ToppersNotes

THE IIT-JEE SECRET

MATHEMATICS VOLUME-I

Sierra Innovations Pvt. Ltd.

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VECTORS

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- Vector is defined as a physical quantity having magnitude as well as direction and satisfying addition law of vectors and it is a directed line segment. $\vec{a}$

- NULL VECTOR
Vector having magnitude zero and no specified direction.

- UNIT VECTOR
Vector having magnitude unity and direction same as $\vec{a}$.
$\hat{n}_a = \hat{a} = \dfrac{\vec{a}}{|\vec{a}|} \Rightarrow \vec{a} = |\vec{a}| \cdot \hat{a}$

- EQUAL VETOR
Two vectors are said to be equal if their magnitude and direction are same and they represents the same physical quantity.

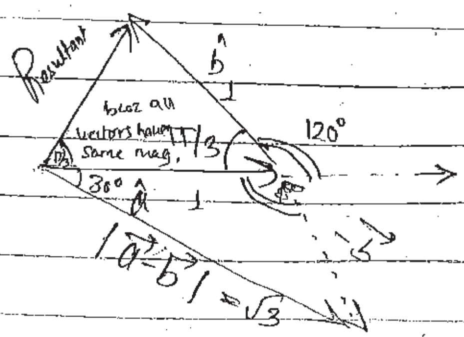
- FREE & LOCALISED VECTOR
Vector if shifted in space keeping its direction and magnitude same its physical effects remains same is called free vector and if its physical ~~xxxx~~ effect is changed is called Localised vector. for eg. velocity is a free vector where as force is localised vector.

Ques If the sum of two unit vectors is a unit vector then find the angle b/w them and the magnitude of their difference.

$\vec{a} \wedge \vec{b} = \dfrac{2\pi}{3}$ ↳ angle

$|\hat{a} - \hat{b}| = \sqrt{3}$

Ans



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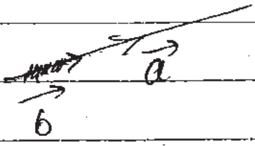
COLINEAR VECTORS

Two vectors are said to be collinear if their line segment are parallel irrespective of their direction.

i.e.

$$\boxed{\vec{b} = \lambda \vec{a}}$$



$\lambda > 0 \Rightarrow$ like vectors or $\parallel^{m}$

$\lambda < 0 \Rightarrow$ Unlike vectors or Anti $\parallel^{m}$

LINEAR COMBINATION OF VECTORS

If $\vec{V} = \lambda_1 \vec{a_1} + \lambda_2 \vec{a_2} + \text{---} + \lambda_n \vec{a_n}$

all the $\lambda_i \rightarrow$ Scalar

implies $\vec{V}$ is linear combination of $\vec{a_1}, \vec{a_2} \text{---}, \vec{a_n}$

LINEARLY INDEPENDENT VECTORS OR BASE VECTORS

If $\vec{a_1}, \vec{a_2} \text{---} \vec{a_n} \neq 0$ and

$\lambda_1 \vec{a_1} + \lambda_2 \vec{a_2} + \text{---} + \lambda_n \vec{a_n} = 0$ and

$\lambda_1 = \lambda_2 = \text{---} = \lambda_n$ implies

$\vec{a_1}, \vec{a_2} \text{----}, \vec{a_n}$ are linearly independent vectors (Base vectors)

NOTE

1. Linearly independent implies no vector can be expressed as the linear combination of other.
2. Linearly dependent implies atleast one vector can be expressed as linear combination of remaining vectors. ($\Rightarrow$ at least one of λ_i must be non zero)
3. If $\vec{a}$ and $\vec{b}$ are two non zero and non colinear vectors and they are linearly independent $\Rightarrow$

If $\boxed{x\vec{a} + y\vec{b} = 0} \Rightarrow \boxed{x = y = 0}$

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Ques If $\vec{a}, \vec{b}$ and $\vec{c}$ are such that every pair is non colinear and $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ are colinear with $\vec{c}$ and $\vec{a}$ respectively find $\vec{a}+\vec{b}+\vec{c}$.

Sol.

$\vec{a}+\vec{b} = \lambda \vec{c}$ —(1)

$\vec{b}+\vec{c} = \mu \vec{a}$ —(2)

(1) − (2)

$\vec{a} - \vec{c} = \lambda \vec{c} - \mu \vec{a}$

$\vec{a}(1+\mu) - (1+\lambda)\vec{c} = 0$

$\vec{a}$ and $\vec{c}$ are non colinear

$\Rightarrow 1+\mu = 0$ and $1+\lambda = 0$

$\Rightarrow \mu = -1$ $\quad \lambda = -1$

put in (1)

$\vec{a}+\vec{b} = -\vec{c} \Rightarrow \vec{a}+\vec{b}+\vec{c} = 0$

Ans.

■ COPLANER VECTOR

$\vec{a_1}, \vec{a_2}, \ldots, \vec{a_n}$ are said to be coplaner if they all are parallel to the same plane. i.e. if

$\vec{n_1}\cdot\vec{a_1} = \vec{n_1}\cdot\vec{a_2} \ldots = \vec{n_1}\cdot\vec{a_n} = 0$ where $n_1 \perp r$ to plane.

NOTE

→ No. of UNIT VECTORS $\perp r$ to a plane is 2. i.e.

$$\boxed{\hat{n} = \underset{\substack{\downarrow \\ \text{into the plane \& out of plane}}}{\pm} \frac{\vec{a_1}\times\vec{a_2}}{|\vec{a_1}\times\vec{a_2}|}}$$

where $\vec{a_1}$ and $\vec{a_2}$ are non colinear.

→ No. of unit vectors $\perp r$ to a line are ∞.

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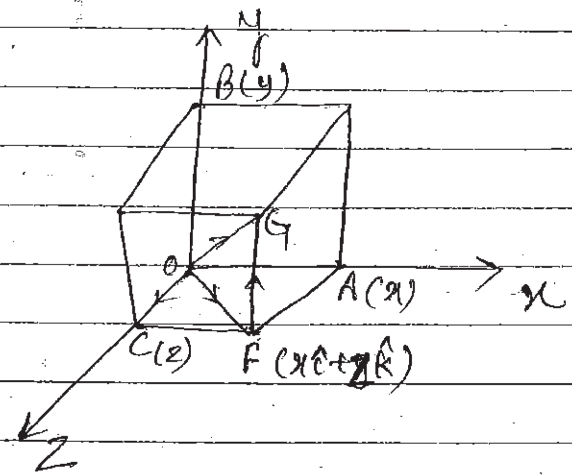
- Two vectors are coplaner but 3 vectors may or may not be coplaner.

- POSITION VECTORS

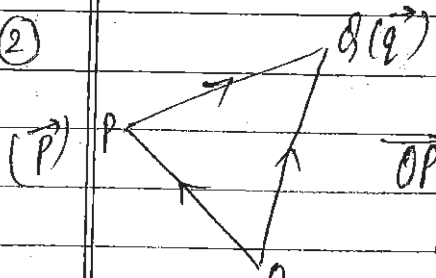
①

$\overrightarrow{OG} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\boxed{|\overrightarrow{OG}| = \sqrt{x^2+y^2+z^2}}$$



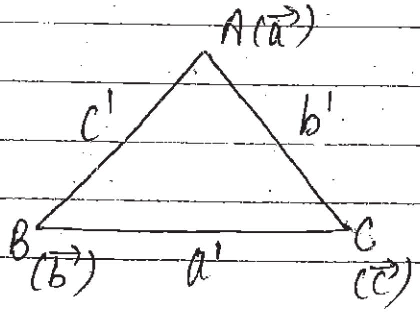
②



$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

$$\boxed{\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}}$$

③



$\overrightarrow{G}$ (Centroid) $= \dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$

$\overrightarrow{I} = \dfrac{a'\vec{a}+b'\vec{b}+c'\vec{c}}{a'+b'+c'}$

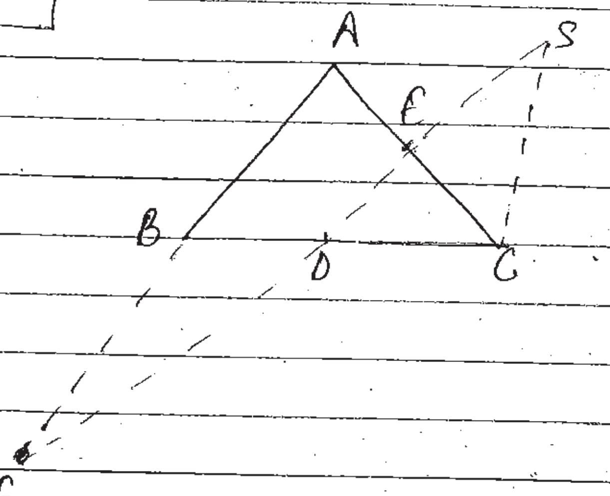
$\overrightarrow{I_1} = \dfrac{-a'\vec{a}+b'\vec{b}+c'\vec{c}}{-a'+b'+c'}$

lication Imp.

④ MENELAUS THEOREM

If the points D, E, F on the sides BC, CA and AB of a $\triangle ABC$ are collinear, then

$$\boxed{\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1}$$



Proof

$\Delta AEF \sim \Delta CES$

$$\frac{AF}{CS} = \frac{AE}{EC} \quad \text{——} (1)$$

$\Delta BDF \sim \Delta CDS$

$$\frac{BD}{DC} = \frac{FB}{CS} \quad \text{——} (2)$$

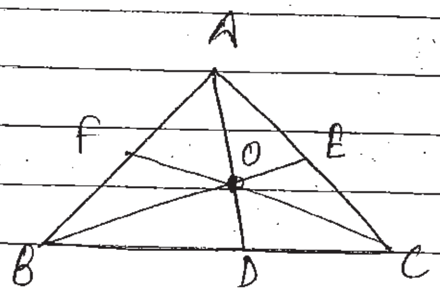
$$(2) \times \frac{1}{(1)} \Rightarrow \frac{BD}{DC} \times \frac{CE}{EA} = \frac{FB}{CS} \times \frac{CS}{AF}$$

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$

CEVA'S THEOREM

If the line joining a point 'O' to the vertices of ΔABC meets the opposite sides in DEF respectively then

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$



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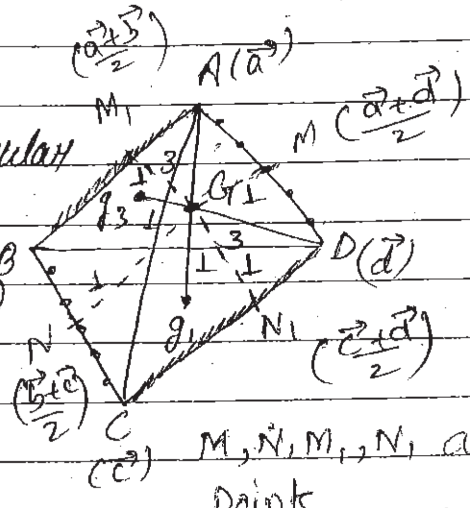
TETRAHADERON

- It is a pyramid of triangular base.
- It has 4 triangular faces
- It has pair of opposite edges as (BC & AD) (AB & CD) etc



M, N, M_1, N_1 are mid point.

- Line joining the vertices to the centroids of the opposite faces are concurrent and the point of concurrency is called centre or centroid of tetrahaderon.

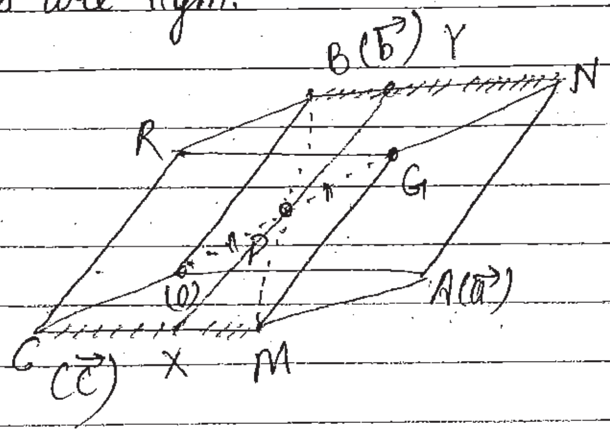
- $$g_1 = \frac{\vec{b}+\vec{c}+\vec{d}}{3}$$ $$\boxed{G = \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}}$$

centre of tetrahaderon.

- The line joining the midpoints of pair of opposite edges are also concurrent at the centre of tetrahaderon.

PARALLELOPIPED :-

- This is a prism whose base is a ||gm.
- All faces are ||gm.



$G = \vec{a}+\vec{b}+\vec{c}$

$P = \dfrac{\vec{a}+\vec{b}+\vec{c}}{2}$

$$\boxed{\overrightarrow{OP} = \frac{\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}}{2}}$$

Pair of opposite edges के mid point को join करने वाली line will pass thru P.

RESULTS

- 4 diagonals of $\parallel$piped are concurrent at the centre of $\parallel$piped and is bisected by it.
- The line joining the mid point of each pair of opposite edges are also concurrent at centre of $\parallel$piped.

Ques If the centre of $\parallel$piped formed by $\overrightarrow{PA}, \overrightarrow{PB}, \overrightarrow{PC}$

$\overrightarrow{PA} = \langle 1,2,2\rangle = \hat{i}+2\hat{j}+2\hat{k}$

$\overrightarrow{PB} = \langle 4,-3,1\rangle$

$\overrightarrow{PC} = \langle 3,5,-1\rangle$ is given by the Position vector

w.r.t origin PV of Centre $= \langle 7,6,2\rangle$ find the position vector of P.

Sol. $\overrightarrow{OR} = 7\hat{i}+6\hat{j}+2\hat{k}$

Let centre be R. Hence P.V. of R w.r.t P

$$\overrightarrow{PR} = \frac{\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}}{2}$$

$$\overrightarrow{OR}-\overrightarrow{OP} = \frac{8\hat{i}+4\hat{j}+2\hat{k}}{2}$$

$$(7\hat{i}+6\hat{j}+2\hat{k}) - (4\hat{i}+2\hat{j}+\hat{k}) = \overrightarrow{OP}$$

$$\overrightarrow{OP} = 3\hat{i}+4\hat{j}+\hat{k}$$

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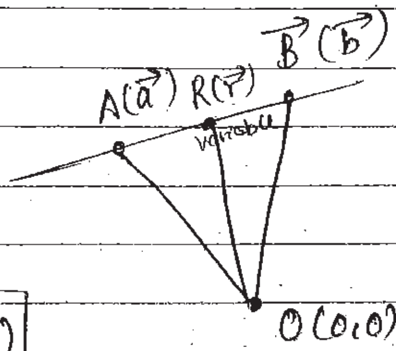
VECTOR EQN OF LINES

Type-I Vector Eq^n of a line passing thru $A(\vec{a})$ and $B(\vec{b})$

$\vec{AR} = \lambda \vec{AB}$

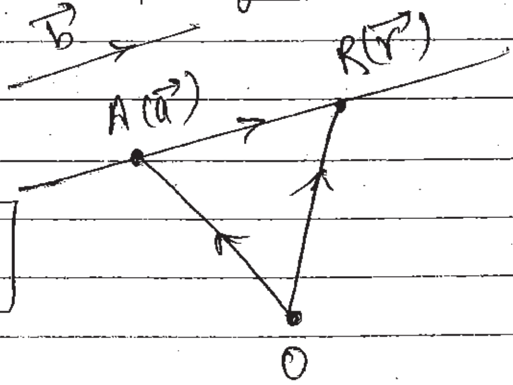


$\vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$

$\boxed{L_{AB}: \vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$

$\Rightarrow$ A line passing thru fixed point $(\vec{a})$ in the direction $(\vec{b} - \vec{a})$

Type-II Vector Eq^n of a line passing thru $A(\vec{a})$ and $\parallel^{r}$ to $\vec{b}$



$\vec{AR} = \lambda \vec{b}$

$\boxed{L_{AR} : \vec{r} = \vec{a} + \lambda \vec{b}}$

NOTE

(i) $L: \vec{r} = (a_1, a_2, a_3) + \lambda(b_1, b_2, b_3)$

a) L is passing thru $(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$ & $\parallel^{r}$ to $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

b) Parametric coordinate of any point P on L is

$\langle a_1 + \lambda b_1, a_2 + \lambda b_2, a_3 + \lambda b_3 \rangle$

(ii) Two lines in a plane are either intersecting or parallel. Conversely two intersecting or $\parallel^{r}$ lines