IES/GATE

Electrical Engineering

VOLUME-V

Signals & Systems

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Signals And System

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1. Signal Definition and its classification
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2. Different operations on signals.
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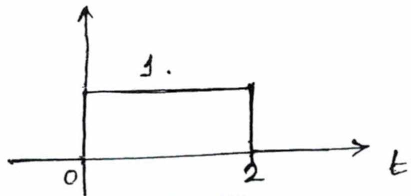
    - c) DCT (Discrete Cosine Transform)

Chapter 2 :

Different Operations on Signal

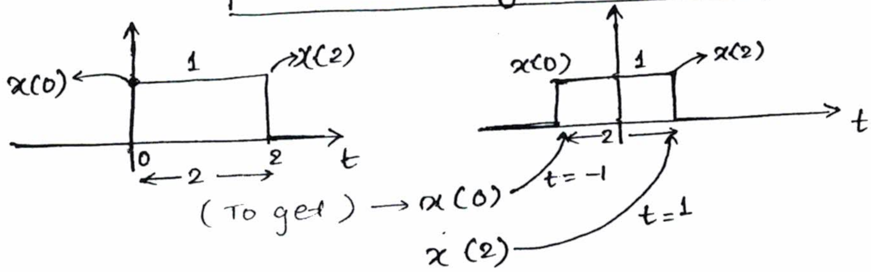
i) Time Shifting :

$$x(t) \longrightarrow y(t) = x(t+k)$$



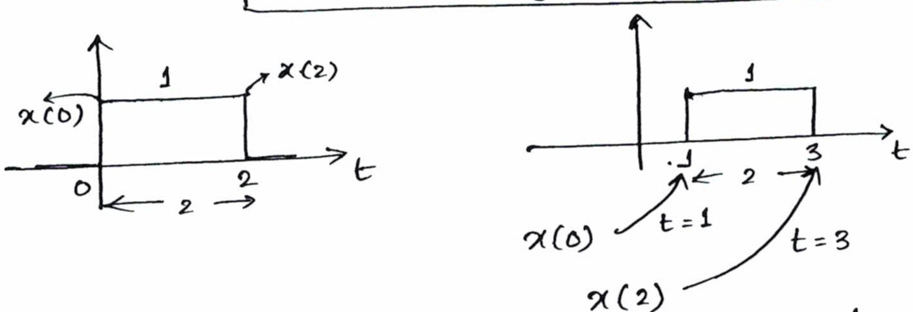
Case (a) : when $\boxed{k > 0}$] → +ve (Left shifting)
ex : k = 1

$$\boxed{x(t) \longrightarrow y(t) = x(t+1)}$$



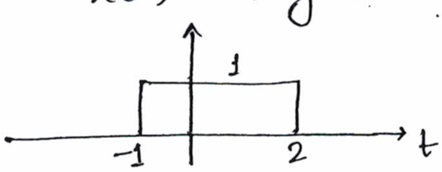
Case (b) : When $\boxed{k < 0}$ } → -ve (Right shifting)
ex k = -1

$$\boxed{x(t) \longrightarrow y(t) = x(t-1)}$$



ii) Amplitude - Shifting : → Upward / → Downward.

$$x(t) \longrightarrow y(t) = k + x(t)$$



Case (a) : When $k > 0$

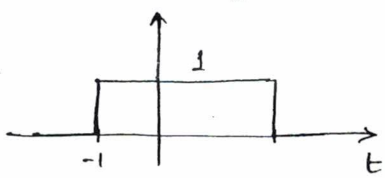
ex $\rightarrow$ $k = 1$.

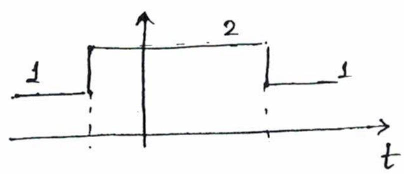
$$\rightarrow x(t) = \begin{cases} 0 ; & t < -1 \\ 1 , & -1 \leq t \leq 2 \\ 0 ; & t > 2 \end{cases}$$

$$\rightarrow y(t) = 1 + x(t)$$

$$= \begin{cases} 1 + 0 \Rightarrow 1 ; & t < -1 \\ 1 + 1 \Rightarrow 2 ; & -1 \leq t \leq 2 \\ 1 + 0 \Rightarrow 1 ; & t > 2 \end{cases}$$

$x(t) \rightarrow y(t) = 1 + x(t)$





Case (b) : when $k < 0$

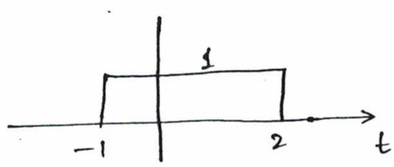
ex : $k = -1$.

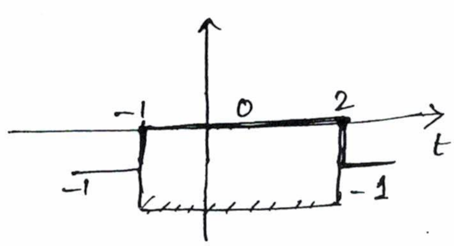
$$\rightarrow x(t) = \begin{cases} 0 , & t < -1 . \\ 1 , & -1 \leq t \leq 2 \\ 0 , & t > 2 \end{cases}$$

$$\rightarrow y(t) = -1 + x(t)$$

$$= \begin{cases} -1 + 0 \Rightarrow -1 ; & t < -1 \\ -1 + 1 \Rightarrow 0 ; & -1 \leq t \leq 2 \\ -1 + 0 \Rightarrow -1 ; & t > 2 \end{cases}$$

$x(t) \longrightarrow y(t) = -1 + x(t)$

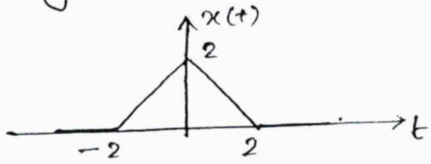




Q. Draw the waveform of

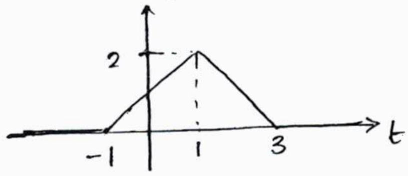
$$y(t) = -4 + x(t-1)$$

where



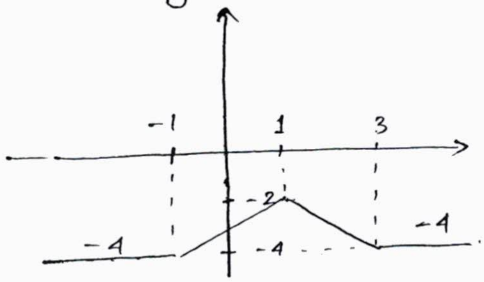
$\Rightarrow$ <i>

$x(t-1) \rightarrow$ RS



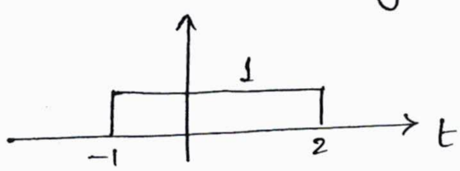
$\Rightarrow$

$y(t) = -4 + x(t-1)$



iii> Time Scaling :

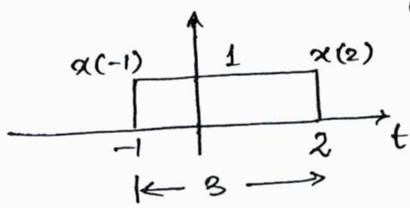
$$x(t) \longrightarrow y(t) = x(at) \; ; \; a \neq 0$$

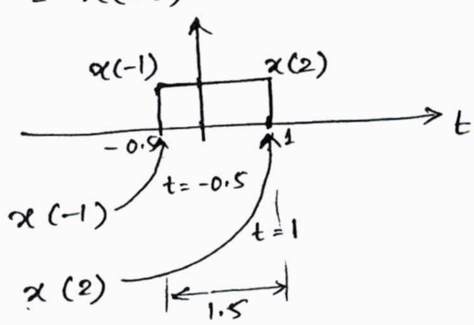


Case a> : when $\boxed{a > 1}$ } $\rightarrow$ Compression.

Ex : $a = 2$

$$x(t) \longrightarrow y(t) = x(2t)$$

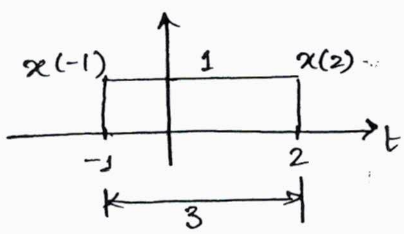


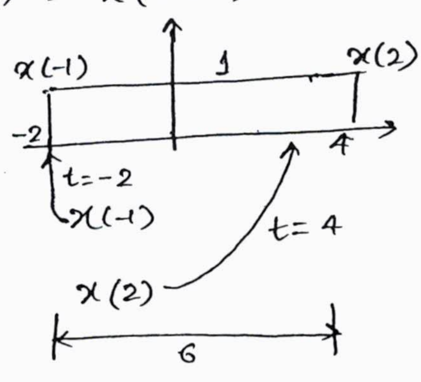


Case b> : when $\boxed{0 < a < 1}$ } $\rightarrow$ Expansion

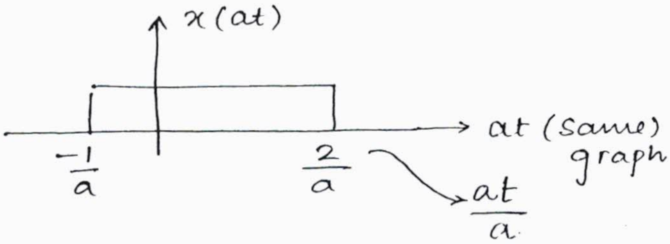
ex : $a = 0.5$

$$x(t) \longrightarrow y(t) = x(0.5t)$$

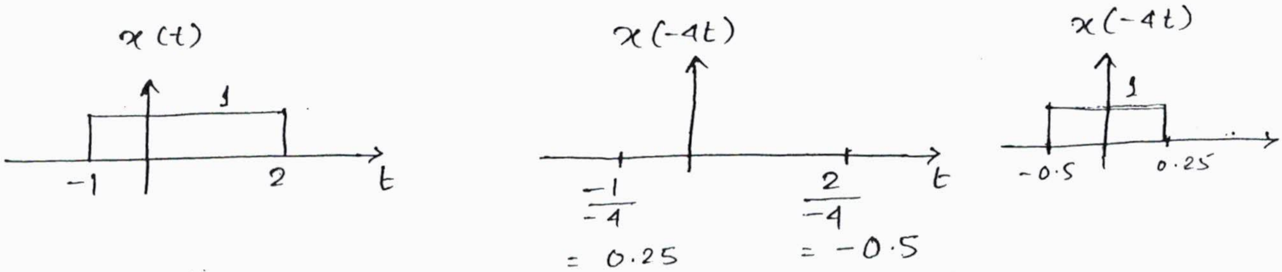




General Rule :



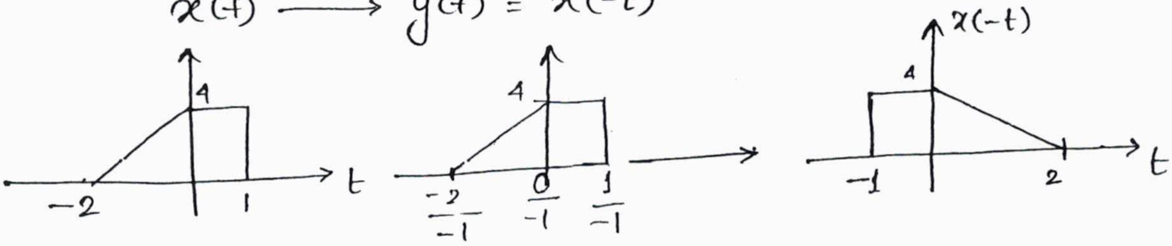
Ex : $x(-4t)$



iv) **Time Reversal :** → folding about Y axis

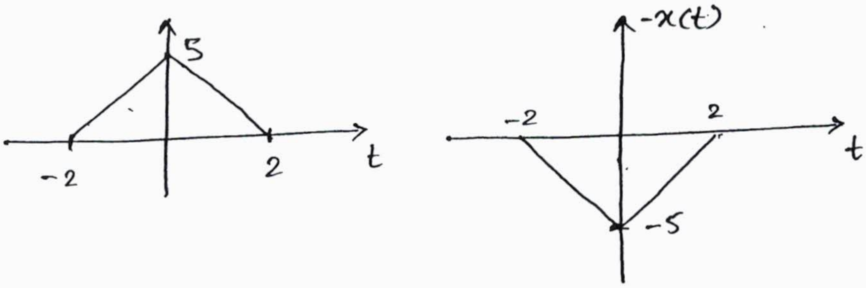
→ Time Scaling $a = -1$

$$x(t) \longrightarrow y(t) = x(-t)$$

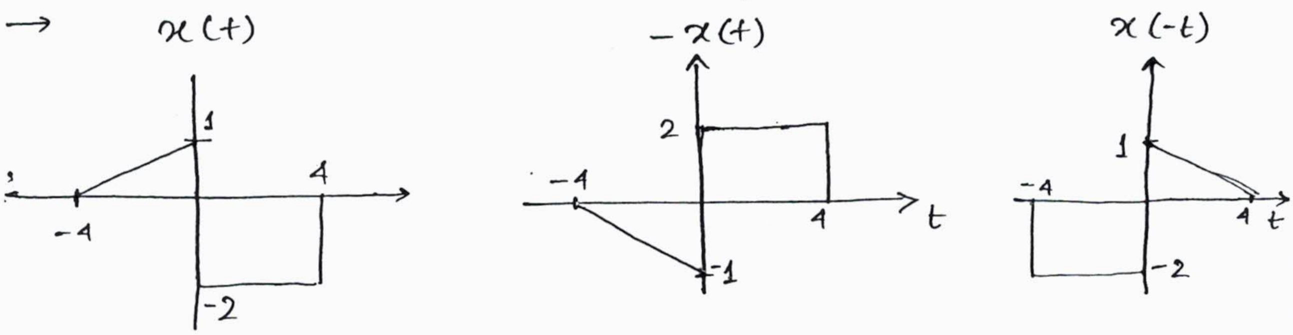


v) **Amplitude Reversal :** → folding about X axis

$$x(t) \longrightarrow y(t) = -x(t)$$

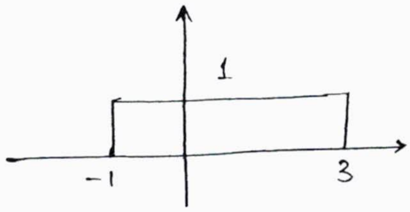


Ex →

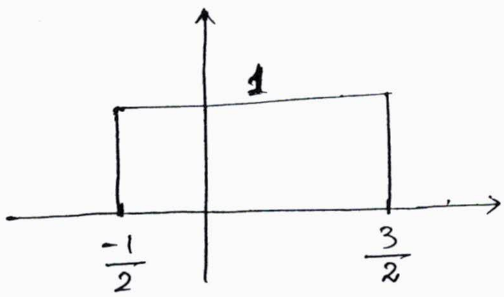


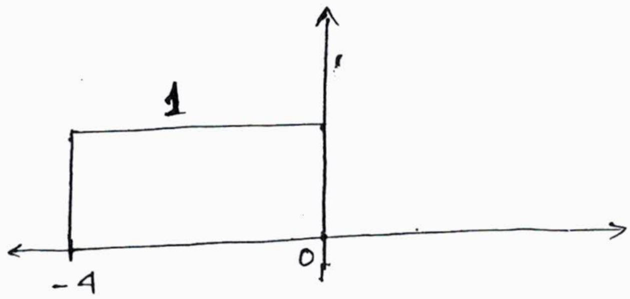
Q. $x(t)$ Draw the waveform of $y(t)$

$y(t) = x(2t+3)$



$x(2t)$ $x(2t+3)$

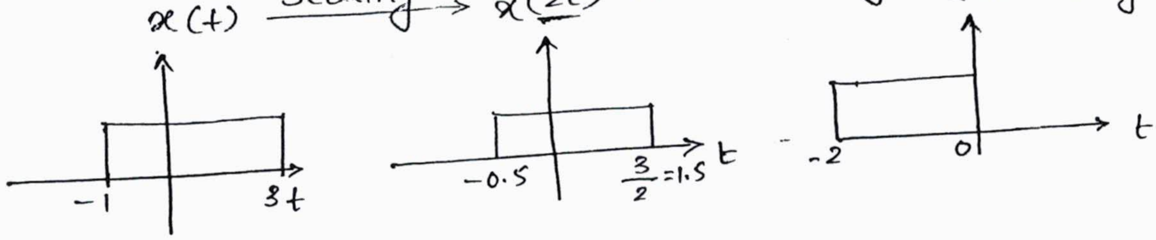




1st Method :

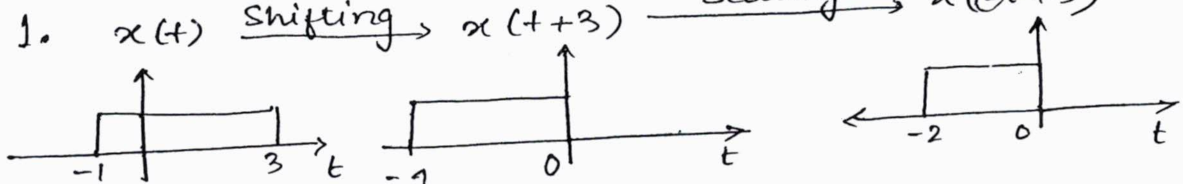
$y(t) = x(2t+3) = x[2(t+1.5)]$

$x(t) \xrightarrow{\text{Scaling}} x(2t) \xrightarrow{\text{Shifting}} y(t) \leftarrow$ LS by 1.5

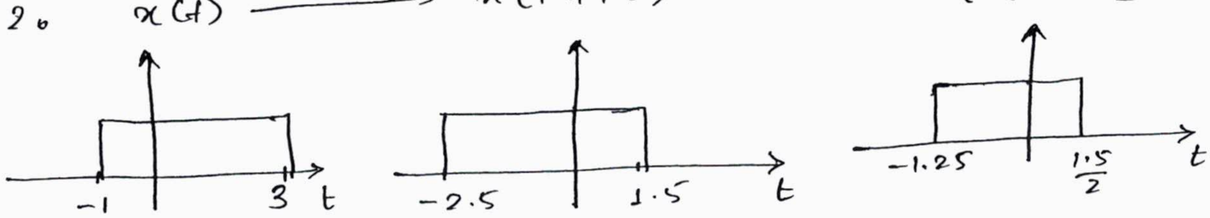


2nd Method : $y(t) = x(2t+3)$

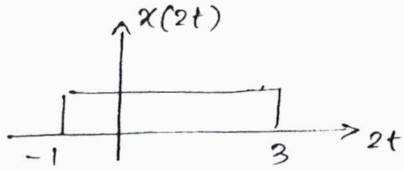
1. $x(t) \xrightarrow{\text{Shifting}} x(t+3) \xrightarrow{\text{Scaling}} x(2t+3)$

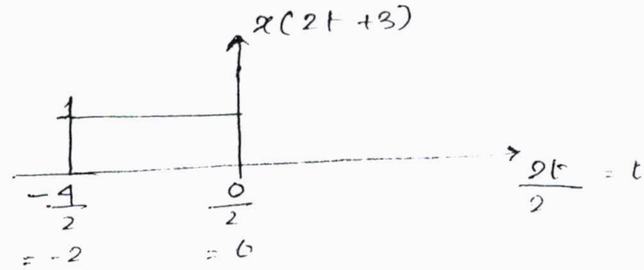


× 2. $x(t) \xrightarrow{\text{Shifting}} x(t+1.5) \longrightarrow x[2(t+1.5)]$

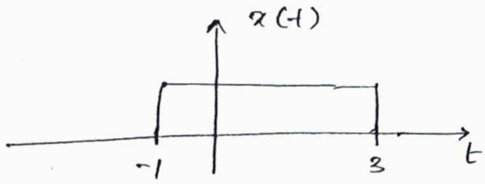


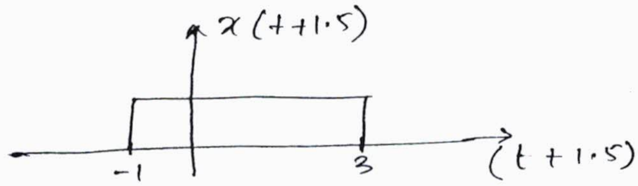
3rd Method :

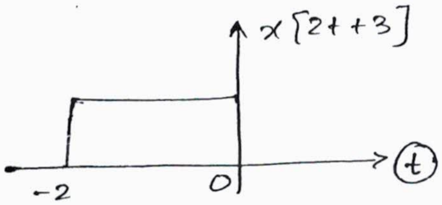


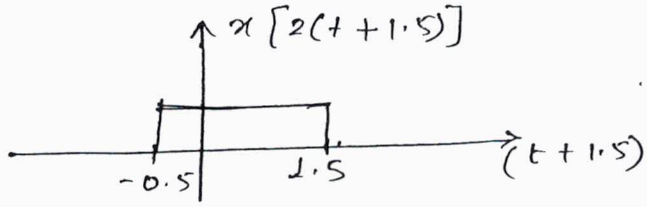


4th Method :

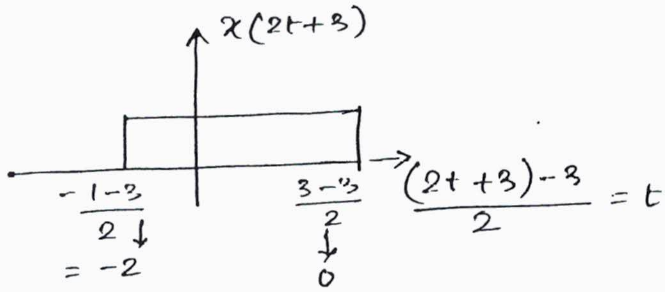




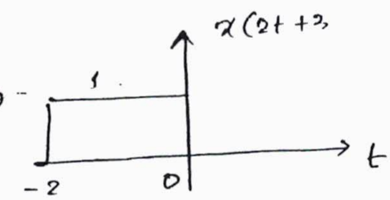




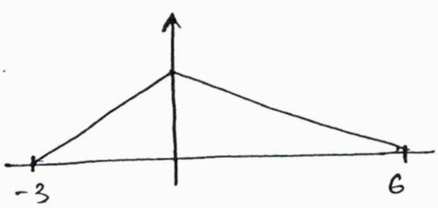
Shortcut :- (for waveform)



$\Rightarrow$



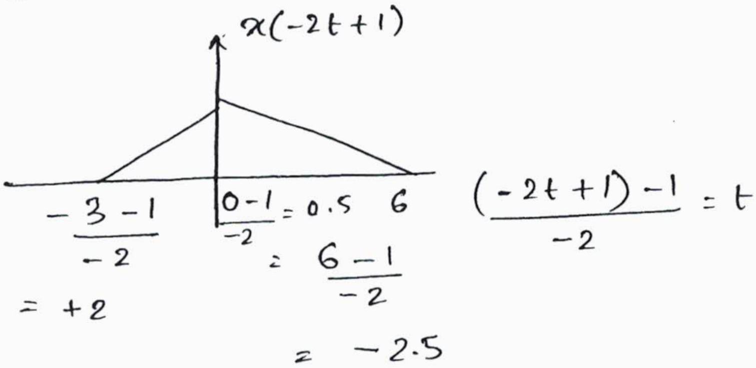
Q. $x(t)$



Draw the waveform of $y(t)$

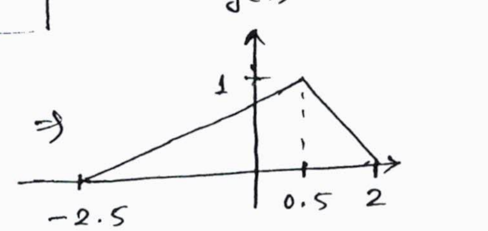
$y(t) = x(-2t+1)$

Shortcut : $\rightarrow$ $\boxed{\text{Slope change} \rightarrow \text{operation}}$



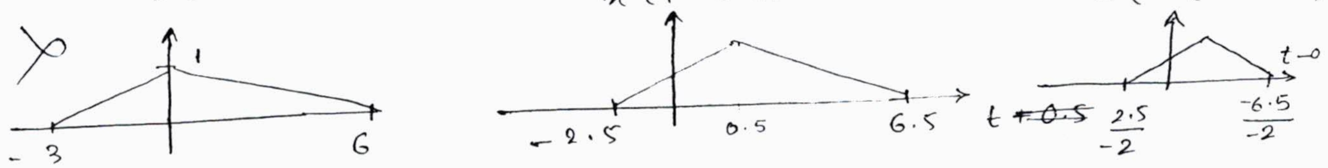
$y(t) = x(-2t+1)$

$\Rightarrow$



4th Method: $y(t) = x(-2(t-0.5))$

$x(t)$ $x(t-0.5)$ $x(-2(t-0.5))$



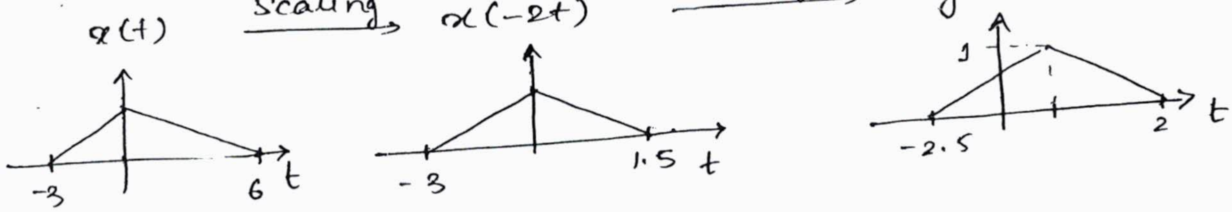
$x(-2t+1)$

1st Method :

RS by 0.5 → t

$y(t) = x(-2t+1) = x[-2(t-0.5)]$

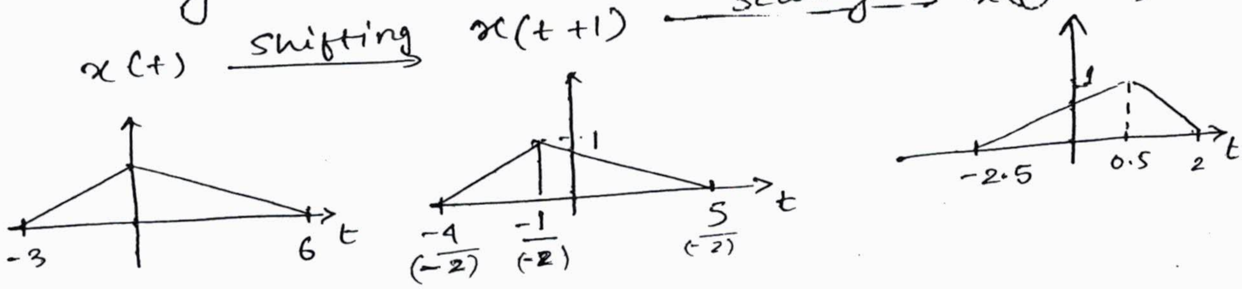
$x(t) \xrightarrow{\text{Scaling}} x(-2t) \xrightarrow{\text{shifting}}$ by 0.5



2nd Method :

$y(t) = x(-2t+1)$

$x(t) \xrightarrow{\text{shifting}} x(t+1) \xrightarrow{\text{Scaling}} x(2t+1)$



1. SIGNAL DEFINITION & ITS CLASSIFICATION:

Signal:
A signal is a fn. which contains some information.

System:
A system is a medium which processes a signal. It is interconnection of devices or components which converts signal from one form to another form.

Classification of Signal:

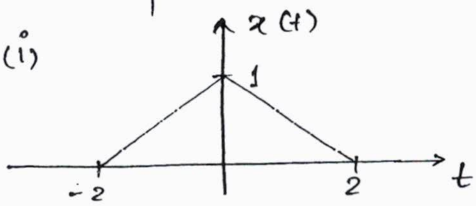
1. Even / Odd Signal:

a) **Even Signal:** Even signals are symmetrical or mirror image about Y axis.

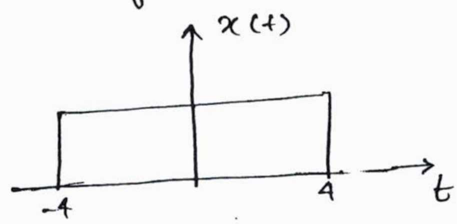
i.e $\boxed{x(t) = x(-t)}$ ⟶ Time Reversal.

* Even signals are independent of time reversal.

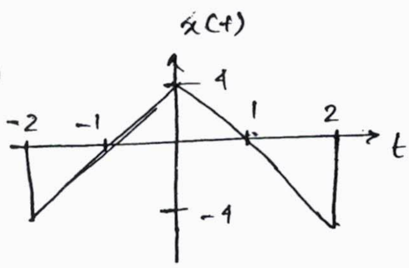
Ex — (i)



ii)



iii)



iv) $x(t) = \boxed{\cos\omega_0 t}$ ⟶ even signal.

$t = -t$

$x(-t) = \cos(-\omega_0 t)$

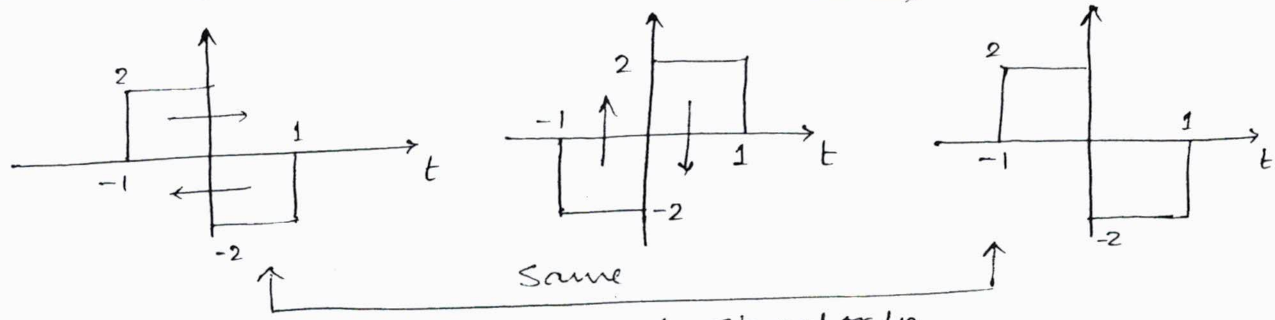
$= \cos\omega_0 t$

$\therefore x(t) = x(-t)$

b) **Odd Signal:** Odd signals are having anti-symmetric LHS and RHS.

i.e
$$\boxed{\begin{array}{c} x(t) = -x(-t) \\ \text{or} \\ x(-t) = -x(t) \end{array}}$$
⟶ Time Reversal
⟶ Amplitude Reversal.

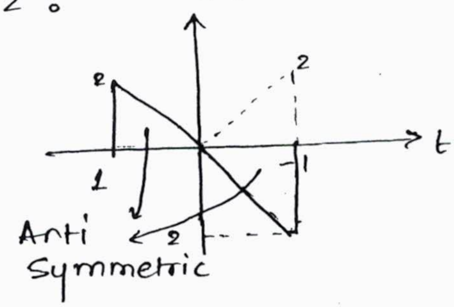
Ex- 1. $x(t) \longrightarrow x(-t) \longrightarrow -x(-t)$



So odd signal or fn.

Antisymmetric $\longrightarrow$ (Mirror of Mirror Image) image

Ex-2 : $x(t) \longrightarrow$ odd.



Ex-3 : $x(t) = \sin\omega_0 t \longrightarrow$ odd signal.

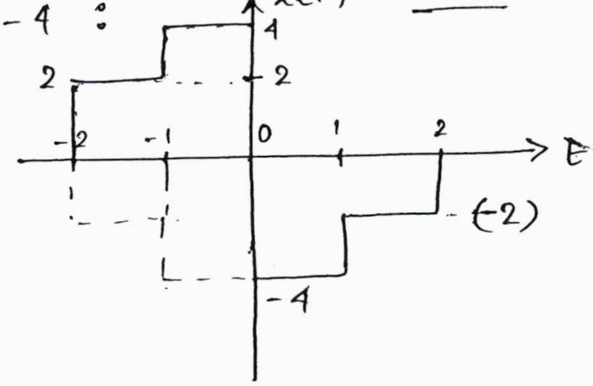
$x(-t) = \sin(\omega_0(-t))$

$= -\sin\omega_0 t$

$= -x(t)$

$\therefore \boxed{x(-t) = -x(t)} \longrightarrow$ odd signal.

Ex-4 : $x(t) \longrightarrow$ odd.



NOTE :

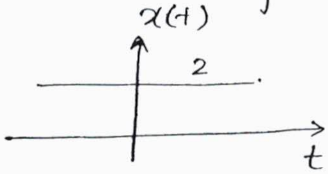
For any odd signal the avg value will be equal to $\boxed{'0'}$

Ex. 5 → $x(t) = 2$ = DC signal → Even Signal

$\downarrow$

$t = -t$

$x(-t) = 2 = x(t)$



Ex. 6 → $x(k) = \sin[k^2]$

$k = -k$

$x(-k) = \sin[(-k)^2]$

$= \sin k^2$ = even signal.

$$\boxed{x(k) = (x(-k))}$$

Ex-7 → $x(z) = \sin\left(\frac{\pi}{2}\right) = 1$ = DC signal → even signal.

NOTE :

Any signal can be divided into 2 parts in which 1 part will be even and the other part will be odd.

i.e $x(t) = x_e(t) + x_o(t)$

where $x_e(t)$ = even part of $x(t)$

$$= \frac{x(t) + x(-t)}{2}$$

$x_o(t)$ = odd part of $x(t)$

$$= \frac{x(t) - x(-t)}{2}$$

15 Some important points :

1. $E \div / \times E = E$

$(t^2 \times t^4 = t^6)$

2. $O \div / \times E = O$

$(t^3 \times t^2 = t^5)$

3. $O \div / \times O = E$

$t^3 \times t = t^4$

4. $E \pm E = E$

ex : $x(t) = \cos t + t^2$

$\downarrow t = -t$

$x(-t) = \cos(-t) + (-t)^2$

$= \cos t + t^2$

$\therefore x(t) = x(-t) = \text{Even}$

5. $O \pm O = O$

ex. $x(t) = \sin t + t^3$

at $t = -t$

$x(-t) = (-t)^3 + \sin(-t)$

$= -t^3 + \sin(-t)$

$= -[t^3 + \sin t]$

$= -x(t) = \text{odd}.$

6. $E \pm O$ = Neither even nor odd.

ex – $x(t) = \cos t - \sin t$

at $t = -t$

$x(-t) = \cos t + \sin t \neq x(t) \neq \text{even}$

$= -(x(t)) \neq \text{odd}.$

Q. find $x_e(t)$ and $x_o(t)$ for signal

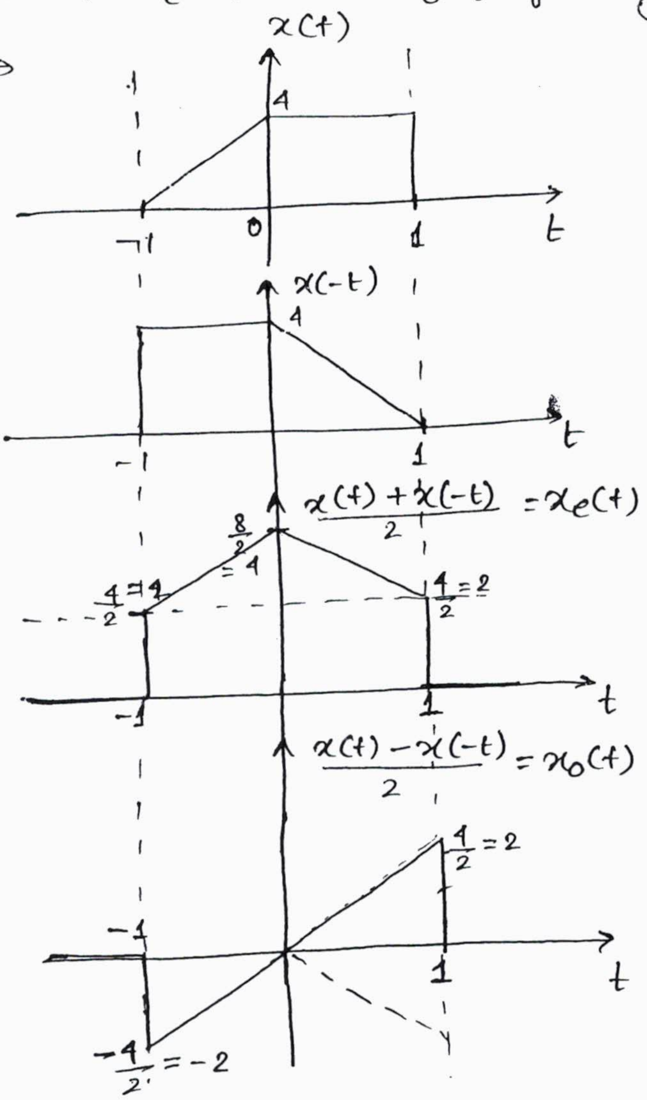
$$x(t) = \underset{E}{2} + \underset{(E \times O) \to O}{t^2 \sin t} - \underset{\frac{O}{E} \to O}{\frac{t^3}{\cos t}} + \underset{\frac{E}{E} \to E}{\frac{\cos^2 t}{t^2}} - \underset{\frac{O}{O} \to E}{\frac{\sin^3 t}{t^5}}$$

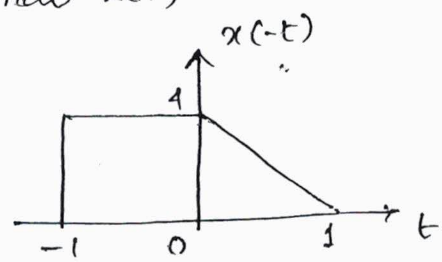
$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$x_o(t) = t^2 \sin t - \frac{t^3}{\cos t} \qquad x_e(t) = 2 + \frac{\cos^2 t}{t^2} - \frac{\sin^3 t}{t^5}$$

Q. find $x_e(t)$ and $x_o(t)$ for signal $x(t)$

Sol^n →





→ Discontinuous

↓

For abrupt change in magnitude.

↓

The value at discontinuous point will be value before the discontinuous point

2. Conjugate Symmetric (CS) and Conjugate - Antisymmetric (CAS) Signal :

a) CS Signal : $\boxed{x(t) = x^*(-t)}$

let $x(t) = a(t) + jb(t)$ —— (1)

$\downarrow$

$t = -t$

$\therefore \quad x(-t) = a(-t) + jb(-t)$

$\downarrow *$

$x^*(-t) = a(-t) - jb(-t)$ —— (11)

for CS : $\longrightarrow \boxed{x(t) = x^*(-t)}$ → for conjugate Symmetry

from (1) & (11)

$$\left.\begin{aligned} a(t) &= a(-t) \rightarrow \text{even} \\ \text{and} \quad b(t) &= -b(-t) \rightarrow \text{odd.} \end{aligned}\right\}$$

Ex : $\underbrace{x(t)}_{CS.} = \underbrace{\cos t}_{E.} + j\underbrace{t^3}_{O}$

b) CAS Signal : $\boxed{x(t) = -x^*(-t)}$

let $x(t) = a(t) + jb(t)$

$\downarrow t = -t$

$x(-t) = a(-t) + jb(-t)$

$\downarrow *$

$x^*(-t) = a(-t) - jb(-t)$

for CAS : $x(t) = -x^*(-t)$

$$\text{CAS} \;— \left[\begin{aligned} a(t) &= -a(-t) \rightarrow \text{odd} \\ b(t) &= b(-t) \rightarrow \text{Even.} \end{aligned}\right.$$

Ex : $\underbrace{x(t)}_{CAS} = \underbrace{t^3}_{O} + j\underbrace{\cos t}_{E}$

Q. Check CS/CAS signals

i) $x(t) = t^2$ = Real → E (CS) = R+E

ii) $x(t) = t^3$ = Real → O (CAS) = R+O

iii) $x(t) = j\cos t$ = Imaginary → E (CAS) = I+E

iv) $x(t) = j\sin t$ = Imaginary → O (CS) = I+O

NOTE :

→ Any signal can be divided into 2 parts in which 1st part will be conjugate symmetric and the other part will be conjugate antisymmetric.

i.e $x(t) = x_{CS}(t) + x_{CAS}(t)$

where $x_{CS}(t) = \dfrac{x(t) + x^*(-t)}{2}$ = CS part of $x(t)$

$x_{CAS}(t) = \dfrac{x(t) - x^*(-t)}{2}$ = CAS part of $x(t)$

→ for conjugate symmetric signal Real part should be Even and imaginary part should be odd.

→ for CAS signal → Real part should be odd and imaginary part should be even.

3. Periodic And Non Periodic Signal :

a) Periodic Signal :

A signal is said to be periodic if it repeats itself after ~~sometime~~ Time period.

i.e $\boxed{x(t) = x(t \pm nT_0)}$

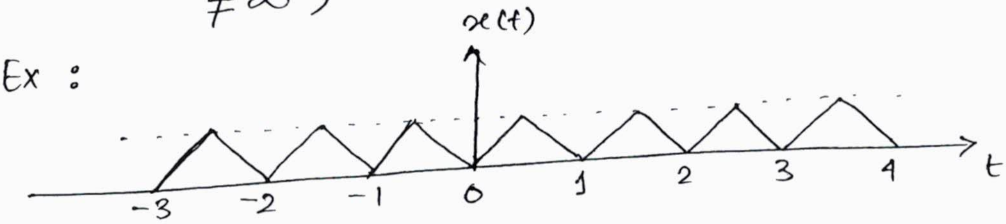
where n = an integer
= 1, 2, 3 . . .

T_0 = FTP (Fundamental Time Period)

= It is the smallest, +ve and fixed value of time for which signal is periodic

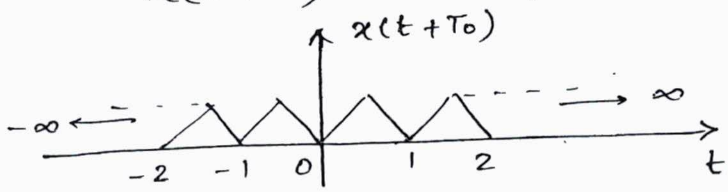
$\left.\begin{matrix}\neq 0 \\ \neq \infty\end{matrix}\right)$

Ex :



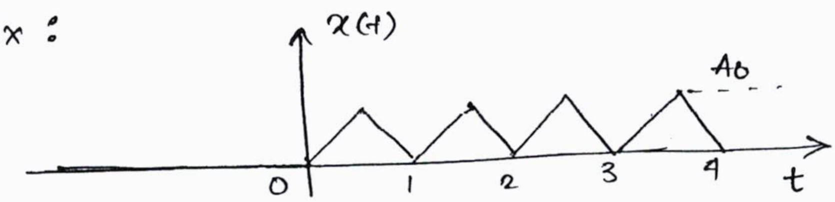
$\longrightarrow$ FTP $= T_0 = 1$.

$x(t + T_0) = x(t+1)$



$\longrightarrow$ Periodic.

Ex :



for a system to be periodic the signal should be from $-\infty$ to ∞

$\longrightarrow$ P $\longrightarrow$ Non periodic

$\longrightarrow$ $T_0 = 1$ $\quad x(t + T_0) = x(t+1)$