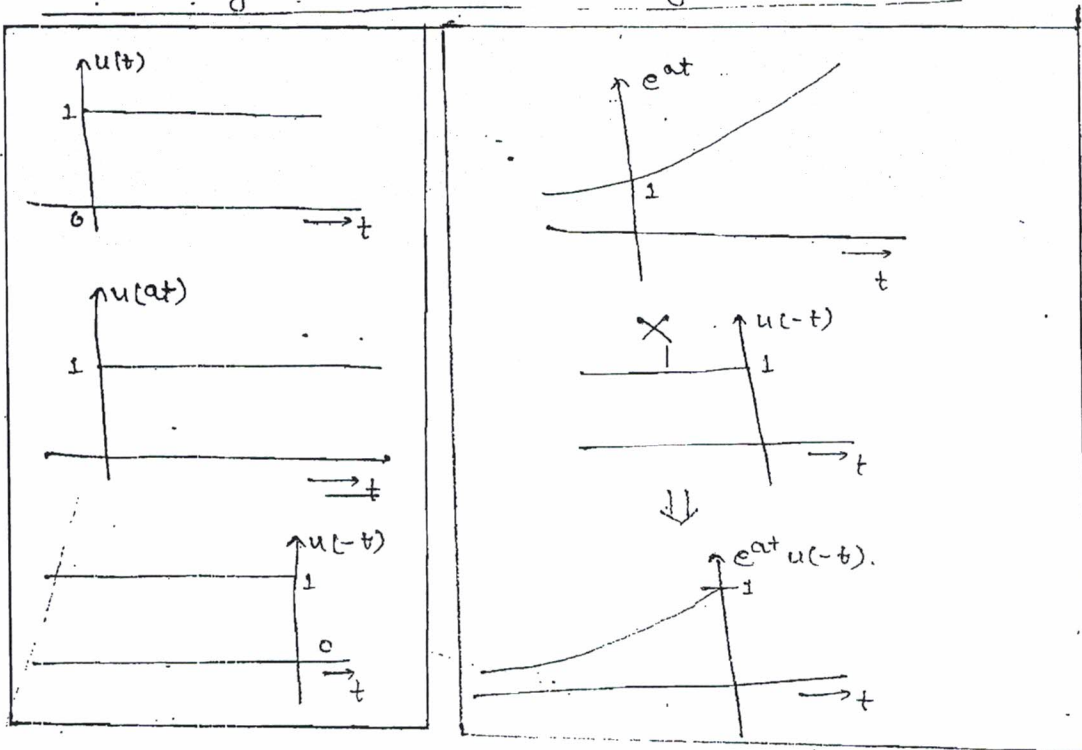


or $2t-4$

$$\begin{aligned}
 f(t) &= 2x(t-2) [u(t-2) - u(t-3)] \\
 &= (2t-4) [u(t-2) - u(t-3)] \\
 &= (2t-4)u(t-2) - (2t-4)u(t-3) \\
 &= (2t-4)u(t-2) - (2t-6+2)u(t-3) \\
 &= 2(t-2)u(t-2) - 2(t-3)u(t-3) - 2u(t-3)
 \end{aligned}$$

$$\boxed{f(t) = 2x(t-2) - 2x(t-3) - 2u(t-3)}$$

Time Scaling + Time reversal change on unit step.



Sketch the Graph of the following signals.

- (a) $e^{at}u(t)$ (d) $\sin t [u(-t)]$ (g) $u(t) - u(-t)$
 (b) $e^{-at}u(-t)$ (e) $\cos t u(t)$
 (c) $e^{at}u(t)$ (f) $u(t) + u(-t)$

Time scaling effect for Ramp Signal

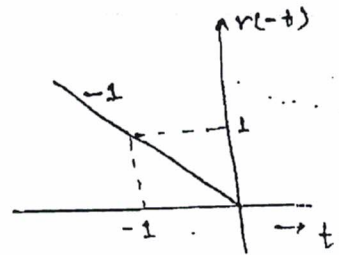
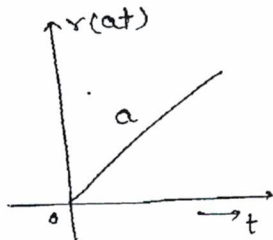
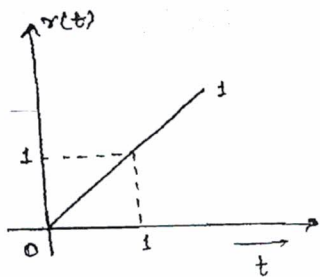
For a ramp signal there is absolutely no difference between time scaling + magnitude scaling

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$r(at) = \begin{cases} at & at \geq 0 \\ 0 & at < 0 \end{cases} \quad \text{or} \quad \begin{cases} at & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$ar(t) = \begin{cases} at & t \geq 0 \\ 0 & t < 0 \end{cases}$$

so $r(at) = ar(t)$



$$r(-t) = \begin{cases} 0 & t > 0 \\ -t & t \leq 0 \end{cases}$$

$$r(3t) = 3r(t)$$

$$r(3t) \neq -3r(t)$$

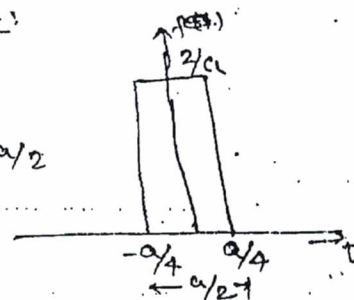
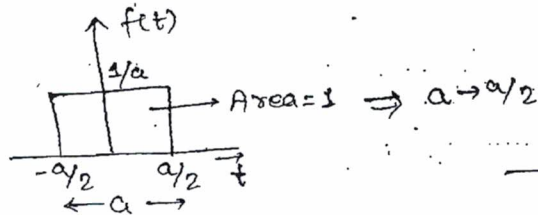
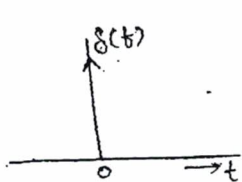
$$r(at) = ar(t)$$

↳ when a is negative only.

Q: Sketch the Graphs of following Signals.

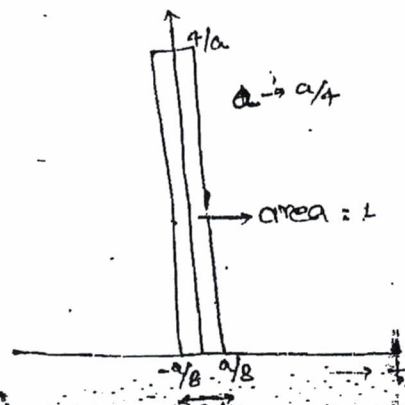
- (1) $r(t+6)$ (2) $r(-t+5)$

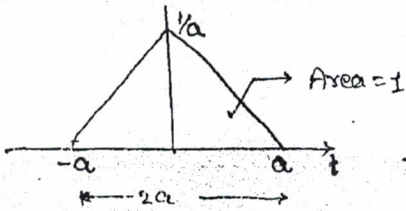
Effect of Scaling on Unit Impulse function:



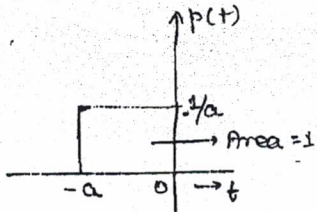
$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$$



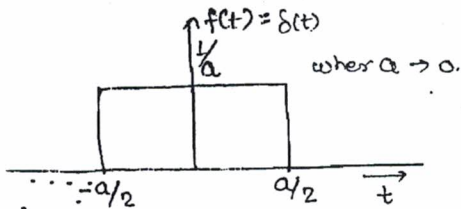


$\lim_{t \rightarrow 0} g(t) = \infty$ (undefined).
 Still Area = 1
 So $g(t) \approx \delta(t)$.

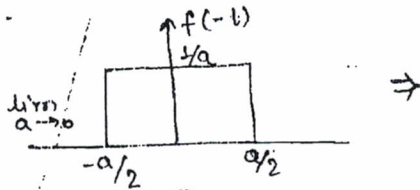
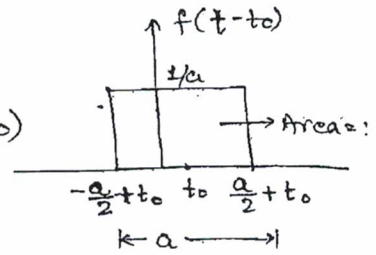


Note:- $\delta(t)$ signal does not have unique definition like other signals, its definition is only generalised by using a wide variety of signals for which all signals the width's & height are inversely proportional and area is equal to 1.

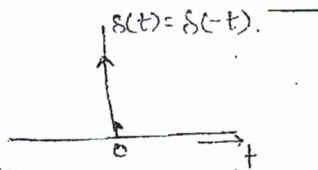
Hence $\delta(t)$ is also called as a generalised function



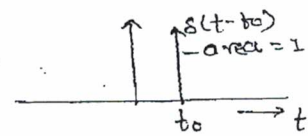
Time shifting $\rightarrow \lim_{a \rightarrow 0} f(t-t_0)$



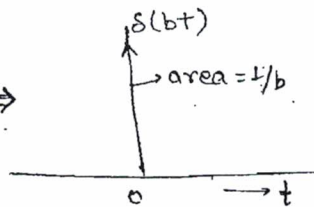
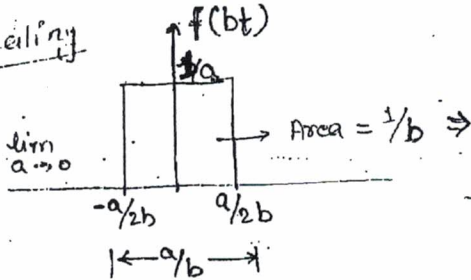
\Rightarrow



or



scaling



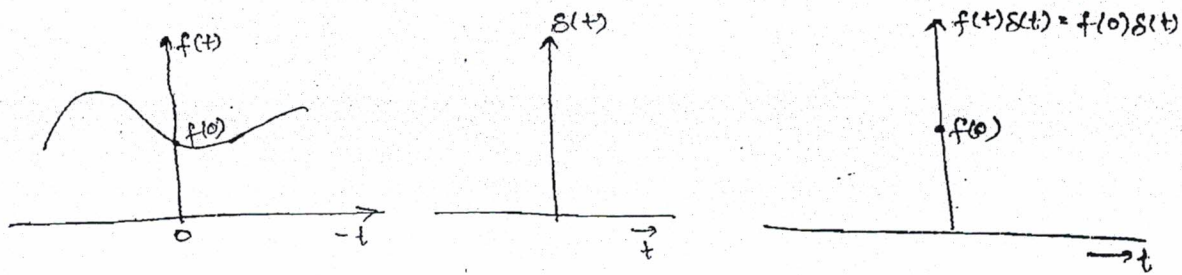
$$A \cdot \delta(t) \Rightarrow \int_{-\infty}^{\infty} A \cdot \delta(t) \cdot dt = A \int_{-\infty}^{\infty} \delta(t) dt = A$$

$$\begin{aligned} r(at) &= a r(t) \\ \delta(bt) &= \frac{1}{b} \delta(t) \end{aligned}$$

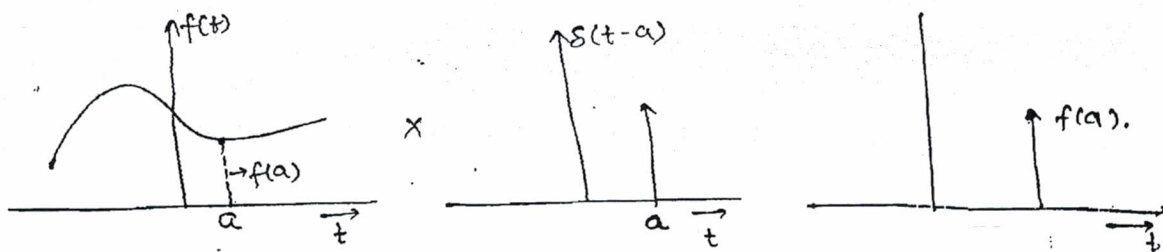
e.g: $\delta(3t) = \frac{1}{3} \delta(t)$
 $\delta(-3t) = \frac{1}{3} \delta(t)$

$$\delta(bt) = \frac{1}{|b|} \delta(t)$$

b can be (-)ve or (+)ve



$$f(t) \cdot \delta(t) = f(0) \delta(t)$$



$$f(t) \cdot \delta(t-a) = f(a) \delta(t-a)$$

This is known as sampling property of delta function.

$$1. \int_{-\infty}^{\infty} f(t) \cdot \delta(t) \cdot dt = f(0) \quad \text{or} \quad \int_{-\infty}^{\infty} f(0) \cdot \delta(t) \cdot dt = f(0) \int_{-\infty}^{\infty} \delta(t) \cdot dt = f(0) \times 1 = f(0)$$

$$2. \int_{-\infty}^{\infty} f(t) \delta(t-a) \cdot dt = f(a) \Rightarrow \text{This property is known as shifting property}$$

$$\int_{-\infty}^{\infty} f(a) \cdot \delta(t-a) \cdot dt = f(a) \int_{-\infty}^{\infty} \delta(t-a) \cdot dt = f(a)$$

Q. Evaluate the value of following integrals.

$$1) \int_{-\infty}^{\infty} \cos t \delta(t) \cdot dt = \cos(0) = 1 \quad \text{Ans} \quad 4) \int_{-\pi/6}^{\pi/6} \sin(t - \pi/2) \delta(3t - \pi) \cdot dt = 0 \quad \text{Ans}$$

$$2) \int_{-\infty}^{\infty} \sin(t - \pi/2) \delta(t - \pi) \cdot dt = \sin(\pi - \pi/2) = \sin \pi/2 = 1 \quad \text{Ans}$$

$$3) \int_{-\infty}^{\infty} \sin(t - \pi/2) \delta(3t - \pi) \cdot dt = \frac{1}{3} \sin(\pi/3 - \pi/2) = \frac{1}{3} \delta(\pi/3 - \pi/2) = \frac{1}{3} \delta(\pi/3 - \pi/2) = \frac{1}{3} \sin(\pi/3 - \pi/2) = \frac{1}{3} (-1/2) = -1/6 \quad \text{Ans}$$

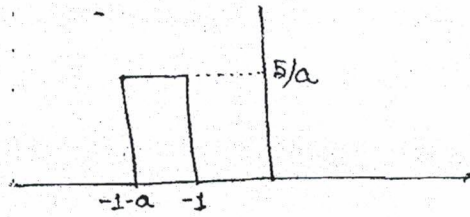
Evaluate the value of the following integral

V. Imp

$$\int_{-\infty}^{\infty} \frac{\sin[\pi/2(t-2)]}{t^2+1} \cdot g(t) \cdot dt$$

where $g(t)$ is the following

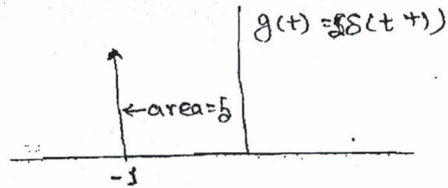
impulse as $a \rightarrow 0$.



$$\int_{-1-a}^{-1} \frac{\sin[\pi/2(t-2)]}{t^2+1} \cdot \frac{5}{a} \cdot dt$$

or

$$5 \int_{-\infty}^{\infty} \frac{\sin[\pi/2(t-2)]}{t^2+1} \cdot \delta(t+1) \cdot dt$$



$$x(t) = A\delta(t)$$

↳ weight of input
or

→ Area of under weighted inputs

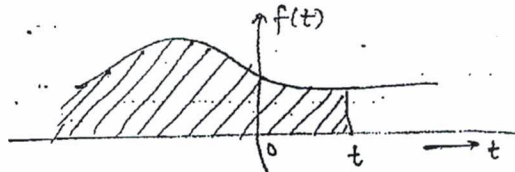
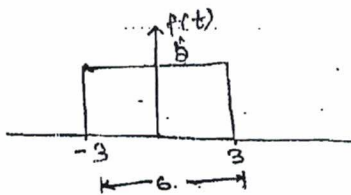
$$5 \frac{\sin[\pi/2(-1-2)]}{(-1)^2+1} = \frac{5}{2} \sin\left[-\frac{3\pi}{2}\right]$$

$$\dots = -\frac{5}{2} \sin \frac{3\pi}{2} = -\frac{5}{2} \times -\sin \pi/2 = \boxed{5/2 \text{ Ans}}$$

Differentiation & Integration of signals :-

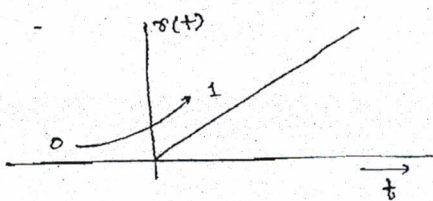
13/08/2013

Integral of any signal is physically the measure of the time varying area under the signal in the limits from $-\infty$ to t , where t can range from $-\infty$ to $+\infty$.



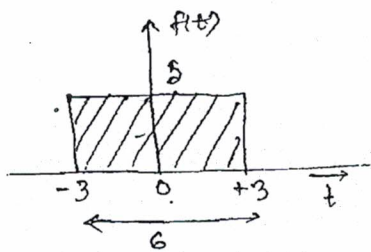
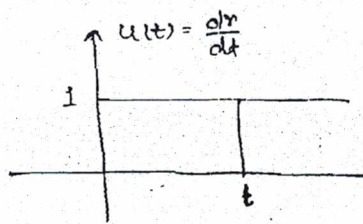
$$\int_{-\infty}^{\infty} f(t) \cdot dt = \left. \begin{matrix} \infty \\ \infty \end{matrix} \right\} \begin{matrix} \text{finite} \\ \text{infinite} \end{matrix}$$

$$\int_{-\infty}^{\infty} f(t) \cdot dt = g(t) \rightarrow \text{Integral of } f(t)$$

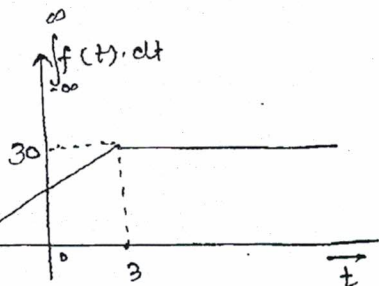


$$\frac{d u(t)}{d t}$$

$$\int dt$$

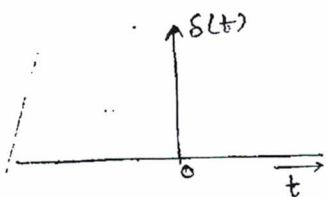


$$\int$$

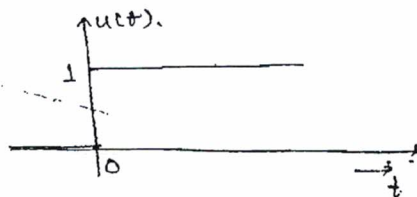


Ans

$$\int_{-\infty}^t f(t) dt = \begin{cases} 0 & t < -3 \\ 5(t+3) & -3 \leq t < 3 \\ 30 & t > 3 \end{cases}$$



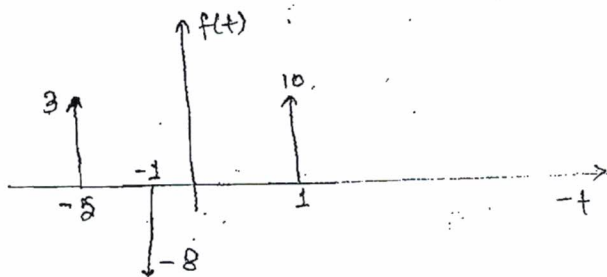
$$\int$$



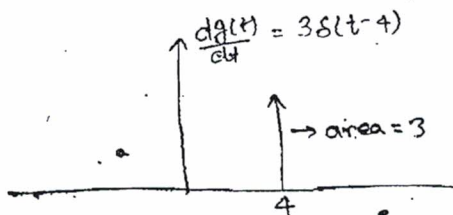
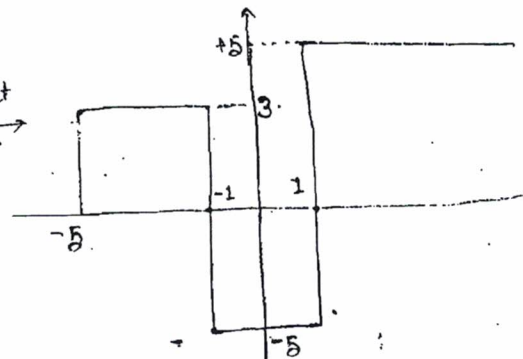
$$u(t) = \int_{-\infty}^t s(t) dt$$

$$s(t) = \frac{du}{dt}$$

Que 8. Q: Find the integral of a signal f(t) given below



$$\int dt$$



$$\int$$

